D-branes, axion monodromy and large-field inflation

Memoria de Tesis Doctoral presentada por

Aitor Landete Marcos

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Tesis Doctoral dirigida por el **Dr. Fernando Marchesano Buznego**,
Científico Titular del Instituto de Física Teórica UAM-CSIC

y

el **Dr. Ángel María Uranga Urteaga**,
Profesor de Investigación del Instituto de Física Teórica UAM-CSIC

Departamento de Física Teórica Universidad Autónoma de Madrid

Consejo Superior de Investigaciones Científicas

Instituto de Física Teórica UAM-CSIC

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Yo sé quién soy,
y sé quién puedo ser.

Don Quijote, El Quijote, Cap V

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[1] La lista continúa pero he de dejar espacio a la tesis.

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Abstract

The cosmological standard model is, at present, one of the most precise theories which describe with high accuracy our universe at large scales. However, this theory presents several shortcomings. Inflationary theories, which are based on a vacuum-like energy density dominating early stages of the universe, are promising candidates to address some of these problems. However, these theories present problems regarding UV sensitivity, which are dramatically enhanced in models which predict measurable tensor modes in the CMB. From a bottom-up perspective it is still a challenge to describe these models in a consistent theory of Quantum Gravity. Besides that, String Theory is a theory of Quantum Gravity and also an outstanding candidate to unify all fundamental forces of nature. Thus, it seems a great opportunity to embed models of large-field inflation in String Theory from both a phenomenological point of view and to analyze Quantum Gravity constraints to these theories. In Part I, we will discuss some fundamental aspects of inflation and its possible embeddings in String Theory. Afterwards, we will review aspects of type II string flux compactifications like: geometrical moduli space, moduli stabilization schemes and the inclusion of the open-string sector.

In Parts II and III we will discuss several models of large-field inflation in string theory based on the presence of D-branes. These models are explicit realizations of the principle of axion monodromy. In Chapter 4 we will propose a model in type IIA string theory whose inflationary potential comes from the presence of D6-branes in Calabi-Yau orientifolds satisfying a topological condition. At large inflaton values the inflationary potential is given by the DBI action while, at low energies, it is described by means of an F-term scalar potential sourced by a open-closed bilinear superpotential. This superpotential is similar to the ones studied in the supergravity literature, classified as chaotic inflation with stabilizer fields. In Chapter 5 we will propose a source of flattening in models described by the DBI action dubbed as Flux-Flattening. This source of flattening is based on the interplay between supersymmetric and non-supersymmetric wordvolume flux components generated on a D-brane which modify the asymptotic behavior of the potential. We will analyze this effect in a well-studied framework of large-field inflation where the inflaton candidate is the position of a D7-brane. We will show that the scalar to tensor ratio of chaotic inflation could be lowered up to $r \sim 0.04$ – in agreement with the recent experimental data given by the joint analysis of BICEP2/Keck and Planck Collaboration – in a model consistent with moduli stabilization.

The last part of this thesis will be focused on moduli stabilization and backreaction in models of large-field inflation since it is crucial to assure the consistency of these models. In chapter 6 we will review the interplay between moduli stabilization, chaotic inflation and supersymmetry breaking which will impose several constraints on the parameter space of the inflationary theory. In Chapter 7 we will analyze in detail backreaction issues regarding the model presented in Chapter 4. This discussion will serve us as the starting point to discuss the viability of embedding stabilizer fields in type II string compactifications. Finally, in Chapter 8 we will analyze in a concrete example the constraints coming from backreaction of the closed-string sector in models of chaotic inflation. Also we will discuss how this is related with the viability of the transplanckian field range.

Resumen

El modelo estándar de cosmología es, a día de hoy, una de las teorías más precisas que describen con gran acierto nuestro universo a grandes escalas. A pesar de eso, esta teoría presenta varios problemas. Las teorías inflacionarias, las cuales están basadas en un universo dominado por una desidad de energía tipo vacío en etapas tempranas, son prometedoras candidatas para solucionar algunos de estos problemas. Pero, estas teorías presentan problemas debido a su sensibilidad a efectos ultravioleta, los cuales se incrementan dramáticamente en modelos que predicen la detección de modos tensoriales en el Fondo Cósmico de Microondas. Desde una perspectiva heurística es un reto describir estas teorías en el marco de una teoría cuántica de la gravedad. Por otra parte, la Teoría de Cuerdas es una teoría de gravedad cuántica candidata a unificar las fuerzas de la naturaleza. Por tanto, parece una gran oportunidad describir modelos de inflación de 'campo grande' en teoría de cuerdas tanto desde un punto de vista fenomenológico como para analizar posibles restricciones a estas teorías por argumentos de gravedad cuántica. En la Parte I, discutiremos algunos aspectos fundamentales de inflación y sus posibles descripciones en Teoría de Cuerdas. Posteriormente, revisaremos aspectos de compactificaciones con flujos en Teorías de cuerdas de tipo II como: *moduli* geométrico, esquemas de estabilización de *moduli* y la inclusión del sector de cuerda abierta.

En las Partes II y III presentaremos varios modelos de inflación de 'campo grande' en teoría de cuerdas en presencia de D-branas, Estos modelos son una realización explícita del principio de monodromía de axiones. En el Capítulo 4 propondremos un modelo en el tipo de cuerdas IIA donde el potencial inflacionario proviene de la presencia de D6-branas en Calabi-Yau *orientifolds* satisfaciendo una condición topológica concreta. Para grandes valores del inflatón, el potencial viene dado por la acción de DBI mientras que, a bajas energías, viene descrito por un potencial F-term originado por un superpotencial bilineal de cuerda abierta-cerrada. Este superpotencial es similar a los estudiados en la literatura de supergravedad, clasificados como inflación caótica con campos estabilizadores. En el Capítulo 5 propondremos una nueva fuente de 'aplanamiento' del potencial en modelos descritos por la DBI que denominamos *Flux-Flattening*. Esta fuente de 'aplanamiento' está basada en la relación entre componentes supersimétricas y no supersimétricas del flujo de *worldvolume* inducido en la D-brana el cual puede modificar el comportamiento asintótico del potencial. Analizaremos este efecto en el conocido contexto de inflación de campo grande generado por D7-branas. Mostraremos que el ratio entre perturbaciones escalares y tensoriales predicho por inflación caótica puede ser disminuido hasta $r \sim 0.04$ – de acuerdo con los recientes datos experimentales ofrecidos por el análisis conjunto de las colaboraciones BICEP2/Keck y Planck – en un modelo consistente con estabilización de *moduli*.

La última parte the esta tesis estará centrada en estabilización de *moduli* y *backreaction* en modelos de inflación de 'campo grande' dado que es crucial para asegurar la consistencia de estos modelos. En el capítulo 6 revisitaremos la relación entre estabilización de moduli, inflación caótica y ruptura de supersimetría la cual impondrá severas restricciones en el espacio de parámetros de la teoría. En el Capítulo 7 analizaremos en detalle *backreaction* en el modelo propuesto en el Capítulo 4. Este análisis nos servirá de punto de partida para discutir la viabilidad de describir

campos estabilizadores en compactificaciones de cuerdas tipo II. Por último, en el Capítulo 8 analizaremos en un ejemplo concreto las restricciones provenientes de *backreaction* del sector de cuerda cerrada en modelos de inflación caótica. Además, discutiremos la relación de esto con la viabilidad del rango transplanckiano del inflatón.

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Part I

Introduction

1
Early Universe Cosmology

1.1 The Cosmological Standard Model

The Standard Model of Cosmology provides a simple, elegant and reliable description of our universe's evolution since the moment of primordial nucleosynthesis until today. It also provides a robust framework in order to discuss earlier moments of our universe.

At large scales our universe is homogeneous and isotropic.[1] The most outstanding measure which points to the smoothness of the universe is provided by the Cosmic Microwave Background (CMB), which is uniform to about a part in 10^5. It is possible to show [1] that the most general metric consistent with homogeneity and isotropy is the Friedman-Lemaître-Robertson-Walker (FLRW) spacetime

$$ds^2 = dt^2 - a^2(t)d\boldsymbol{x}^2 = dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right) , \tag{1.1.1}$$

where $a(t)$ is the scale factor and $d\Omega = \sin\theta d\theta d\phi$. The constant k defines the curvature of the spacetime, with $k = 0$ corresponding to flat spatial sections, and $k = \pm 1$ corresponding to closed and open spatial sections, respectively. We will model the energy content of the homogeneous and isotropic universe by a perfect fluid. Its associated stress-energy tensor is given by

$$T^\mu_\nu = \mathrm{diag}\left(\rho(t), -p(t), -p(t), -p(t)\right) , \tag{1.1.2}$$

where ρ is the energy density and p the pressure for any kind of energy source. The perfect fluid will be described by the equation of state $\rho = \omega p$ where ω will specify the nature of the energy source. Einstein's field equations:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} , \tag{1.1.3}$$

[1] A homogeneous space is translationally invariant, i.e. looks the same at every point. An isotropic space is rotationally invariant, i.e. looks the same in every radial direction. The two are not the same: a space which is everywhere isotropic is necessarily homogeneous, but a space which is homogeneous is not necessarily isotropic.

worked out for the system described above, reduce to a set of two non-linear, coupled differential equations:

$$H^2 := \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\rho - \frac{k^2}{a^2}\,, \tag{1.1.4}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}\left(\rho + 3p\right)\,, \tag{1.1.5}$$

called Friedmann equations. The parameter H represents the expansion rate of the universe and is called the Hubble parameter, which typically sets the fundamental scales of our universe, i.e. the Hubble length $d_H \sim H^{-1}$. From the conservation of the stress-energy tensor we see,

$$T^{\mu\nu}_{;\nu} = 0 \rightarrow \dot{\rho} = -3H\left(\rho + p\right)\,, \tag{1.1.6}$$

which could also be derived from the Friedmann equations. A usual convention is to define Ω as the ratio between the actual energy density and the critical density ρ_c, which is defined as the density for which $k = 0$, and thus corresponding to a flat universe. Using the first Friedmann equation (1.1.4) it is easy to see that the critical density is given by:

$$\rho_c := 3H^2M_P^2 \text{ and } \Omega := \frac{\rho}{\rho_c} = \frac{1}{3M_P^2}\frac{\rho}{H}\,, \tag{1.1.7}$$

and with these definitions we see that we can rewrite the Friedmann equations (1.1.4) as

$$\Omega = 1 + \frac{k^2}{(aH)^2}\,. \tag{1.1.8}$$

We summarize the solution for the Friedmann equation for three different kinds of energy sources in the following table [2]

matter	$\omega = 0$	$\rho_m \sim a^{-3}$	$a \sim t^{2/3}$
radiation	$\omega = \frac{1}{3}$	$\rho_\gamma \sim a^{-4}$	$a \sim t^{1/2}$
vacuum	$\omega = -1$	$\rho_\Lambda \sim \Lambda$	$a \sim e^{Ht}$

Table 1.1: Solutions to the Friedmann equations depending the energy source

Note that the third type of energy source could be understood in terms of the introduction of a cosmological constant in Einstein's equations (1.1.3).[2] This vacuum-like energy makes the universe expand exponentially quick with a time constant given by the Hubble constant

$$H = \sqrt{\frac{\rho_\Lambda}{3M_P^2}}\,, \tag{1.1.9}$$

[2]This could be easily seen from the identity $D_\nu g_{\mu\nu} = 0$ which means that the conservation of the energy momentum (1.1.6) is unchanged. $D_\nu T^{\mu\nu} \rightarrow D_\nu\left(T^{\mu\nu} + \Lambda g_{\mu\nu}\right) = 0$

and such a spacetime is called de Sitter spacetime. Looking at the solutions of the Friedmann equations it is straightforward to see that the energy density associated to radiation is diluted faster than the energy coming from matter. We also see that a vacuum energy source is not diluted over time. This tells us that the universe at early times was radiation-dominated during which the universe cooled down. After that it changed to a matter-dominated phase during which galaxies, stars and planets formed. At later times, due to the existence of a non-vanshing vacuum-like energy, it will eventually dominate and, the universe will expand in an accelerated way.[3] This time evolution is one of the cornerstones of the ΛCDM model supported by the most recent experimental evidence [4]

$$\Omega_m \sim 0.3 \; , \; \Omega_\gamma \sim 10^{-4} \; , \; \Omega_\Lambda \sim 0.7 \, . \tag{1.1.10}$$

Predictions of the Cosmological Standard model As we mentioned before, the cosmological standard model provides a reliable and elegant description of our universe. It provides a tested account of the history of our universe from the time of nucleosynthesis until today. It predicts that our universe is around 13 billion years old, starting from the Big Bang singularity which expanded and cooled down until today. The early universe was filled by a plasma of radiation and fundamental matter at high temperature with same number of particles and anti-particles. After the process of baryogenesis took place the observed asymmetry between matter and antimatter. As the universe cooled down the quarks confined into hadrons and, afterwards protons and neutrons and finally formed the lightest elements. This process is called Nucleosynthesis. Finally the universe cooled down enough for atoms to form in a process called recombination. After recombination photons were no longer subject to scattering with electrons and protons at which point they can room freely through the universe forming the CMB. The last step was the transition to a matter dominated universe. CMB anisotropies seeded the large scale structures which we observe today.

Shortcomings of the Standard Model of Cosmology

As we have seen, we have been able to describe the evolution of our universe considering a FRW metric with a perfect fluid evolving through Einstein's equations. But despite the success of this framework there are still several problems which remain unsolved.

i) In the first place, the Big Bang singularity is a clear indication that this classical description fails. This can be seen at $t = 0$ where our universe is filled with an infinite energy density and at infinite temperature. In order to obtain physical insight of this initial point it is necessary to have a quantum description of gravity.

[3]Experimental evidence of the accelerated expansion of our universe relies on the redshift of the standard candles type IA supernovae [3].

ii) As we described above, recent experimental data suggest that our universe is in a phase of accelerated expansion. The source of this process is unclear. As we saw this process could be described by a cosmological constant in Einstein's equations, whose present value is $\Lambda \sim 10^{-120} M_P^4$. Explaining this tiny value remains one of the biggest unsolved questions in fundamental physics. A cosmological constant is not the only explanation for the accelerated expansion, it could also be described by means of quintessence, which is essentially a scalar field evolving over time.

iii) From (1.1.10) we see that the matter content of our universe is around 30 percent, but observable matter is around a 5 percent of the tonal energy in our universe. This means that the remaining content of matter is unknown. This is called dark matter which can, for instance, be found in halos surrounding galaxies and galaxy clusters[4] and galaxy clusters in halos. Nowadays, hot dark matter is discarded by experimental evidence pointing us to cold dark matter, whose candidates are WIMPS or axions. Regrettably the standard model of particle physics does not contain any particle which could be a viable dark matter candidate.

iv) With this description of an homogeneous and isotropic universe, the standard model of cosmology is not able to explain the source of the anisotropies observed in the CMB.

v) Finally, from the point of view of unified gauge theories different stable and heavy particles should have been produced in the early universe, thus contributing to the present energy density if they are not bound to annihilate. There is no way to explain the absence of this unwanted relics, of which monopoles are one of the clearest examples.

Flatness problem From the Friedmann equations one can deduce the evolution of the energy density through

$$\frac{d\Omega}{d\log a} = (1+3\omega)\,\Omega\,(\Omega-1)\ . \tag{1.1.11}$$

From the former equation is straightforward to see that a flat universe $\Omega = 1$ will be flat at all times. But we see that for a non-flat universe, the energy density of our universe is time-dependent, and the evolution depends on the energy source ω. Thus, we can conclude that a flat universe is an unstable fixed point. Any deviation from a flat geometry of our universe will be amplified through cosmological expansion. Current experimental evidence [5] points that the universe is nearly flat $|\Omega - 1| < 0.02$. Tracking back the energy density, one can see that at the time the CMB was emitted $\Omega_{\rm rec} = 1 \pm 0.0004$, and at the time of primordial nucleosynthesis, $\Omega_{\rm nuc} = 1 \pm 10^{-12}$. The standard model of cosmology does not provide any hint

[4]The existence of dark matter solves the problem regarding the velocity of rotation of galaxies.

about why the universe was incredibly flat at early times. Without any mechanism to explain this, a nearly flat universe should be a severely fine-tuned situation.

Horizon Problem This problem arises from the fact that the universe has a finite age. Namely, photons can only have traveled a finite distance since the Big Bang, and such that our universe has a horizon. The initial singularity is a surface of constant conformal time [5] $\tau = 0$, and the comoving [6] horizon size is the width of the past light cone projected on that surface. The key insight is that two events on the conformal spacetime diagram are causally connected only if they share a causal past, which means that past light cones overlap. If we consider two points in the CMB sufficiently separated, we will see that their past light cones do not overlap, and thus they are causally disconnected. So, if those two points on the CMB correspond to two completely separate, disconnected observable universes it is a mystery why those points reach the observed thermal equilibrium to a few parts in 10^5. This is called the horizon problem and it is summarized as the universe reaching a perfect equilibrium on scales much larger that the size of any local horizon.

From the Friedmann Equation (1.1.11) it is easy to show that the horizon problem and the flatness problem are related: consider a comoving length scale λ. It is easy to show that for $\omega = cte$, the ratio of λ to the horizon size dH is related to the curvature by a conservation law

$$\left(\frac{\lambda}{d_H}\right)^2 |\Omega - 1| = \mathrm{cte}\,, \tag{1.1.12}$$

therefore, for a universe evolving away from flatness

$$\frac{d\,|\Omega-1|}{d\log a} > 0 \rightarrow \frac{d}{d\log a}\left(\frac{\lambda}{d_H}\right) < 0\,, \tag{1.1.13}$$

which means that the horizon size gets bigger in comoving units.

Solving the horizon and flatness problems naively

Illustratively, we will show what should characterize the energy source in order to solve the horizon and flatness problems. This naive approach would give us insight about the underlying physics in order to build inflationary models. In order to solve the horizon and flatness problems, we see that we need a universe which evolves towards flatness, rather than from it. Paying attention to the equation (1.1.11), we see that a sufficient condition for that is

$$\frac{d\,|\Omega-1|}{d\log a} < 0 \iff (1+3\omega) < 0\ . \tag{1.1.14}$$

[5]Conformal time is defined as $d\tau := \frac{dt}{a(t)}$. In these coordinates the FLRW metric is written as $ds^2 = a^2(\tau)\left(d\tau^2 - |d\boldsymbol{x}|^2\right)$. This means that in this frame the photon geodesics are just described by $d|\boldsymbol{x}| = d\tau$ and in a diagram the photons travel along angles of 45 degrees

[6]The transformation between comoving distance and proper distance is given by $d_{\mathrm{prop}}(t) = a(t)d_{\mathrm{com}}(t)$

We have seen in Table 1.1 that when the universe is dominated by matter or radiation we cannot achieve $(1+3\omega) < 0$. The energy source to achieve that should be able to generate a sufficiently negative pressure $p < -\rho/3$ to render the universe flatter. Also from (1.1.5) we see that this condition is exactly equivalent to an accelerating expansion:

$$\frac{\ddot{a}}{a} \sim -(1+3\omega) > 0 \ . \tag{1.1.15}$$

Thus, we can conclude that for a universe dominated by matter or radiation the expansion of the universe slows down and the curvature evolves away from flatness. But, on the contrary, if the universe is accelerating its expansion, the universe gets flatter. From (1.1.12) we see that this negative pressure solution also solves the horizon problem, since an accelerating expansion means that the horizon size is shrinking in comoving units

$$\frac{d}{d\log a}\left(\frac{\lambda}{d_H}\right) > 0 \ , \ (1+3\omega) < 0 \ . \tag{1.1.16}$$

When the expansion accelerates, distances initially smaller that the horizon are redshifted to scales larger than the horizon at late times. This accelerating cosmological expansion is called inflation. As a naive approach one could consider the simplest example where the source of negative presure is a vacuum energy, for which we have seen that the scale factor is e^{Ht}. For that naive example we see that the universe is driven exponentially towards a flat geometry

$$\frac{d\log\Omega}{d\log a} = 2\,(1-\Omega) \ . \tag{1.1.17}$$

We can see that the horizon problem is also solved by looking at the conformal time:

$$d\tau = e^{-Ht}dt \rightarrow \tau = -\frac{1}{aH} < 0 \ . \tag{1.1.18}$$

Therefore, we see that the example of de Sitter evolution, prior to the epoch of radiation-domination, gives a qualitative picture of how inflation, or accelerated expansion, solves the horizon, flatness and monopole problems of the standard model of cosmology. But, obviously this approach is not realistic for different reasons: first of all we know that vacuum-like energy density does not dilute with the expansion of the universe. So, obviously, a universe dominated at early times by vacuum energy will be dominated by this energy source at late times. This means that considering a pure de Sitter era for inflation does not satisfy the ΛCDM model. Also, this model would not be able to explain the anisotropies seen at the CMB. As a final remark, note that inflation takes place in a negative conformal time and $\tau = 0$ will represent the transition from inflationary expansion to radiation-dominated era.

1.2 Inflation

In the previous section we briefly reviewed the virtues and drawbacks of the standard model of cosmology. Inflation is the mechanism which explains in a simple and elegant way how to solve the horizon and flatness problem and the absence of unwanted relics. Moreover, quantum fluctuations of the inflaton can explain the anisotropies of the CMB, which are responsible for large-scale structure formation. The necessary requirements to fix the above problems and fit the experimental data are translated into a setup with a time-dependent vacuum-like energy source, whose energy density and scale factor correspond to a quasi-de Sitter space. The suitable candidate will be a scalar field whose potential is nearly flat and whose quantum fluctuations will source the CMB anisotropies.

1.2.1 Inflation from scalar fields

In this section we will describe briefly what physics is responsible for this accelerated expansion at early times.

The qualitative picture of scalar field-driven inflation is the following: At early times, the energy density of the universe is dominated by the field ϕ which is slowly evolving on a nearly constant potential, so that it approximates a cosmological constant. During this period, the universe is exponentially driven toward flatness and homogeneity. Inflation ends as the potential steepens and the field begins to oscillate about its vacuum state at the minimum of the potential. In order to transition to a radiation-dominated hot Big Bang cosmology, the energy in the inflaton field must decay into Standard Model particles, a process generically termed reheating. On top of that, since the energy density of the universe during inflation is dominated by the inflaton field, quantum fluctuations, $\delta\phi$, couple to the spacetime curvature and result in fluctuations in the density of the universe. Since the process of inflation shrinks the Hubble radius, this primordial perturbations where generated by fluctuations larger than the horizon which are needed to explain the CMB anisotropies.[7]

We will consider that inflation is driven by a homogeneous scalar field, at a scale comparable to H^{-1}, which means that spacial gradients, $\overline{\nabla}\phi$, are negligible. The universe we live in today is homogeneous, but only when averaged over very large scales. Large structure formations were created by gravitational instabilities acting on tiny seed perturbations, so in top of that we will add quantum perturbations

$$\phi\,(t) = \phi + \delta\phi \ , \ \delta\phi \ll \phi \, , \tag{1.2.1}$$

where quantum fluctuations follow the Klein-Gordon equation in a curved background. The simplest action that we can assume including gravity for the background evolution is

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm P}^2}{2}R + F\left(\phi, g^{\mu\nu}\partial_\mu\phi\partial\nu\phi\right) - V\left(\phi\right)\right] \, , \tag{1.2.2}$$

where R is the Ricci scalar. In the former action is assumed a minimally coupled theory. In practice, many such non-minimally coupled theories can be transformed

[7]The perturbations we observe in the CMB exhibit correlations on scales which corresponds to an angular multipole of $l \sim 100$, or about 1^o as observed on the sky today. Thus, this perturbations exhibit correlations on scales much larger than the horizon size at that time.

to a minimally coupled form by a Jordan transformation, which will modify non-trivially the kinetic terms and, thus, upon canonical normalization the shape of the scalar potential. Also, another way would be to modify the gravitational sector by replacing the Ricci scalar R with $f(R)$ theories.

First of all we will focus on the classical background evolution, which will give us the insight of how inflation works. Afterwards we will describe briefly how to connect inflation with the observables in the CMB through the quantum fluctuations of this field.

Background Evolution Considering a FRW spacetime, at this level of approximation the equation of motion for a scalar field is given by

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0\,, \tag{1.2.3}$$

where the dot implies a derivative with respect the time t. Considering that the friction term in the equation of motion dominates, $\ddot{\phi} \ll 3H\dot{\phi}$, the former equation could be approximated to

$$3H\dot{\phi} + V' \approx 0\,. \tag{1.2.4}$$

Computing the stress-energy tensor for this concrete case we find that the energy density and the pressure are

$$\begin{aligned} \rho &= \frac{1}{2}\dot{\phi} + V(\phi)\,, \\ p &= \frac{1}{2}\dot{\phi} - V(\phi)\,, \end{aligned} \tag{1.2.5}$$

we see that in the de Sitter limit $p \approx -\rho$, is just the limit in which the potential energy of the field dominates the kinetic energy, $V(\phi) \gg \dot{\phi}$. Plugging the energy and pressure (1.2.5) into the Friedmann equations (1.1.4) and (1.1.5) we see

$$H^2 = \frac{1}{3M_{\mathrm{P}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \approx \frac{1}{3M_{\mathrm{P}}^2} V(\phi)\,, \tag{1.2.6}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6}(\rho + 3p) = H^2(1-\varepsilon)\,, \tag{1.2.7}$$

where the parameter ε specifies the equation of state

$$\varepsilon = -\frac{1}{H}\frac{dH}{dN} = \frac{1}{6}\frac{\dot{\phi}}{H}^2 \to \varepsilon_V := \frac{M_{\mathrm{P}}^2}{2}\left(\frac{V'}{V}\right)^2\,. \tag{1.2.8}$$

Equation (1.2.6) and (1.2.4) are together referred to as the slow roll approximation. The condition $\ddot{\phi} \ll 3H\dot{\phi}$ can be expressed in terms of the parameter η as

$$\eta := -\frac{\ddot{\phi}}{H\dot{\phi}} = \varepsilon + \frac{1}{2\varepsilon}\frac{d\varepsilon}{dN} \to \eta_V := M_{\mathrm{P}}^2\frac{V''}{V}\,. \tag{1.2.9}$$

The parameters ε and η are referred to as slow roll parameters, and the slow roll approximation is valid as long as both are small $\varepsilon, |\eta| \ll 1$. Note that we have defined ε_V and η_V[8] which are the slow-roll parameters in terms of the shape of the inflationary potential. This limit is then just equivalent to a field evolving on a flat potential $V' \ll V$. This is a useful parametrization because the condition for accelerated expansion $\ddot{a} >> 0$ is equivalent to $\varepsilon < 1$. Note that the de Sitter limit corresponds to $\varepsilon \to 0$. Also, under such conditions the universe expands quasi-exponentially $a(t) \sim \exp\left(\int H dt\right) := e^{-N}$, where we have defined the number of e-folds N as $dN := -Hdt$. This quantity measures the amount of inflation and, as a function of the field is

$$N = -\int H dt = \frac{1}{M_{\mathrm{P}}^2} \int \frac{d\phi}{\sqrt{2\varepsilon}} \approx \frac{1}{M_{\mathrm{P}}^2} \int \frac{V}{V'} d\phi \,. \tag{1.2.10}$$

The number of e-folds needed in order to solve the flatness and horizon problems is constrained by a lower bound due to thermodynamic arguments related with primordial nucleosynthesis, baryon asymmetry among others giving us $N \simeq 50$ [2]. Not having upper bound on the number of e-folds of inflation is related with the idea of eternal inflation [6, 7], in which inflation, once initiated, never completely ends, with reheating occurring only in isolated patches of the cosmos.

This simple single-field picture we have discussed is therefore an effective representation of a large variety of underlying fundamental theories. All of the physics important to inflation is contained in the shape of the potential $V(\phi)$ while the microscopical details of the theory are important for understanding the epoch of reheating

CMB Observables from Inflation In this section, we will review briefly how quantum fluctuations of the inflaton field could give rise to the CMB anisotropies, making contact between the CMB observables and inflation. In this section we will focus only, for the sake of simplicity , on single-field models and gaussian fluctuations (for more details see [8,9]). Quantum fluctuations will follow the Klein Gordon equation in a curved spacetime whose vacuum is defined by the Bunch-Davies vacuum. Decomposing the Fourier modes and applying the KG equation in conformal time we see that the equation reduces to the harmonic oscillator

$$u_k'' + \left(k^2 - \frac{a''}{a}\right) u_k = 0 \,. \tag{1.2.11}$$

The frequency of each mode is given by $\omega(t) \sim \frac{k}{a(t)}$. At sufficiently early times there will be modes whose frequency $\omega \gg H$. In this regime we can neglect the expansion of the universe and therefore any time dependence. In this regime $\frac{\dot{\omega}}{\omega} \sim H$ and, the two point function follows adiabatically the value in the vacuum, until $\omega \sim H$. At this transition, called freeze-out, the adiabatic approximation breaks down and the two point functions can no longer evolve as the two points are separated from each other at a distance longer than the Hubble scale. Hence, the two points in a

[8] Note to define these quantities we have approximated the background evolution as $\dot{\phi} \approx -\frac{V'}{3H}$ from (1.2.6) and $H^2 \approx \frac{1}{3} V$ from (1.2.4).

two-point function are separated by the event horizon, which allows as to define the horizon exit point as

$$\omega \sim H \to k = aH\,. \tag{1.2.12}$$

Thus, we have seen that the modes develop a large-scale invariant two-point function at scales longer than Hubble scale during inflation. In the following discussion we will connect this to the CMB observables. The CMB will be sensitive to perturbations of a different nature, like scalar density and tensor perturbations. Since we want to relate them and the splitting made is not unique, the study has to be performed in terms of gauge invariant combinations of matter and density perturbations. During inflation we will focus on

$$\mathcal{R} = \Psi + \frac{H}{\dot{\bar{\phi}}}\delta\phi\,, \tag{1.2.13}$$

where $\mathcal{R}$ is the comoving curvature perturbation.[9] The key point is that the gauge invariant quantities, $\mathcal{R}$ and ψ, are conserved for superhorizon scales.[10] The power spectrum of the curvature perturbation is given by

$$\left\langle \mathcal{R}_{\overline{k}}\mathcal{R}_{\overline{k}'}\right\rangle = \frac{H^2}{\dot{\phi}^2}\left\langle \delta\phi_{\overline{k}}\delta\phi_{\overline{k}'}\right\rangle = (2\pi)^3\,\delta^3\left(\overline{k}+\overline{k}'\right)P_{\mathcal{R}}\left(k\right)\ ,\ \Delta_s^2 = \frac{k^3}{2\pi^2}P_{\mathcal{R}}\left(k\right)\ , \tag{1.2.14}$$

and thus, in this case

$$P_{\mathcal{R}}\left(k\right) = \frac{H^4}{\dot{\phi}^2}\frac{1}{k^3} \to \Delta_s^2 = \frac{H^4}{(2\pi)^2\,M_{\mathrm{P}}^2\dot{\phi}^2}|_{k=aH} = \frac{H^2}{M_{\mathrm{P}}^2\left(8\pi^2\right)\varepsilon}|_{k=aH}\,, \tag{1.2.15}$$

where $\Delta_{\mathcal{R}}$ is the dimensionless power spectrum. Note that in the last equality we have used (1.2.8). We would like to emphasize that H and $\dot{\phi}$ depend slightly on the position of the scalar field. The best approximation is to evaluate those at the moment when the mode crossed the Hubble radius and became constant. A measure of the scale dependence of the power spectrum is given by the tilt n_s, defined such that the k-dependence of the power spectrum is approximated by a power-law[11] of the form

$$\Delta_s^2 = A_s\left(k_\star\right)\left(\frac{k}{k_\star}\right)^{n_s-1+\ldots}\,, \tag{1.2.16}$$

[9]There exists another quantity like the curvature perturbation, namely $\psi = \Psi + \frac{H}{\dot{\bar{\rho}}}\delta\rho$, but it could be proven that for adiabatic, slow-roll perturbations both coincide during inflation and also equal on superhorizon scales $k \ll aH$.

[10]Heuristically, this is because the universe looks locally homogeneous, with the same energy everywhere. The scale factor will evolve as in an unperturbed universe, and therefore the invariant quantities will be conserved. For a proof see [10]. This will happen until gradients become shorter than the Hubble length again, and so that local dynamics will be able to feel that the iniverse is not really unperturbed and this quantity will start evolving.

[11]This can be see naively from exact computation in the de Sitter limit for long-wavelength modes [2].

where $k_\star$ is a pivot scale of reference and the dots represent subleading corrections like the running of the spectral index $\alpha_s = \frac{dn_s}{d\log k}$. Therefore we have that

$$n_s - 1 = \frac{d\log\Delta_s^2}{d\log k}|_{k=aH} = -4\varepsilon_\star + 2\eta_\star\,. \tag{1.2.17}$$

Next, we will focus on gravitational wave modes, where the transverse and longitudinal polarization states of the gravity waves evolve as independent scalar fields. Using perturbations in the metric we can then compute the power spectrum in gravity waves (or tensors) as the sum of the two-point correlation functions for the separate polarizations

$$\Delta_T^2 = 2\frac{H^2}{\pi^2 M_{\rm P}^2}|_{k=aH} = A_T\left(k_\star\right)\left(\frac{k}{k_\star}\right)^{n_T}\,, \tag{1.2.18}$$

and the spectral index

$$n_T = \frac{d\log\Delta_T^2}{d\log k} = -2\varepsilon_\star\,. \tag{1.2.19}$$

If the amplitude of tensor perturbations is large enough, such a spectrum of primordial gravity waves will be observable in the CMB.

Therefore, for any particular choice of inflationary potential we have four mesurable quantities: the amplitudes Δ_T and Δ_s of the tensor and scalar power spectra, and the spectral indices. However, not all of these parameters are independent. In particular the ratio r between scalar and tensor amplitudes is given by the parameter ε, as is the tensor spectral index n_T

$$r = \frac{\Delta_T^2}{\Delta_s^2} = 16\varepsilon_\star = -8n_T\,. \tag{1.2.20}$$

This relation is known as the consistency condition for single-field slow roll inflation, and is in principle testable by a sufficiently accurate measurement of the primordial perturbation spectra. In the slow-roll approximation the Hubble and potential slow roll parameters are related as

$$\varepsilon \approx \varepsilon_V\ ,\ \eta \approx \eta_V - \varepsilon_V\,. \tag{1.2.21}$$

Note that any deviation from scale invariance, $n_s = 1$ and $n_T = 0$ would point as an indirect probe of the inflationary dynamics. As a final remark, one could consider deviations of gaussian fluctuations taking into account the three-point function of the perturbations. Single-field models predict a negligible amount of non-gaussianities in the power spectrum, in agreement with observations. Multifield models, predict typically larger values for non-gaussianities due to isocurvature perturbations. Nowadays, non-gaussianities are highly constrained by the Planck and WMAP data.

Energy Scale of Inflation Tensor fluctuations are often normalized relative to the amplitude of scalar fluctuations for which the Planck Collaboration [5] gives

$$A_s = (2.20 \pm 0.10) \times 10^{-9} \,. \tag{1.2.22}$$

Since this measurement is fixed and from (1.2.18), $\Delta_T^2 \sim H^2 \approx V$ shows us that the scalar-to-tensor ratio is a direct measure of the energy scale of inflation

$$V^{1/4} \sim \left(1.88 \times 10^{16}\right) \left(\frac{r}{0.10}\right)^{\frac{1}{4}} \text{ GeV} \,. \tag{1.2.23}$$

Note that a measurable scalar-to-tensor ratio, $r \geq 0.01$, implies that inflation occurs at GUT energy scales.

Types of inflationary models Now we will classify the set of possible single-field potentials into the following groups:

Large-field inflation: In this type of models, the field is displaced from the vacuum at the origin by $\Delta\phi \geq M_{\mathrm{P}}$ and rolls down the potential toward the origin. Large-field models are typically characterized by $n_S < 1$ and a scalar-to-tensor ratio $r \geq 0.01$

Small-field inflation: In this type of potentials, typically the inflaton rolls down from an unstable equilibrium point $V' = 0$ toward a displaced vacuum. These models are characterized by a spectral index $n_S < 1$ and a scalar to tensor ratio $r \leq 0.01$ and a field displacement $\Delta\phi \leq M_{\mathrm{P}}$.

Hybrid models: These models involve a second field at the end of inflation in order to stop it. Typically they predict a negligible scalar-to-tensor ratio $r \ll 0.01$ and a spectral index $n_S > 1$. These models are strongly disfavored by the CMB data.

The Lyth bound As we have seen a large primordial gravitational wave signal implies a high scale for inflation and, thus, increased sensitivity to ultraviolet physics. The Lyth bound [11] relates observable tensor modes to field displacement of the inflaton. From the definition of the slow roll parameter (1.2.8) and its relation with the scalar-to tensor ratio (1.2.20) we see that

$$r = \frac{8}{M_{\mathrm{P}}^2} \left(\frac{d\phi}{dN}\right)^2 \,. \tag{1.2.24}$$

Integrating the former expression we can obtain the total field range between the time when CMB fluctuations exited the horizon at the end of inflation

$$\frac{\Delta\phi}{M_{\mathrm{P}}^2} = \int \sqrt{\frac{r\,(N)}{8}} dN \,. \tag{1.2.25}$$

We can relate this expression with the scalar-to-tensor ratio measured in the CMB, $r_\star$ obtaining the well-known Lyth bound:

$$\frac{\Delta\phi}{M_{\mathrm{P}}} \approx \mathcal{O}\,(1) \times \left(\frac{r_\star}{0.01}\right)^{1/2} \,, \tag{1.2.26}$$

where $\mathcal{O}(1)$ takes into account the variations of the scalar-to-tensor ratio during inflation. Large values of the scalar-to-tensor ratio, $r > 0.01$, therefore correlate with $\Delta\phi > M_{\mathrm{P}}^2$ or large-field inflation.

1.2.2 UV Sensitivity

We have observed so far that single-field inflationary models are able to explain in a simple and elegant way several problems that appear in the standard model of cosmology through the slow-roll approximation. All the predictions obtained by these models are based on the shape of the potential and the motion of the scalar field along it but, all we discussed is at the level of an effective field theory without ultraviolet completion. These models are based on a vacuum-like energy coming from a scalar field rolling down a nearly-flat potential. From the point of view of quantum field theory coupled to general relativity this type of setup seems quite unnatural.

Before analyzing the details of the microscopic description of single-field inflationary models we will see that at the level of effective field theory adding a cutoff scale, Λ, above the Hubble scale, which we have seen that is the scale of inflation, will introduce corrections which spoil the flatness (1.2.8), (1.2.9) of the potential regardless of the type of model, i.e. large or small-field. Also, we will see that these problems are even more dramatic in the case of large-field inflation. The UV completion of a single-field inflationary theory will introduce naturalness problem and non-renormalizable operators which, one should address in order to have a consistent theory.

Finally, we will see that a way to overcome these two problems is the introduction of a symmetry in the lagrangian which forbids the presence of dangerous corrections in the theory. We will see that axion-like fields will be promising candidates for the inflaton.

The eta problem

The eta problem is inherent to all single-field inflationary models and appears naturally when we try to achieve a UV completion. This problem appears due to quantum corrections that renormalize coupling constants in the effective theory and corrections due to higher-order non-renormalizable operators. At leading order both UV corrections will add a contribution to the mass of the inflaton of order $\Delta m_\phi^2 \sim H^2$ and thus leads to a violation of one of the slow-roll conditions since $\eta \sim 1$.

First, we will discuss the naturalness problem in inflation. An effective field theory with a cutoff scale, Λ, is typically characterized by operators which will renormalize the coupling constants. The mass of the scalar field will run to the cutoff scale unless it is protected by some symmetry. Since in a UV completed theory of inflation the cutoff scale is $\Lambda \geq H$ so that, typically, quantum corrections will drive the mass of the inflaton

$$\Delta m_\phi^2 \sim \Lambda^2 \to \Delta\eta \sim \frac{\Delta m_\phi^2}{3H^2} \sim \frac{\Lambda^2}{3H^2} \geq 1\,, \tag{1.2.27}$$

preventing prolonged inflation. In absence of symmetries that protect the mass of the inflaton the avoidance of the eta-problem would require a severe fine-tuning between the bare mass of the inflaton and quantum corrections. This naturalness problem is analogous to the Higgs hierarchy problem, which can be solved by supersymmetry. In this case it alleviates but does not suffice to stabilize the inflaton mass. During inflation supersymmetry is spontaneously broken by the positive vacuum energy and the resulting splittings in supermultiplets are of order H, so the corrections to the inflaton mass will imply $\Delta\eta \sim \mathcal{O}(1)$.

Next, we will focus on the presence of non-renormalizable operators. Integrating out particles of mass $M \geq \Lambda$ give rise to operators in the lagrangian of the form

$$\mathcal{L}_{\text{eff}}(\phi) = \mathcal{L}_{\text{inf}}(\phi) + \sum_\delta a_\delta \frac{\mathcal{O}_\delta}{\Lambda^{\delta-4}}\,, \tag{1.2.28}$$

where δ denotes the mass dimension of the operator. In general, these operators are negligible unless we approach energies close to the cutoff scale. However, due to the flatness of the potential, inflationary models will be sensitive, in general, to $\delta \leq 6$ suppressed operators such as

$$\mathcal{O}_6 = a_6 V_{\text{inf}}(\phi)\frac{\phi^2}{\Lambda^2}\,, \tag{1.2.29}$$

where V_{inf} is the inflationary potential and contains all renormalizable corrections. Since $V_{\text{inf}} = 3H^2$ the mass term would receive a correction of order the Hubble scale as in the former case.

One possible way out is to impose an additional weakly broken global symmetry preserved by the effective lagrangian which forbids higher dimensional operators that correct the mass of the inflaton. One possibility would be a continuous shift symmetry of the inflaton, weakly broken by non-perturbative effects. In absence of any symmetry a severe fine-tuning will be necessary in order to avoid the eta-problem but this solution seems unnatural. Thus, we see that assumptions are necessary in the UV theory in order to ensure that the theory supports at least 60 e-folds of inflationary expansion.

Transplanckian fields

We have seen that in the absence of any symmetry, integrating out fields whose mass is above the cutoff scale Λ with couplings to the inflaton ϕ will lead to an effective theory of the form (1.2.28). Thus, whenever ϕ traverses a distance of order Λ, or M_{P} in an optimist completion, along a direction that is not protected by a suitable symmetry, the effective Lagrangian receives substantial corrections from an infinite series of higher-dimension operators. In order to mantain the slow-roll conditions of inflation, the potential should of course be approximately flat over a transplanckian range. If this is to arise by accident or by fine-tuning, it requires a conspiracy among infinitely many coefficients. For the moment, we will see how to address this problem from a bottom-up perspective. Such remedies do not necessarily have to hold in a UV complete theory of gravity. In following sections we will see how one can address these full quantum gravity effects in a UV completion like string theory.

The leading idea for implementing large-field inflation is to use a symmetry to suppress the dangerous higher-dimension contributions. For example an unbroken continuous shift symmetry

$$\phi \to \phi + a\,, \tag{1.2.30}$$

where a is some constant, forbids all non-derivative operators in (1.2.28), including the desirable parts of the inflaton potential, while a suitable weakly-broken shift symmetry can give rise to a radiatively stable model of large-field inflation. This means that the inflaton should be a pseudo-goldstone boson. Whether such a shift symmetry can be UV-completed is a subtle and important question for a Planck-scale theory like string theory.

Using this philosophy, axion models like [12, 13] were promising inflationary theory candidates. Axions are equipped with a continuous shift symmetry to all orders in perturbation theory, weakly broken spontaneously or explicitly. The spontaneous way of breaking the symmetry is by introducing periodic corrections, which might arise from instantons, which will lead to a discrete shift symmetry. However, high-scale inflation generated by such potentials requires a transplanckian axion decay constant f. There are various indications that such decay constants will not occur in a consistent theory of quantum gravity [14].

The Kaloper-Sorbo formalism

We have seen in the last section that if the inflaton enjoys a shift symmetry to all orders in perturbation theory, it will help to avoid, or at least mitigate, the presence of dangerous UV corrections which will spoil the flatness of the inflaton potential and thus spoiling inflation. But, from the standard lore, in a UV completion of quantum gravity global symmetries will be broken by gravity unless we promote them to gauge symmetries. The usual method to gauge a shift symmetry is due to Stückelberg mechanism, where introducing a gauge field A_μ the shift symmetry is promoted to a local gauge symmetry and the axion becomes the Stückelberg field for the gauge field with a gauge invariant mass.

The mechanism introduced by Kaloper and Sorbo [15] was designed for the standard quadratic chaotic inflation model at the level of the effective field theory. It establishes a natural way to gauge the shift symmetry, giving a mass for the axion in a shift-symmetrically invariant way, and to keep under control the dangerous Planck-suppressed operators that we have seen. In [15–17] the authors propose to couple the axion to a gauge three-form $C_{\mu\nu\rho}$ (see also [18, 19]). As we know, a three form has no propagating degrees of freedom in four dimensions, so we are not introducing any new degrees of freedom. The action is given by:

$$S = \int d^4x\sqrt{-g}\left(\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{48}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} - \frac{\mu}{24}\phi\varepsilon_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right)\,, \quad (1.2.31)$$

where $F_4 = dC_3$ is the field-strength of the three-form. Note that even if the three-form has no propagating degrees of freedom, it can still yield a non-vanishing field strength in the vacuum. Upon integrating out the four-form field via its equation of motion we get the following scalar potential

$$V = \frac{1}{2}(q+\mu\phi)^2\,, \quad (1.2.32)$$

where q is an integration constant related to the charge of the membranes charged under the three-form field. The variation on ϕ is absorbed by a shift of the four-form background q.

$$\phi \to \phi + 2\pi f\ ,\ q \to q - 2\pi f\,. \quad (1.2.33)$$

Thus, we see that the axion gets a mass but the shift symmetry still remains unbroken. Selecting a value of q (choosing a specific vacuum, and thus a branch) the shift symmetry will be spontaneously broken. This structure underlies the models of monodromy inflation, where the scalar potential is multivalued with a multi-branched structure given by the underlying discrete shift symmetry. When we select a branch, we can go up in the potential away from the minimum and travel a distance $\Delta\phi$ larger than the fundamental periodicity f and thus avoiding transplanckian decay constants.

Considering this natural framework for large-field inflation one could avoid the problems related with the eta-problem without a severe fine-tuning. The underlying discrete shift symmetry protects the axion from radiative corrections since the axion could only be coupled to massive particles via derivative couplings, and thereby, solving the naturalness problem. Also it protects the potential from dangerous Planck-suppressed operators. Since they have to satisfy the underlying gauge symmetry, they can only appear as powers of the gauge-invariant field strength over the cutoff scale $\frac{F_4^n}{\Lambda^{2n}}$. Integrating out the four-form the corrections to the scalar potential will be of the form [16, 17]

$$\delta V \sim \sum_n \frac{V^n}{\Lambda^{4n}}\,, \quad (1.2.34)$$

contrary to the one given in (1.2.28). Thus, we see that this mechanism is suitable to keep Planck-suppressed corrections in inflationary models under control, since the potential remains subplanckian during inflation $V = 3H^2 < M_{\rm P}^4$. Hence, these corrections will be subleading, For a generalization for Minkowski three forms in flux string vacua see [20].

We will see that this mechanism is naturally embedded in models of F-term axion monodromy inflation [21], where the authors also gave an alternative description for this mechanism in which the scalar is dualized into a 2-form. The 3-form gets massive by eating up the 2-form in a gauge invariant way.

2

String Inflation

We have seen that the inflationary process is driven by a nearly flat potential and, if tensor modes are observed in the CMB, at energies close to the GUT scale. This implies, as we have observed, that the theory is UV sensitive and corrections coming from pure quantum gravity effects could be important. Since string theory is a candidate to give a UV completion of particle physics and gravity, it seems that we are facing a great opportunity to build inflationary models within this framework since it could give us more insights about fundamental aspects of quantum gravity. Also, due to the high energies involved, which are not comparable to any known experiment, maybe we would be able to observe purely stringy effects which cannot be decoupled from the low energy effective field theory, like cosmic strings. Also inflation could shed some light on the vacuum solution of the string landscape [22,23] that describes our world. The way to build inflationary models in string cosmology starts specifying a consistent string compactification, which includes the geometry of the compact manifold, orientifold planes, D-branes, background fluxes and localized sources. This configuration specifies a four-dimensional effective field theory limited by the accuracy of the dimensional reduction, for instance α' and g_s corrections, or backreaction effects from the localized sources. Then this low-energy effective Lagrangian should be capable of producing inflation that is consistent with current observations.

2.1 Models of string inflation

As we have seen, four-dimensional effective field theories coming from string compactifications are plagued of moduli, i.e. massless scalar fields with gravitational couplings. Models of string inflation could be classified according to the nature of the modulus which we identify with the inflaton while the rest of moduli are stabilized (for a review see [24]). This rough classification could be as follows

Brane moduli In this case the inflaton candidate comes from the position modulus of a space-filling brane or the distance between two types of branes. If the brane is placed in a warped region there are models where the inflaton is the motion of a single D3-brane or the distance between a pair $D3-\overline{D3}$ [25,26]. If the branes are in unwarped regions we have examples like the motion between $D3-D7$ branes [27,28] and fluxbrane inflation [29, 30] or the motion of a single brane like a D6 [31] or a D7 [32,33]. In the case of non-relativistic branes the scalar potential comes from the DBI. Also one can consider relativistic branes [34,35], and in that case the resulting flat scalar potential will be controlled by the kinetic terms coming from the DBI action.

Kähler moduli In these models the inflaton candidate is associated with a time-dependent Kähler modulus or an axion paired with the volume form. Prototypical examples are blow-up inflation [36], racetrack inflation [37], and fibre inflation [38].

Complex structure In these models the inflaton candidate comes typically through a single complex structure or a linear combination of them sourced by background fluxes [39] or branes [40]. These models are usually built in the context of type IIB with O3/O7-planes and background fluxes in the large-complex structure limit. For other special points in moduli space see [41–43]

2.2 General Challenges on String Inflation

In this section we will see, briefly, some actual challenges that string inflation models, regardless its microscopic origin, should address in order to be consistent. Addressing these fundamental issues will bring us the chance to face fundamental questions of a UV complete theory of gravity like: landscape, fine-tuning, swampland, etc.

2.2.1 Supergravity eta-problem

This problem arises in the context of four-dimensional $\mathcal{N}=1$ supergravity effective field theories, so it appears, generically, as a low-energy problem in stringy models of inflation. The F-term scalar potential is defined through a Kähler potential, K, and a superpotential W. If the inflaton candidate Φ appears in the Kähler potential and expanding K for small Φ, i.e. $\Phi=\Phi_0+\varphi$, we see

$$K = K|_{\Phi=\Phi_0} + \frac{\partial^2 K}{\partial\Phi\partial\bar{\Phi}}|_{\Phi=\Phi_0}\varphi\bar{\varphi} + \dots \ . \tag{2.2.1}$$

If we expand the F-term scalar potential, V_F, for small values of Φ and obtain the canonically-normalized inflaton φ_c, which is given by

$$\partial\varphi_c\partial\bar{\varphi}_c \approx \frac{\partial^2 K}{\partial\Phi\partial\bar{\Phi}}|_{\Phi=\Phi_0}\partial\varphi\partial\bar{\varphi} \, , \tag{2.2.2}$$

we can observe that the following mass term for φ_c arises

$$\Delta m^2_{\varphi_c} \approx \frac{V_F|_{\Phi=\Phi_0}}{M_P^2} = 3H^2 \to \Delta\eta \approx 1 \ . \tag{2.2.3}$$

A way out to this problem is the presence of a protective shift symmetry in the Kähler potential forbidding the presence of the inflaton candidate in the Kähler potential, for a review on these issues see [44]. Note that, as happened before, in absence of any protective symmetry, generically, a severe fine-tuning would be needed in order to avoid this problem.

2.2.2 Mass hierarchies and the cosmological moduli problem

This problem appears generically in any model of string inflation due to the appearance of moduli [1] in the effective field theory after compactification. In absence of any source to stabilize them (branes, fluxes, non-perturbative effects, etc.) these scalar fields remain massless. The mass of these fields, due to moduli stabilization procedures, cannot be arbitrary for consistency with inflation due to the so-called cosmological moduli problem [45–47] which we will explain briefly in the following.

This problem could be illustrated as follows, if we consider a scalar field with mass below the Hubble scale it will undergo quantum fluctuations during inflation. These fluctuations carry the field away from its minimum and hence lead to storage energy. After inflation, this field behaves as a non-relativistic matter and, as we saw, its energy density decreases with the temperature as T^{-3}, whereas radiation decreases faster. This implies that these fields will dominate the energy density of the universe faster as it evolves. The signatures of this problem depend on the mass of this scalar field. If the scalar field only couples gravitationally and it is nearly massless it would have not decayed by the present day, and they will populate the Universe. On the other hand, if the field is heavier than 30 TeV, it would have decayed during or after the nucleosynthesis and, thus, it would spoil the delicate predictions of the light element abundances. To address the cosmological moduli problem one should be able to stabilize all the moduli above the Hubble scale. This challenge is warning us that the moduli stabilization problem cannot be decoupled from the inflationary dynamics. Related to this problem we will see that single-large-field inflation models arising from string compactifications show an inherent mass hierarchy problem. This problem consists on that, for consistency, all the moduli should be stabilized in this narrow range of energies $M_{\rm inf} < H < M_{\rm mod} < M_{\rm KK} < M_{\rm s} < M_{\rm P}$. Also if the hierarchy between the stabilized moduli, $M_{\rm mod}$, and the iinflationary scale, H, is not sufficiently large backreaction effects would spoil our model. We will review these problems in detail in Part IV and also we will see how we address this problems in the models proposed.

[1] Moduli are zero-energy deformations arising from the plethora of topologically distinct cycles in typical Calabi-Yau manifolds. From the 4d EFT perspective they are scalar fields with gravitational-strength couplings that have vanishing potential. For more details see Chapter 3.

2.2.3 The Weak Gravity Conjecture

The Weak Gravity Conjecture, first proposed in [48], has turned out to be a vey powerful tool for constraining phenomenological models, but no full formal proof of the conjecture has been given yet. Its power relies on its generality: the WGC demands the existence of certain charged states in any theory in order to be consistent with quantum gravity. Arguably, then if a theory does not include such states it is in trouble with quantum gravity, likely belonging to the Swampland. Once we try to embed inflationary models in a consistent theory of gravity, WGC arguments could constrain the available field range of the inflaton candidate.

It has been argued that in a consistent theory of quantum gravity one cannot have global symmetries since that would imply infinitely many black hole remnants and a pathological theory. Thus, considering in a theory a U(1) symmetry, this implies that it has to be gauged. This symmetry is described by a gauge coupling g and we consider it charged under a charge q. Turning $g \to 0$ would lead to trouble, since the gauge field will be decoupled from the theory but it will preserve the symmetry acting on charged fields. Thus, we see that this limit is problematic and points us that in a consistent theory of quantum gravity one cannot perform this kind of tuning of a gauge coupling of this form. The WGC explores what happens in this regime.

We will describe briefly its different versions for a single U(1), while it could be extended for p-forms straightforwardly. For the case of multiple U(1)'s see [49–52]

Black hole evaporation First of all we will see the WGC in its electric form. In order to obtain more insight about that we will use black hole evaporation arguments. The paradigmatic arena to analyze the WGC is the study charged black holes described by the well-known Reissner-Nordstrom metric [53]. The casual structure of this black holes is controlled by

$$\Delta = \frac{4}{M_{\mathrm{P}}^2}\left(M^2 - (gQM_{\mathrm{P}})^2\right) \ . \tag{2.2.4}$$

For $\Delta > 0$ the black hole is called subextremal. At $\Delta = 0$ the black hole is said to be extremal. Classically it is a stable object which Bekenstein-Hawking entropy is vanishing. Note that in supersymmetric theories this condition is the BPS bound. For $\Delta < 0$ the black hole is superextremal. In these black holes there is no event horizon and, thus contain a naked singularity. These objects violate the Cosmic Censorship hypothesis [140,141] which states that no naked singularities can form dynamically in a classical theory of general relativity. So, given a subextremal black hole of mass M and charge Q, it will lose both charge and mass via Hawking evaporation [54], in such a way that it will only stop radiating when approaching the extremal limit. If we take the limit $g \to 0$ one can see that extremal black holes will contribute to the Unruh temperature without control. This issue could be solved assuming that extremal black holes could decay once we take into account quantum fluctuations. Thus, an extremal black hole of mass M and charge Q decays via emission of a particle of mass m and charge q. We require the final black hole to also be subextremal. We have then

$$gM_{\rm P}\,(Q-q) \geq M - m \rightarrow m \leq qgM_{\rm P}\ . \tag{2.2.5}$$

Mild Form The mild form of the WGC precisely comes from (2.2.5). It states that there should be a charged superextremal particle in the theory. We would be able to extend this case to d dimensions, in that case the gauge coupling has $g^{D/2-2}$. Following a similar reasoning we see that the WGC states the existence of a particle with mass $m < g/\sqrt{G}$ where G is the Newton's constant in d dimensions.

Strong forms There are other two versions of the WGC more restrictive that the one that we have seen. The first one states that the state of least charge under the U(1) satisfies (2.2.5). We can choose without lose of generality the least charge as $Q = 1$, so it implies that there is a superextremal state with charge unity.

The second form states that the lightest state charged under the U(1) field satisfies (2.2.5). One could see that the first strong from implies the second, and both imply the mild one.

Magnetic version Since a U(1) gauge theory can couple to both, electric and magnetic sources, we can follow the previous reasoning considering Reissner-Nordstrom solutions with magnetic charges. In this case the extremality condition states $M \leq 2\pi M_{\rm P}/g$. The magnetic WGC conjecture comes from assuming that monopole charge is non-zero at infinity. For a weakly-coupled U(1) the magnetic field diverges at the origin, so we set a cutoff scale Λ. Taking the mass of the monopole as the 't Hooft-Polyakov monopole one can set the so-called magnetic version of the WGC

$$\Lambda \leq gM_{\rm P}\ . \tag{2.2.6}$$

Thus, the effective field theory must have a cutoff lower than the mass of the magnetic particle. Like the electric form this also extends to higher dimensions.

WGC and inflation As we have seen, inflationary models are typically built from a bottom-up perspective. Therefore, there remains the question about if they could be embeddable in a consistent theory of quantum gravity. Paying attention to models of inflation based on axions, we see that the WGC does not apply straightforwardly. This is because the WGC does not affect the axion by itself. In order to apply the WGC to these kind of models we need to couple the axion to gravitational instantons.

These instantons are effective descriptions of configurations in concrete models of quantum gravity. In string theory it could be reassembled as non-perturbative effects whose microscopic description is given by D-brane instantons. An important aspect, is that the corresponding instantons may not correspond to BPS instantons in the vacuum. In non-supersymmetric scenarios, like inflation, non-BPS instantons will contribute to the scalar potential. Typically non-perturbative contributions to the scalar potential are of the form

$$V_{\text{inst}} \sim \Lambda^4 e^{-S_{\text{D-inst}}} \left(1 - \cos\left(n\frac{\phi}{f}\right)\right) , \tag{2.2.7}$$

where $S_{\text{D-inst}}$ is the action of the D-brane action. If we want the full range of the axion f to be available for inflation we need V_{inst} very suppressed. In other words $S_{\text{D-inst}} >> 1$. This means that $M_{\text{P}} \gg f$. Thus, effects of gravitational instantons constrain the effective axion decay constant. If $f \sim M_{\text{P}}$, the gravitational instantons with low n will not be suppressed. If the contribution from gravitational instantons is sufficiently strong they could spoil the transplanckian field range introducing modulations on the potential (for different proposals see [55–65]).

2.3 String inflation and axions

As we have seen from a bottom-up perspective, axion-like fields are promising candidates to drive inflation due to the fact that its inherent shift symmetry alleviates all the problems that arise once we try to embed inflation in a consistent UV complete theory. Also, four-dimensional effective field theories coming from string compactifications are plagued of axions coming from dimensional reduction of p-form gauge fields integrated over p-cycles, where the continuous shift symmetry comes from the gauge invariance in higher dimensions. Thus, it seems that string theory could provide a microscopic description of inflation in a UV complete theory of quantum gravity.

Nowadays there are in string theory two different groups of models of large-field inflation based on axions: models based on multiple axions and single-field models based on axion monodromy.

2.3.1 Models based on multiple axions

In these models the scalar potential is generated by the breaking of the continuous shift symmetry of the axion by instantons like (2.2.7). They alleviate the problem of transplanckian decay constant by introducing multiple axions with subplanckian decay constants in a intricate way. Summarizing, there are two models that fit in this classification: models based on two aligned axions and models based on a large number of axions, so-called N-flation.

The first type of models are based on only two axions coupled to linear combinations of two confining non-abelian gauge groups, see [66] for an example. Assuming a suitable relation on these couplings, it could be proved that a particular linear combination of the axions is unlifted and which effective decay constant is transplanckian, regardless the decay constant of the original axions are subplanckian. These models are, nowadays, under stress by the WGC.

The second type of models are called N-flation [67] and are based on a large number of axions ϕ_i, each one with subplanckian decay constant and a scalar potential generated by non-perturbative effects where there are no couplings between the different axions. From the equation of motion for each axion (1.2.3) one can see that each axion feels enhanced Hubble friction and the naive potential is $3H^2 \sim \sum_{i=1}^{N} m_i\phi_i^2$ and effectively one could have $V = m\Phi^2$, with $\Phi^2 = \sum_i \phi_i^2$. Thus, we see that in order to describe large-field inflation one needs $\Delta\Phi > M_{\mathrm{P}}$ while the field-range of each axion is subplanckian, the typical number of axions needed is around 10^3.

2.3.2 Axion Monodromy

In this section we will review the framework of axion monodromy which describes in an elegant way how to drive single-field inflation with axions in string theory.

Naive attempts to achieve single-field inflation The first attempts in string theory to build single-large-field models of inflation described models of natural inflation [12,13], were the potential for the axion was generated through the breaking of the continuous shift symmetry into a discrete one through non-perturbative effects giving the following effective action

$$\mathcal{L} = \frac{1}{2}\left(\partial\phi\right)^2 - \Lambda^4\left(1 - \cos\left(\frac{\phi}{f}\right)\right)\,, \tag{2.3.1}$$

where f is the axion decay constant. In order to be compatible with experimental data, these models need a transplanckian decay constant and, thus, being not compatible with a UV completion of quantum gravity [14].

The main idea of axion monodromy, proposed on [40, 68], is to weakly-break the discrete shift symmetry of the axion by branes or fluxes where every time the axion completes a circuit $\phi \to \phi + 2\pi f$ the system reaches a new configuration which compensates this shift and thus preserves the gauge invariance of the lagrangian. Every new configuration of the system will define a branch, where the potential energy for the axion is unbounded and thus, it could roll down. One could imagine this system as a spiral staircase, where the symmetry breaking ingredient (branes or fluxes) unwraps the fundamental domain of the axion.

Illustratively, we will describe briefly one of the first attempts [40]. One simple setup for axion monodormy is to consider type IIB compactifications with O3/O7-planes, with the axion as the scalar arising from the KK reduction of the two-form C_2 over a 2-cycle Π_2 in the compactification space, and introducing a NS5-brane wrapped on Π_2 to break the shift symmetry and thus, generate the monodromy[2].

[2]In the first proposal [40] the authors considered the inflaton candidate the NSNS two-form B_2 and the monodromy was generated by a D5-brane. This model cannot avoid the eta-problem. It could be easily seen in $\mathcal{N} = 1$ supergravity, where the inflaton candidate appears in the Kähler potential, what we saw from (2.2.1) to (2.2.3) applies straightforward.

The inflationary process will proceed considering in first place taking a large initial vev for the axion c and continue by reductions of this vev, until finally $\int_{\Pi_2} C_2 = 0$. Every time when the axion completes one period, the c-field ends up inducing one unit of D3-brane charge on the worldvolume of the NS5 due to the CS coupling. In order to satisfy RR tadpole conditions, this forces us to consider pairs of NS5 - $\overline{\text{NS5}}$ branes wrapped on homologous two-cycles on different throats. The scalar potential comes from the dimensional reduction of the DBI action for a NS5-brane wrapping a two-cycle Π_2

$$V = \frac{\varrho}{(2\pi)^6 g_s^2 \alpha'^2} \sqrt{(2\pi)^2 l_{\Pi_2}^4 + g_s^2 c^2}\,, \tag{2.3.2}$$

where $l_{\Pi_2}^2$ is the size of the two-cycle Π_2 in string units and ϱ is a dimensionless number associated to the dependence on the warp factor. We see that the brane energy is clearly not invariant under the shift symmetry $c \to c + 2\pi$, although this is a symmetry of the corresponding compactification without the wrapped NS5-brane. Thus the DBI action leads directly to monodromy for c. Moreover, when $c \gg l_{\Pi_2}^2$, the potential is asymptotically linear in the canonically-normalized field ϕ. It has been argued that, in general, axion monodromy models are described in general as

$$\mathcal{L} = \frac{1}{2} (\partial\phi)^2 - \Lambda^4 \left(1 - \cos\left(\frac{\phi}{f}\right)\right) - \mu^{4-p}\phi^p \ . \tag{2.3.3}$$

The non-perturbative effects will be negligible for large-initial vevs of the inflaton but it will introduce modulations in the scalar potential at the end of inflation. These modulations could be measured in the future and thus offering a signal in favor of string theory.

The models we presented were built on non-supersymmetric configurations of NS-branes-antibrane pairs, just because it was needed to cancel D3-tadpoles, this makes the stability of this models more difficult to handle.

F-term axion monodromy Inflation

Here we will review a subclass of axion monodromy models, called F-term axion monodromy [21]. These models are capable to realize axion monodromy inflation with spontaneously broken supersymmetry where the monodromy is induced by an F-term potential for the axion. As we will discuss in the following, there are plenty of string theory setups where this idea can be realized.

Typical examples involve closed string axions whose potential is created by the presence of background fluxes. A further novelty of this framework is that one can also implement the monodromy idea to axions associated to massive Wilson lines or their T-dual, D-brane position moduli. Compactifications with background fluxes lead to superpotentials which can stabilize moduli, in particular the components which correspond to the axions from p-forms. This essentially follows from the fact that the increase of energy upon axion monodromy is due to the appearance of extra fluxes, whose contributions to the superpotential can be understood in terms of domain walls.

Finally, it was argued that these constructions have a built-in mechanism to prevent the appearance of the axions in the Kähler potential, and thus avoiding the supergravity eta-problem in the underlying $\mathcal{N}=1$ SUSY structure (2.2.3).

These appealing features points us to see F-term axion monodromy inflation as one of the most natural ways to realize axion monodromy with a four-dimensional supersymmetric structure. In particular, in the context of flux compactifications, F-term axion monodromy is an elegant setup to build axion monodromy inflation.

Another advantage of F-term axion monodromy is that it allows to connect with the four-dimensional axion monodromy framework [16,17]. It was found in [21] that upon dimensional reduction one obtains an effective Kaloper-Sorbo Lagrangian describing the coupling of an axion with a non-dynamical four-form. As we have seen in Section 1.2.2 the presence of this four-form creates a quadratic potential for the inflaton which is protected against dangerous corrections to the slow-roll potential that arise upon UV completion of the theory. Illustratively we will see that in the context of type IIB, with O3-planes and quantized RR, F_3, and NSNS, H_3, fluxes, Kaloper-Sorbo protection arises naturally. The four-dimensional flux superpotential is given by

$$W=\int_{\boldsymbol{X}_6}(F_3-\tau H_3)\wedge\Omega=\int_{\boldsymbol{X}_6}\left(F_3-\frac{i}{g_s}H_3-C_0H_3\right)\wedge\Omega\,, \tag{2.3.4}$$

where $\tau=C_0+\frac{i}{g_s}$ with C_0 the type IIB axion and g_s the string coupling. Considering $\phi=C_0$ the axion, which continuous shift symmetry is broken to a discrete one by D(-1)-brane instantons. As the axion completes a period $\phi\to\phi+1$ the system reaches a new configuration, which compensates the shift, due to a non-trivial shift of the background fluxes [3]. The four-dimensional coupling needed to achieve the Kaloper-Sorbo realization comes from KK reduction of the CS coupling between the inflaton and the domain wall associated to the increase in the tension every time the system completes a period [4]. In this concrete case

$$\int_{4\mathrm{d}}C_0\int_{\boldsymbol{X}_6}H_3\wedge F_7=\int_{4\mathrm{d}}C_0\int_{\Pi_{\mathrm{d.w}}}F_7=\int_{4\mathrm{d}}C_0F_4\,, \tag{2.3.5}$$

where domain walls described above are $\boldsymbol{Z}_k$ valued, as mentioned above.

Challenges in F-term axion monodromy

In this section we will review some challenges that arise typically in F-term axion monodromy models. As we have seen F-term axion monodromy models are protected of the eta-problem since it incorporates a built-in mechanism to prevent the appearance of the inflaton candidate in the Kähler potential. We summarize

[3]The fluxes shift as $F_3\to F_3+H_3$ i.e. $(n_i,n'_i,m_i,m'_i)\to(n_i-m_i,n'_i-m'_i,m_i,m'_i)$ and thus keeping the gauge invariance.

[4]The CS coupling resposible is $\int_{10\mathrm{d}}C_0H_3\wedge F_7$ and the domain wall is given by a D5-brane wrapping on the 3-cycle Poincare dual to $[H_3]$, namely $\Pi_{\mathrm{d.w.}}$

the main challenges of this type of models as: mass hierarchy problems, tuning and backreaction problems and, recently appeared, some authors pointed out that maybe it is not possible to achieve transplanckian field ranges due to the Refined Swampland Conjecture [69].

The problem of achieving a consistent mass hierarchy in F-term axion monodromy models was suggested in [70] in the context of type IIB flux compactifications. There, the authors pointed the difficulty of lowering the mass of the inflaton candidate with respect the other closed string moduli. Also, there is another difficulty regarding the consistency of the effective field theory since, it is also difficult to achieve the appropriate hierarchy of scales[5]

$$M_{\mathrm{inf}} < H < M_{\mathrm{K}} < M_{\mathrm{cx}} < M_{\mathrm{KK}} < M_{\mathrm{s}} < M_{\mathrm{P}}\,, \tag{2.3.6}$$

since there is no much room in energies between the Hubble scale and the Planck scale. This means that if, for example, the complex structure scale is stabilized above the KK scale the consistency of the four-dimensional model is compromised.

Other important challenge is related with the amount of tuning necessary to mitigate backreaction issues in F-term axion monodromy. It was pointed out in [71] in the context of type IIB. The authors consider a model where the complex structure moduli are integrated out and thus, in terms of a 4d $\mathcal{N}=1$ supergravity description they are treated as constants in the coefficients of superpotential. In order to achieve a sufficiently large mass hierarchy between the Kähler moduli and the inflaton sector one should tune this coefficients. But considering its dynamical nature, and thus considering its backreaction, the tuning of coefficients is dramatically enhanced. This issue could make unfeasible models of F-term axion monodromy due to a severe fine-tuning

Finally, it seems that achieving a parametrically large field-range for the inflaton is under stress due to the Refined Swampland Conjecture [69,72]. The arguments are based on the Swampland Conjecture [73] which states that a field-range with a parametric logarithmic behavior, as it appears in some cases in string theory, could not be embeddable in a consistent theory of quantum gravity and thus, belonging to the swampland. The authors of [69,72,74] show that integrating out consistently all moduli except the inflaton candidate will modify the kinetic term of the inflaton, $K_{\Phi\bar{\Phi}}$ in such a way that the field range of the canonically normalized inflaton $\Delta\varphi = \int \sqrt{K_{\Phi\bar{\Phi}}} \sim \log(\alpha\phi)$ where α depends on the details of the compactification and the mass hierarchy between the inflaton and the rest of the moduli. The authors suggest that in any compactification the parameter $\alpha \sim \mathcal{O}(1)$ and thus, F-term axion monodromy models belong to the swampland. Also, it was suggested in [69] that the KK scale is lowered as the inflaton rolls down, i.e. $m(\varphi + \Delta\varphi) = e^{\alpha\frac{\Delta\varphi}{M_{\mathrm{P}}}} m(\varphi)$. This means that for $\alpha \sim \mathcal{O}(1)$, our effective field theory will not be trustable for transplanckian displacements of the inflaton. We will review this issues in concrete examples in Part IV.

[5]In this context M denotes energy scale where the subindices make reference to:inf (typical mass of the inflaton), K (Kähler moduli) , cx (complex structure), KK (lightest mass of the KK models), s (String scale)

New Data and necessity of flattening

Nowadays, models of large-field inflation which predict a large gravitational wave contribution are being constraint due to recent experimental data coming from the joint analysis done by Planck and BICEP2/Keck collaborations [75]. They set an upper bound of the scalar-to-tensor ratio $r < 0.07$ at 95 percent of confidence level, while the constraints for the spectral index are the same. This important feature puts under stress paradigmatic models like quadratic chaotic inflation $m^2\phi^2$. The only way to reconcile chaotic inflation with new experimental data is through flattening of the potential.

The mechanism of flattening affects the asymptotic form of the scalar potential for large values of the inflaton candidate. There are different sources of flattening. First of all, one could consider coupling the inflaton candidate to heavy fields with appropriate couplings and integrate them all. This is a common feature that arises when one computes the backreaction of heavy fields in inflationary models [76]. We will review this mechanism in Part IV. Other mechanism of flattening is to introduce non-minimal couplings of the inflaton candidates with the Ricci scalar. Once we transform our model into Einstein frame and canonically normalize we could modify the asymptotic form of the scalar potential.

However, whether flattening occurs depends on how the inflaton couples to the heavy fields, and hence a diagnostic is possible only if the UV completion of inflation is known. For instance, in string theory constructions with D-branes [24, 77, 78], flattening can follow from the structure of the DBI+CS action [21, 40, 68, 79–83]. However, the degree of flattening that one finds in this context is to date rather limited, e.g., a quadratic potential gets flattened to a linear potential through the α' effects included in the DBI action. For instance, it was argued in [80] that the linear scalar potential obtained in axion monodromy (2.3.3) is an example of flattening. The main argument is that, in that case, the C_2 axion has a coupling with H_3 of the form

$$S \supset \int d^{10}X \, |C_2 \wedge H_3|^2 \, , \tag{2.3.7}$$

which naively points that the inflaton should appear quadratically in the scalar potential, but the potential in that case is linear. The claim is that backreaction of localized D3-brane charge, which shifts the moduli vevs, is responsible for the flattening from $p = 2$ to $p = 1$. This is a common feature of axion monodromy models, where the monodromy is induced by a D-brane. Since the scalar potential comes from the DBI and the inflaton candidate typically is quadratic inside the square root for large vevs of the inflaton this will tend to linear. But, as we commented before, this flattening effect is rather limited if we want to fit with experimental data. We will propose a new way to flatten the scalar potential which we call flux flattening. We will review this issues in Part III.

3
Type II flux compactifications

In this chapter we will review the basics about type II flux compactifications. We will start reviewing, briefly, some basic concepts about compactifications in Calabi-Yau manifolds. Afterwards, we will obtain the four-dimensional massless spectrum of type II string theory compactified on these manifolds and, finally, we will review the basics about type II orientifold flux compactifications. We will finish this chapter giving some insights about moduli stabilization in both scenarios.

3.1 Compactification toolkit

The aim of this section is to give some basic concepts regarding Calabi-Yau compactifications which will be useful in order to obtain the four-dimensional massless spectrum of type II theories. For more technical details about this topic we encourage the reader to see [84, 85].

First of all we assume that the ten-dimensional spacetime is a product of the four-dimensional Minkowski spacetime and a real six-dimensional compact manifold $\mathbb{R}^{1,3} \times \mathbf{X}_6$. Our first aim is to analyze the number of preserved supersymmetries after the compactification.

Preserving supersymmetry After compactification on $\mathbf{X}_6$, the number of remaining supersymmetries in four dimensions correspond to globally well-defined supercharges on the compact manifold, and thus, preserving some supersymmetries corresponds to the existence of non-trivial 6d Killing spinors which are covariantly constant in $\mathbf{X}_6$. This could be seen in terms of the holonomy group of $\mathbf{X}_6$.[1] In general, the Lorentz group in the ten ten-dimensional spacetime decomposes into $SO(1,3) \times SO(6)$ and, in $SO(6)$ does not transform any spinor as a singlet. In conclusion, we see that compactification on a generic holonomy space breaks all the supersymmetries. In order to preserve some supersymmetries we should focus on manifolds of special holonomy, i.e. with a reduced structure group

[1]In the following discussion we will assume absence of background fluxes. For more details in this topic see [86].

$SU(3) \subset SO(6) \cong SU(4)$. Hence starting from an $\mathcal{N}=1$ theory in ten dimensions, and then compactifying on a six-dimensional manifold with SU(3) holonomy one obtains an $\mathcal{N}=1$ theory in four dimensions. Now, that we have understood how to count the number of supersymmetries in four dimensions, it is straightforward to realize that a compactification manifold with $SU(2)$ holonomy gives $\mathcal{N}=2$ supersymmetries in four dimensions. Similarly compactifications on a T^6 will give $\mathcal{N}=4$. Thus, we have seen that compactifications that preserve the minimal amount of supersymmetries, with our ansatz, are the ones with $SU(3)$ holonomy. This makes Calabi-Yau threefolds, with $SU(3)$ holonomy, a perfect candidate to be the compactification manifold of string theory.

In general, a Calabi-Yau N-fold (where N denotes the complex dimension of the manifold) is characterized to be Kähler and to have a first vanishing Chern class, which means that is Ricci-flat. They also admit a non-vanishing closed (N,0)-form Ω. Also, since it is a complex manifold, we are able to define a (1,1)-form

$$J = g_{i\bar{j}} dz^i \wedge dz^{\bar{j}} \,, \tag{3.1.1}$$

and due to the fact that this manifold is Kähler this form is closed, i.e. $dJ = 0$, and for this reason it is called Kähler form. Both forms describe the manifold and are related. In the case of Calabi-Yau three-folds we see that

$$J \wedge J \wedge J = \frac{3i}{4} \Omega \wedge \bar{\Omega} \ , \ J \wedge \Omega = 0 \,. \tag{3.1.2}$$

In the case of Calabi-Yau three-folds we see that the massless modes in the four-dimensional theory will satisfy $\nabla_6 \phi = 0$ [2] (note that ∇_6 is the Laplace operator in the internal manifold) and are in one-to-one correspondence with harmonic forms of $\mathbf{X}_6$. These forms are in one-to-one correspondence with the elements of the cohomology group $H^{p,q}(X)$.[3] The dimension of these groups are given by the Hodge numbers $h^{p,q}$, which usually are arranged in the so-called Hodge diamond. The Hodge numbers in Calabi-Yau three-folds satisfy three plus one symmetries: complex conjugation, Poincaré duality, holomorphic duality and mirror symmetry (this one has been only proven on a subspace of Calabi-Yau manifolds). In this case the only non-trivial Hodge numbers are $h_{1,1}$ and $h_{1,2}$ and the symmetries could seen as

$$\begin{aligned} \text{Complex conjugation} \quad &\rightarrow \quad h^{p,q} = h^{q,p} && (3.1.3)\\ \text{Poincaré duality} \quad &\rightarrow \quad h^{p,q} = h^{n-q,n-p} && (3.1.4)\\ \text{Holomorphyc duality} \quad &\rightarrow \quad h^{0,q} = h^{0,3-q} \,,\, h^{p,0} = h^{3-p,0} && (3.1.5)\\ \text{Mirror symmetry} \quad &\rightarrow \quad h^{2,1}(\mathbf{X}_6) = h^{1,1}\left(\tilde{\mathbf{X}}_6\right) \,,\, h^{1,1}(\mathbf{X}_6) = h^{2,1}\left(\tilde{\mathbf{X}}_6\right) && (3.1.6) \end{aligned}$$

where $\tilde{\mathbf{X}}_6$ denotes the mirror dual manifold of $\mathbf{X}_6$ and $n = 3$ in Calabi-Yau three-folds.

[2]The same applies to spinors.

[3]The elements of $H^{p,q}(X)$ are defined as the set of closed (p,q)-forms quotiented out by the set of exact (p,q)-forms, where (p,q) is denoting the number of holomorphic and anti-holomorphic differential forms

3.1.1 Geometrical moduli space

We have seen that the massless modes in the compactified theory are in one-to-one correspondence with the harmonic forms of the Calabi-Yau manifold. The geometrical moduli space will be constituted by all the scalar fields obtained in the effective field theory resulting from deformations of the metric, g, of the manifold that preserve the Calabi condition. The multiplicity of these zero modes is counted by the dimension of the non-trivial cohomology groups. More precisely, we will take the ten dimensional metric to be block diagonal

$$ds^2 = \eta_{\mu\nu}(x)\, dx^\mu dx^\nu + g_{i\bar{j}} dy^i dy^{\bar{j}} \,. \tag{3.1.7}$$

Moduli could be understood as the coordinates of the moduli space which parametrize the size and shape of the manifold. There are two types of geometrical moduli: Kähler moduli and complex structure moduli

Kähler moduli These moduli correspond to cohomologically non-trivial deformations of the Kähler form (3.1.1) and thus correspond to harmonic (1,1)-forms. This corresponds to $h^{1,1}$ real scalar fields, v^a, which are expanded in a basis of $H^{1,1}(X)$, ω_a:

$$J = v^a \omega_a \,, \tag{3.1.8}$$

where J is the Kähler form of $\mathbf{X}_6$ in the string frame. These complex scalars will define the so-called Kähler cone due to the consistency conditions that the Kähler form has to satisfy

$$\int_C J > 0 \,, \int_S J \wedge J > 0 \,, \int_X J \wedge J \wedge J > 0 \,, \tag{3.1.9}$$

for all complex curves C and surfaces S on the Calabi-Yau X. Kähler moduli are complexified in type II string theory in order to obtain the standard low energy $\mathcal{N} = 2$ effective field theory. To do so it is combined with the scalar field arising from the compactification of the NSNS two-form $\hat{B}_2$ and thus

$$t^A = b^A + i v^A \,. \tag{3.1.10}$$

These variables span a complex manifold, $\mathcal{M}^K$, that admits a metric given by a Kähler potential, K^K, determined by a holomorphic prepotential $\mathcal{F}(t^a)$. Manifolds that satisfy this condition are called special Kähler. Thus, the metric of the manifold will be given by

$$G_{a\bar{b}} = \frac{3}{2\mathcal{K}} \int \omega_a \wedge *\omega_b = \partial_{t^a} \partial_{\bar{t}^b} K^K \,, \tag{3.1.11}$$

where K^K is the Kähler potential

$$K^K = -2 \log \left(\mathcal{K}_{abc} v^a v^b v^c \right) \;,\; \mathcal{F}(t) = \mathcal{K}_{abc} t^a t^b t^c \,, \tag{3.1.12}$$

and $\mathcal{F}$ is the prepotential. Also, note that $\mathcal{K}_{abc}$ are topological intersection numbers defined by

$$\mathcal{K}_{abc} = \int_{\mathbf{X}_6} \omega_a \wedge \omega_b \wedge \omega_c \,. \tag{3.1.13}$$

For completitute we also define the following useful relations

$$\mathcal{K}_a = \int \omega_a \wedge J \wedge J = \mathcal{K}_{abc} v^b v^c \ , \ \mathcal{K}_{ab} = \int \omega_a \wedge \omega_b \wedge J = \mathcal{K}_{abc} v^c \,. \tag{3.1.14}$$

Note that the volume of the compactification manifold in the string frame will be given by

$$\mathcal{V} = \frac{1}{6} \int_{\mathbf{X}_6} J \wedge J \wedge J = \frac{1}{6} \mathcal{K} \,. \tag{3.1.15}$$

Complex structure moduli These moduli come from deformations of purely holomorphic or purely antiholomorphic components of the metric. They are related with harmonic (2,1)-forms χ_K[4] and described by a set of $h^{2,1}$ complex scalar fields z^K

$$\delta_{\bar{i}\bar{j}} = \frac{-1}{|\Omega|^2} \bar{\Omega}^{kl}_{\bar{i}} \left(\chi_K\right)_{kl\bar{j}} z^K \,. \tag{3.1.16}$$

The forms χ_K constitute a basis of (2,1)-forms which are related to the variation of the three-form Ω via Kodaira's formula

$$\chi_K = \partial_{z^K} \Omega\left(z\right) + \Omega\left(z\right) \partial_{z^K} K^{\text{cs}} \,. \tag{3.1.17}$$

The metric of the complex structure moduli space is defined by

$$G_{K\bar{L}} = -\frac{\int_Y \chi_K \wedge \bar{\chi}_L}{\int_Y \Omega \wedge \bar{\Omega}} \,, \tag{3.1.18}$$

and thus, we see that the holomorphic three-form Ω could be expanded in a real and symplectic basis of H^3, $\left(\alpha_K, \beta^L\right)$

$$\Omega = Z^K \alpha_K - \mathcal{F}_L \beta^L \,, \tag{3.1.19}$$

where

$$\int_{\mathbf{X}_6} \alpha_K \wedge \beta^L = \delta^L_K \ , \ \int_{\mathbf{X}_6} \alpha_K \wedge \alpha_L = 0 = \int_{\mathbf{X}_6} \beta^K \wedge \beta^L \,. \tag{3.1.20}$$

One can show that $G_{K\bar{L}}$ is a special Kähler metric determined by the periods of Ω

$$G_{K\bar{L}} = \partial_{z^K} \partial_{\bar{z}^L} K^{\text{cs}} \ , \ K^{\text{cs}} = -\log\left(i \int \Omega \wedge \bar{\Omega}\right) = -\log i \left(\bar{Z}^{\hat{K}} \mathcal{F}_{\hat{K}} - Z^{\hat{K}} \bar{\mathcal{F}}_{\hat{K}}\right) = -\log\left(i \Pi^T \Sigma \Pi\right) \ , \ , \tag{3.1.21}$$

and the holomorphic periods $Z^{\hat{K}}, \mathcal{F}_K$ are defined as

$$Z^{\hat{K}}\left(z\right) = \int_Y \Omega(z) \wedge \beta^{\hat{K}} \ , \ \mathcal{F}_{\hat{K}}(z) = \int_Y \Omega(z) \wedge \alpha_{\hat{K}} \,, \tag{3.1.22}$$

[4]At first sight, the moduli arising from these deformations should correspond to (2,0)-forms. But, since $h^{2,0} = 0$ in a Calabi-Yau, they are related with (2,1)-forms which are in one-to-one correspondence with $H^{2,0}$ via the holomorphic three-form Ω using Kodaira's formula.

where $\mathcal{F}_{\hat{K}}$ is the first derivative with respect $Z^{\hat{K}}$ of the prepotential $\mathcal{F} = \frac{1}{2} Z^{\hat{K}} \mathcal{F}_{\hat{K}}$. Note that in (3.1.21) we have introduced the so-called period vector defined as

$$\Pi = \begin{pmatrix} \mathcal{F}_0 \\ \vdots \\ \mathcal{F}_{h^{2,1}} \\ Z^0 \\ \vdots \\ Z^{h^{2,1}} \end{pmatrix} , \tag{3.1.23}$$

and defining Σ as the symplectic matrix

$$\Sigma = \begin{pmatrix} 0 & \mathbf{1_3} \\ -\mathbf{1_3} & 0 \end{pmatrix} .. \tag{3.1.24}$$

In this form, the invariance of the Kähler potential (3.1.21) under $\mathrm{Sp}(2\,(h^{2,1}+1)\,, \mathbb{Z})$ transformations of the periods is manifest.

On the other hand, one could see that Ω is only defined up to complex rescaling by a holomorphic function $e^{-h(z)}$ which via (3.1.21) also changes the Kähler potential by a Kähler transformation

$$\Omega \to \Omega e^{-h(z)}\ , \ K^{\mathrm{cx}} \to K^{\mathrm{cx}} + h + \bar{h}\,. \tag{3.1.25}$$

This symmetry allows us to choose a Kähler gauge where $Z^0 = 1$. The complex strucuture deformations can thus be identified with the remaining $h^{1,2}$ periods Z^K by defining the special coordinates $z^K = \frac{Z^K}{Z^0}$.

In practice, one way to compute the periods (3.1.22) in terms of the complex structure moduli, z^K, is to solve a system of coupled partial differential equations called Picard-Fuchs equations. These arise from the relations among the derivatives of Ω with respect to the complex structure moduli, due to the fact that the dimension of the third cohomology group of $\mathcal{M}$ is finite.

As in the case of the Kähler moduli, the parameters z^K span a special Kähler manifold $\mathcal{M}^{\mathrm{cs}}$ called the complex structure moduli space which is a subset of the quaternionic moduli space $\mathcal{M}^{Q}_{h_{1,2}}$. At tree-level, the total moduli space $\mathcal{M}$ factorizes and takes the form of a direct product

$$\mathcal{M} = \mathcal{M}^{cs}_{h_{1,2}} \times \mathcal{M}^{K}_{h_{1,1}}\,. \tag{3.1.26}$$

We finally stress that these metric deformations which give rise to moduli, are then seen in the four dimensional effective theory as massless scalar fields. Giving them a mass via the generation of a scalar potential for these fields, corresponds to fixing the size and the shape of the Calabi-Yau three-fold and this task is what we call moduli stabilization.

After studying the geometrical moduli appearing in type II compactifications on Calabi-Yau manifolds, it is worthy to mention again mirror symmetry (3.1.6). It is straightforward to see that this symmetry exchanges Kähler with complex structure moduli, as well as their complexified moduli spaces. So in this case the following mirror symmetry manifests itself has the famous T-duality [87], which relates type IIA with type IIB in a mirror symmetric background. In other words, the following equivalence holds:

$$\text{type IIA } \mathbb{R}^{1,3} \times \mathbf{X}_6 \equiv \text{ type IIB } \mathbb{R}^{1,3} \times \tilde{\mathbf{X}}_6 \,. \tag{3.1.27}$$

3.2 $\mathcal{N} = 2$ type II compactifications

We have reviewed the basics about Calabi-Yau compactifications, focusing on the geometrical moduli space. Now we will describe briefly type II string theories compactificacions on Calabi-Yau threefolds $\mathbf{X}_6$ with the ansatz (3.1.7).

After performing the dimensional reduction, we will obtain a $\mathcal{N} = 2$ four-dimensional effective field theory. In the following we will consider only the bosonic massless spectrum with a UV cutoff in our theory given by the string scale, M_s. This spectrum will consist on two sectors: Neveu-Schwarz/Neveu-Schwarz (NSNS) and Ramond/Ramond (RR).

3.2.1 Type IIA compactified on Calabi-Yau three-folds

First of all, we consider the ten-dimensional type IIA supergravity action in the Einstein frame

$$S^{10}_{\text{IIA}} = \int -\frac{1}{2}\hat{R}*\mathbf{1} - \frac{1}{4}d\hat{\phi}\wedge *d\hat{\phi} - \frac{1}{4}e^{-\hat{\phi}}\hat{H}_3\wedge *\hat{H}_3 - \frac{1}{2}e^{\frac{3}{2}\hat{\phi}}\hat{F}_2\wedge *\hat{F}_2 - \frac{1}{2}e^{\frac{1}{2}\hat{\phi}}\hat{F}_4\wedge *\hat{F}_4 + \mathcal{L}_{\text{top}}\,, \tag{3.2.1}$$

where

$$\mathcal{L}_{\text{top}} = -\frac{1}{2}\left[\hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - \left(\hat{B}_2\right)^2 \wedge \hat{d}C_3 \wedge d\hat{A}_1\right] \,. \tag{3.2.2}$$

The field strengths are defined by

$$\hat{H}_3 = -d\hat{B}_2 \;,\; \hat{F}_2 = d\hat{A}_1 \;,\; \hat{F}_4 = d\hat{C}_3 - \hat{A}_1 \wedge \hat{H}_3 \,. \tag{3.2.3}$$

The NSNS sector is given by the dilaton $\hat{\phi}$, the ten-dimensional metric $\hat{g}$ and a two-form $\hat{B}_2$. On the other hand, in this case, the RR sector will be described by $\hat{A}_1$ and $\hat{C}_3$. The ten-dimensional dilaton is defined by

$$e^D = e^{\phi}\left(\mathcal{K}/6\right)^{-\frac{1}{2}} \,. \tag{3.2.4}$$

As we have previously seen the massless spectrum in the compactified theory will be related with the harmonic forms in the Calabi-Yau threefold. Thus, expanding the gauge potentials (3.2.3) in terms of harmonic forms we see that each sector is reduced in the following form

NSNS sector

$$\hat{B}_2 = B_2 + b^a\omega_a \ , \ A = 1, \dots, h^{1,1} \, , \tag{3.2.5}$$

where $\hat{b}^a$ are four dimensional scalars and B_2 is a two-form[5].

RR sector

$$\begin{aligned} \hat{A}_1 &= A^0 \, , & (3.2.6)\\ \hat{C}_3 &= A^a \wedge \omega_a + \xi^{\hat{K}}\alpha_{\hat{K}} - \tilde{\xi}_{\hat{K}}\,(x)\,\beta^{\hat{K}} \ , \ \hat{K} = 0, \dots, h^{2,1} \, , & (3.2.7) \end{aligned}$$

where $\xi^{\hat{K}}, \tilde{\xi}_{\hat{K}}$ are four-dimensional scalars and A^0, A^a are one forms. As we have seen before the harmonic forms ω_a form a basis of $H^{1,1}(Y)$ on the internal manifold while the $\left(\alpha_{\hat{K}}, \beta^{\hat{K}}\right)$ form a real symplectic basis of $H^3(Y)$.

These massless modes are completed by the ones coming from deformations of the Calabi-Yau metric. All these fields assemble into $\mathcal{N}=2$ multiplets which are given in the following table

Multiplet	Number	Bosonic content
Gravity	1	$(g_{\mu\nu}, A^0)$
Vector	$h^{1,1}$	(A^a, v^a, b^a)
Hyper-	$h^{2,1}$	$\left(z^K, \xi^K, \tilde{\xi}_K\right)$
Tensor	1	$\left(B_2, \phi, \xi^0, \tilde{\xi}_0\right)$

Table 3.1: $\mathcal{N}=2$ four-dimensional supergravity multiplets in type IIA compactifications on Calabi-Yau threefolds

The next step, is to obtain the four-dimensional effective action. Plugging the field expansion obtained (3.2.3), (3.2.5) and (3.2.7) into the ten-dimensional action and performing the dimensional reduction we obtain

$$S^4_{\mathrm{IIA}} = \int -\frac{1}{2}R{*}\mathbf{1} + \frac{1}{2}\mathrm{Im}\mathcal{N}_{\hat{A}\hat{B}}F^{\hat{A}}\wedge{*}F^{\hat{B}} + \frac{1}{2}\mathrm{Re}\mathcal{N}_{\hat{A}\hat{B}}F^{\hat{A}}\wedge F^{\hat{B}} - G_{A\bar{B}}dt^A\wedge{*}d\bar{t}^{\bar{B}} - h_{uv}d\tilde{q}^u\wedge{*}d\tilde{q}^v \, , \tag{3.2.8}$$

where $F^A = dA^A$ and $\mathcal{N}_{\hat{A}\hat{B}}$ is the gauge-kinetic coupling matrix. Note that $G_{A\bar{B}}$ is the metric defined by the Kähler moduli (3.1.11). Also, we denote h_{uv} as the quaternionic metric which encodes the couplings of the hypermultiplet sector. Analyzing $h_{uv}d\tilde{q}^u \wedge *d\tilde{q}^v$ one could see that the kinetic terms for the complex structure moduli z^K are given by the metric $G_{K\bar{L}}$ which is the one obtained for the complex structure moduli space (3.1.21).

We see that the $\mathcal{N}=2$ moduli space could be written as a factorization $\mathcal{M} = \mathcal{M}^K \times \mathcal{M}^Q$ where $\mathcal{M}^K$ is a special Kähler manifold spanned by the scalars in the vector multiplets v^a and b^a, which are complexified following (3.1.10). In

[5] Note that with our conventions $\hat{B}_2$ denote a ten-dimensional two-form while B_2 for a four-dimensional two-form. This convention will apply on all the text.

the other hand $\mathcal{M}^Q$ is spanned by the scalars in the hypermultiplet sector. This manifold has a special Kähler submanifold, $\mathcal{M}^{\rm cs}$, spanned by the complex structure moduli z^K and thus it can be written as

$$\mathcal{M} = \mathcal{M}^K \times \mathcal{M}^{\rm cs} \,. \tag{3.2.9}$$

3.2.2 Type IIB compactified on Calabi-Yau three-folds

As we have done before, here we will describe briefly the $\mathcal{N}=2$ four-dimensional low-energy effective field theory obtained from dimensional reduction of type IIB string theory. In this case the ten-dimensional supergravity action for type IIB in the Einstein frame is given by

$$\begin{aligned} S^{10}_{\rm IIB} &= -\int \frac{1}{2}R*1 + \frac{1}{4}d\hat{\phi}\wedge *d\hat{\phi} + \frac{1}{4}e^{-\hat{\phi}}\hat{H}_3\wedge *\hat{H}_3 & (3.2.10)\\ &- \frac{1}{4}\int e^{2\hat{\phi}}d\hat{F}_1\wedge *d\hat{F}_1 + e^{\hat{\phi}}d\hat{F}_3\wedge *d\hat{F}_3 + \frac{1}{2}d\hat{F}_5\wedge *d\hat{F}_5 + \mathcal{L}_{\rm top}\,, & (3.2.11)\end{aligned}$$

where

$$\mathcal{L}_{\rm top} = -\frac{1}{4}\int \hat{C}_4\wedge\hat{H}_3\wedge\hat{F}_3\,. \tag{3.2.12}$$

The self-duality condition $\hat{F}_5 = *\hat{F}_5$ is imposed at the level of equations of motion. The field strengths are defined as

$$\hat{H}_3 = d\hat{B}_2\ ,\ \hat{F}_1 = d\hat{C}_0\ ,\ \hat{F}_3 = d\hat{C}_2 - \hat{l}d\hat{B}_2\ ,\ \hat{F}_5 = d\hat{C}_4 - \frac{1}{2}d\hat{B}_2\wedge\hat{C}_2 + \frac{1}{2}\hat{B}_2\wedge d\hat{C}_2\,. \tag{3.2.13}$$

As in type IIA the NSNS sector will be given by the dilaton $\hat{\phi}$, the ten-dimensional metric $\hat{g}$ and a two-form $\hat{B}_2$. The RR sector, in this case will be given by the axion, $\hat{C}_0$, a two-form $\hat{C}_2$ and a four-form $\hat{C}_4$. Following the same steps as before, we will compute the massless spectrum coming from the RR and NSNS sector. To do that first of all we expand the gauge potentials (3.2.13) into harmonic forms in the Calabi-Yau

NSNS sector

$$\hat{B}_2 = B_2 + b^a\omega_a\,. \tag{3.2.14}$$

RR sector

$$\begin{aligned}\hat{C}_2 &= C_2 + c^a\omega_a\,, & (3.2.15)\\ \hat{C}_4 &= D_2^a\wedge\omega_a + V^{\hat{K}}\wedge\alpha_{\hat{K}} - U_{\hat{K}}\wedge\beta^{\hat{a}} + \rho_a\tilde{\omega}^a\,, & (3.2.16)\end{aligned}$$

where $a = 1,\dots,h^{1,1}$ and $\hat{K} = 0,\dots,h^{1,2}$. As before ω_a is a basis of (1,1)-forms of the cohomolgy group $H^{1,1}$ of the three-fold, $\left(\alpha_{\hat{K}},\beta^{\hat{K}}\right)$ is a real symplectic basis of H^3 and $\tilde{\omega}_A$ is the basis of $H^{2,2}$. Note that, in the former expansions b^a, c^a and ρ_a

are scalars, $V^{\hat{K}}$ and $U_{\hat{K}}$ are one-forms and B_2, C_2 and D_2^a are two-forms in the four-dimensional theory. We would be able to eliminate D_2^a and $U_{\hat{K}}$ in favor of ρ_A and $V^{\hat{K}}$ since the self-duality condition of $\hat{F}_5$ allows us to eliminate half of the degrees of freedom of $\hat{C}_4$. Finally, $\hat{\phi}$ and $\hat{C}_0$, which are scalars in ten dimensions, also appear as scalars in $d = 4$. Completing the massless modes with the ones coming from metric deformations of the manifold we can assemble all these fields into $\mathcal{N} = 2$ multiplets which are given in the following table

Multiplet	Number	Bosonic content
Gravity	1	$(g_{\mu\nu,V^0})$
Vector	$h^{1,2}$	$\left(V^K, z^K\right)$
Hyper-	$h^{1,1}$	(v^a, b^a, c^a, ρ_a)
Tensor	1	(B_2, C_2, ϕ, C_0)

Table 3.2: $\mathcal{N} = 2$ four-dimensional supergravity multiplets in type IIB compactifications on Calabi-Yau threefolds

The next step, is to obtain the four-dimensional effective action. Plugging the field expansion obtained (3.2.13), (3.2.14) and (3.2.16) into the ten-dimensional action and computing the dimensional reduction gives us

$$S^4_{IIB} = \int -\frac{1}{2}R{*}\mathbf{1}+\frac{1}{4}\mathrm{Re}\mathcal{M}_{\hat{K}\hat{L}}F^{\hat{K}}\wedge F^{\hat{L}}+\frac{1}{4}\mathrm{Im}\mathcal{M}_{\hat{K}\hat{L}}F^{\hat{K}}\wedge{*}F^{\hat{L}}-G_{K\bar{L}}dz^K\wedge{*}d\bar{z}^{\bar{L}}-h_{\hat{A}\hat{B}}dq^{\hat{A}}\wedge dq^{\hat{B}}\,, \tag{3.2.17}$$

where $\mathcal{M}_{\hat{K}\hat{L}}$ is the gauge-kinetic matrix and is related to the metric on $H^3(Y)$ in terms of the periods of the holomorphic three-form. Note that $G_{K\bar{L}}$ is the metric defined by the complex structure moduli (3.1.21). As before, we denote $h_{\hat{A}\hat{B}}$ as the quaternionic metric which encodes the couplings of the hypermultiplet sector. Finally, we can see that the moduli space could be obtained as a factorization between the special Kähler manifold spanned by the complex structure moduli $\mathcal{M}^{\mathrm{cs}}$ and the one spanned by the scalars $q^{\hat{A}}$ in the hypermultiplets, $\mathcal{M}^Q$,

$$\mathcal{M} = \mathcal{M}^{\mathrm{cs}} \times \mathcal{M}^Q\,. \tag{3.2.18}$$

3.3 The closed-string sector in type II orientifolds

We have seen that type II string theory compactifications on Calabi-Yau three-folds gives an $\mathcal{N} = 2$ four-dimensional effective field theory. We will see that the amount of supersymmetry preserved by the compactification could be reduced to $\mathcal{N} = 1$ through orientifold action, $\mathcal{O}$, since it projects out a subset of the original $\mathcal{N} = 2$ fields. The orientifold action is a discrete symmetry which includes: worldsheet parity Ω_p, spacetime fermion number $(-1)^{F_L}$ in the left-moving sector and σ, which is an involutive symmetry of $\mathbf{X}_6$, which satisfies $\sigma^2 = 1$ and which acts trivially on the Minkowski spacetime.

$$\mathcal{O} = \Omega_p \, (-1)^{F_L} \, \sigma \,. \tag{3.3.1}$$

In order to preserve $\mathcal{N} = 1$ in the case of type IIA, σ has to be antiholomorphic and in type IIB has to be holomorphic. Due to the fact that the four-dimensional Minkowski spacetime is left invariant by σ, it means that the orientifold has to be space-filling. This means that, in the case of type IIA they have to be odd dimensional, and thus it will select O6-planes. In type IIB it has to be even dimensional and will select O3-,O5-,O7- and O9-planes. Note that the dimension of the orientifold plane is determined by the dimensionality of the fixed point set of σ in $\mathbf{X}_6$. Finally in the case of type IIB the combinations of different O-planes are fixed due to its action on the three-form Ω. Thus, we can classify the different types of type II orientifolds as

Type IIA with O6-planes	$\sigma^* J = -J$	$\sigma^*\Omega_3 = e^{2i\theta}\bar{\Omega}_3$
Type IIB with O3/O7-planes	$\sigma^* J = J$	$\sigma^*\Omega_3 = -\Omega_3$
Type IIB with O5/O9-planes	$\sigma^* J = J$	$\sigma^*\Omega_3 = \Omega_3$

Table 3.3: Summary of type II orientifolds

Finally, we will obtain the massless bosonic spectrum in each theory. As we know, the spectrum is related with the harmonic forms of the Calabi-Yau. After performing the orientifold projection we will see that the space of harmonic forms will split into even and odd eigenspaces of σ^*

$$H^{p,q}(Y) = H^{p,q}_+ \oplus H^{p,q}_- \,. \tag{3.3.2}$$

The $\mathcal{O}$-invariant states will be either in H^p_+ or in H^p_- and thus we see naively that the total number of states obtained in the $\mathcal{N} = 2$ theory will be reduced after performing the orientifold action.

3.3.1 Type IIA orientifolds

In this section we will obtain the $\mathcal{N} = 1$ low-energy effective field theory action of type IIA compactifications. To do so, first of all we will need to define the appropriate chiral field variables. As we have anticipated before, in type IIA the involution, σ has to be antiholomorphic in order to preserve $\mathcal{N} = 1$ supersymmetry. Using this, its action over the three-form Ω will be constrained due to the relation between the Kähler form and Ω and will satisfy

$$\sigma^* J = -J \,, \tag{3.3.3}$$
$$\sigma^* \Omega = e^{2i\theta}\bar{\Omega} \,. \tag{3.3.4}$$

In this case the fixed point set σ on the internal components will be three-cycles, where the O6-plane will be wrapped. These will be special Lagrangian (sLag) three-cycles which we denote as $\hat{\Pi}_3$. We see that from (3.3.3) and (3.3.4)

$$J|_{\hat{\Pi}_3} = 0 \ , \ \mathrm{Im}\left(e^{-i\theta}\Omega\right)|_{\hat{\Pi}_3} = 0 \,, \tag{3.3.5}$$

and thus obtaining straightforwardly the calibration condition

$$\mathrm{vol}\left(\hat{\Pi}_3\right) \sim \int_{\hat{\Pi}_3} \mathrm{Re}\left(e^{-i\theta}\Omega\right) . \tag{3.3.6}$$

In the following, we will determine the $\mathcal{O}$-invariant states. To do so, we will need the transformations under worldsheet parity and left-moving fermion number. We will summarize it in the following table

	$(-1)^{F_L}$	Ω_p
$\hat{B}_2$	+	-
$\hat{g}$	+	+
$\hat{\phi}$	+	+
$\hat{A}_1$	-	+
$\hat{C}_3$	-	-

Table 3.4: Summary of transformations under $(-1)^{F_L}$ and Ω_p of ten-dimensional type IIA NSNS and RR fields

As a consequence of the former transformations, NSNS and RR fields have to transform under σ in the following way in order to be $\mathcal{O}$-invariant

$$\sigma^*\hat{\phi} = \hat{\phi} \ , \ \sigma^*\hat{g} = \hat{g} \ , \ \sigma^*\hat{B}_2 = -\hat{B}_2 \ , \ \sigma^*\hat{A}_1 = -\hat{A}_1 \ , \ \sigma^*\hat{C}_3 = \hat{C}_3 \, . \tag{3.3.7}$$

Now, we will focus on the splitting of the harmonic forms (3.3.2). In this case, the volume form (3.1.2) is odd and it could be seen that it implies $h^{0,0}_{\pm} = 0$, $h^{3,3}_{+} = 0$ and $h^{3,3}_{-} = 1$. Also, by Hodge duality one can see that $h^{1,1}_{\pm} = h^{2,2}_{\mp}$. Finally, from (3.3.2) one can see that the decomposition of H^3 will show $h^3_+ = h^3_- = h^{2,1} + 1$. This means that for each element $\alpha_{\hat{K}} \in H^3_+$ there is a dual element $\beta^{\hat{L}} \in H^3_-$ with the intersections

$$\int \alpha_{\hat{K}} \wedge \beta^{\hat{L}} = \delta^{\hat{L}}_{\hat{K}} \ , \ \hat{K}, \hat{L} = 0, \ldots h^{2,1} \, . \tag{3.3.8}$$

This fact is pointing us that the orientifold projection is breaking the symplectic invariance. Thus $\left(\alpha_{\hat{K}}, \beta^{\hat{L}}\right)$ is one possible choice among others. Computations in the most general case are reviewed in [88]. The prepotential, and thus, the Kähler potential will depend on the choice of the symplectic basis. It is possible to define a generic basis where we assume that

$$\begin{aligned} h^3_+ &= h^{2,1} + 1 \text{ basis elements } \left(a_k, b^\lambda\right) \text{ span } H^3_+ \qquad (3.3.9) \\ h^3_- &= h^{2,1} + 1 \text{ basis elements } \left(a_\lambda, b^k\right) \text{ span } H^3_- \, . \qquad (3.3.10) \end{aligned}$$

After the orientifold projection the total number of complex structure is $h^{2,1} + 1$, so in order to work in full generality we choose the symplectic basis $\left(\alpha_k, \beta^\lambda\right)$ where $k = 0, \ldots, \tilde{h}$, $\lambda = \tilde{h} + 1, \ldots, h^{2,1}$. This generic choice is telling us how many α's are even.

$$\int_{\mathbf{X}_6} \alpha_k \wedge \beta^l = \delta^l_k \ , \ \int_{\mathbf{X}_6} \alpha_k \wedge \beta^\lambda = \delta^\lambda_k \, . \tag{3.3.11}$$

Now, we will see how the orientifold projection affects to the three-form Ω (3.1.19). From (3.3.4) it is clear to see that the number of complex structure deformations will be reduced. Expanding Ω in the basis of $H^p_+ \oplus H^p_-$ and applying the orientifold condition (3.3.4) one finds that

$$\mathrm{Im}\left(e^{-i\theta}Z^k\right) = 0\ ,\ \mathrm{Re}\left(e^{-i\theta}Z^\lambda\right) = 0\ ,\ \mathrm{Re}\left(e^{-i\theta}\mathcal{F}_k\right) = 0\ ,\ \mathrm{Im}\left(e^{-i\theta}\mathcal{F}_\lambda\right) = 0\,. \tag{3.3.12}$$

We see that that the former expression sets $h^{2,1}+1$ real conditions for the complex scalars, this fact and using the scale invariance of Ω, allows us to project out $h^{2,1}$ complex scalars. One useful convention is to define the so-called "compensator" field

$$C = e^{-D-i\theta}e^{K^{\mathrm{cs}(z)}/2}\ ,\ C \to Ce^{\mathrm{Re}h(z)}\,. \tag{3.3.13}$$

Now we will expand into harmonic forms the RR sector. We see from (3.3.7) that $\hat{A}^1$ is odd, and since a Calabi-Yau manifold does not have harmonic one-forms σ will project it out. On the other hand

$$\hat{C}_3 = c_3 + A^a \wedge \omega_a + C_3\,, \tag{3.3.14}$$

where A^a are $h^{1,1}_+$ one-forms and c_3 is a three-form in four dimensions and thus does not have physical degrees of freedom. Now we will expand C_3 and $C\Omega$ in the real symplectic basis that we have shown before

$$C_3 = \xi^k\alpha_k - \tilde{\xi}_\lambda\beta^\lambda\,, \tag{3.3.15}$$

$$C\Omega = \mathrm{Re}\left(CZ^k\right)\alpha_k + i\mathrm{Im}\left(CZ^\lambda\right)\alpha_\lambda - \mathrm{Re}\left(C\mathcal{F}_\lambda\right)\beta^\lambda - i\mathrm{Im}\left(C\mathcal{F}_k\right)\beta^k \tag{3.3.16}$$

Applying the orientifold constraint one concludes that

$$\mathrm{Im}\left(CZ^k\right) = \mathrm{Re}\left(C\mathcal{F}_k\right) = 0\ ,\ \mathrm{Re}\left(CZ^\lambda\right) = \mathrm{Im}\left(C\mathcal{F}_\lambda\right) = 0\,. \tag{3.3.17}$$

The appropriate complex fields arise form the combination

$$\Omega_c = C_3 + 2i\mathrm{Re}\left(C\Omega\right)\ , \tag{3.3.18}$$

and expanding Ω_c in the basis of H^3_+ we see that

$$\Omega_c = \left(\xi^k + 2i\mathrm{Re}\left(CZ^k\right)\right)\alpha_k + \left(\tilde{\xi}_\lambda + 2i\mathrm{Re}\left(C\mathcal{F}_\lambda\right)\right)\beta^\lambda\,. \tag{3.3.19}$$

The new Kähler coordinates are determined by the periods of Ω_c and given by

$$N^k = \frac{1}{2}\int\Omega_c\wedge\beta^k = \frac{1}{2}\xi^k + i\mathrm{Re}\left(CZ^k\right)\ , \tag{3.3.20}$$

$$T_\lambda = i\int\Omega_c\wedge\alpha_\lambda = i\tilde{\xi}_\lambda - 2\mathrm{Re}\left(C\mathcal{F}_\lambda\right)\ . \tag{3.3.21}$$

Note that, with this at hand, one could define the scale invariant variables

$$l^k = \mathrm{Re}\left(CZ^k\right)\ , \tag{3.3.22}$$

$$l_\lambda = 2\mathrm{Re}\left(C\mathcal{F}_\lambda\right)\ . \tag{3.3.23}$$

The important fact to note here is that the moduli space of the complex structure sector is equipped with a new complex strucutre and the corresponding Kähler coordinates coincide with half of the periods of Ω_c. This contrasts with the situation in $\mathcal{N}=2$ where one of the periods is a gauge degree of freedom and the Kähler coordinates are the special coordinates. The $\mathcal{N}=1$ contraints given by the orientifold destroy this complex structure and force us to combine $\mathrm{Re}\,(C\Omega)$ with the RR three-form C_3 into Ω.

The special Kähler manifold spanned by the complex structure moduli, in analogy with (3.1.21), has the following Kähler potential

$$\begin{aligned} K^Q &= -2\log\left(\int \mathrm{Re}\,(C\Omega)\wedge *(C\Omega)\right) &(3.3.24)\\ &= -2\log\left(\frac{1}{4}\left[\mathrm{Re}\,(C\mathcal{F}_\lambda)\,\mathrm{Im}\left(CZ^\lambda\right)-\mathrm{Re}\left(CZ^k\right)\mathrm{Im}\,(C\mathcal{F}_k)\right]\right)\,. &(3.3.25)\end{aligned}$$

Alternatively, using the expression of the compensator (3.3.13), we see that

$$K^Q = -\log e^{-4D}\,. \tag{3.3.26}$$

Once we have seen under complete generality the chiral variables that define the complex structure moduli, we will focus mostly on the basis for $\tilde{h}=h^{2,1}$. Recall that this means chosing the simplectic basis $\left(\alpha_{\hat{K}},\beta^{\hat{L}}\right)$ and the complex structure variables will be given by $N^{\hat{K}}$ (3.3.20). The kinetic terms of the complex structure moduli are given by

$$2e^{2D}\mathrm{Im}\mathcal{M}_{\hat{K}\hat{L}} = \partial_{N^{\hat{K}}}\partial_{N^{\hat{L}}}K^{\mathrm{cs}}\,, \tag{3.3.27}$$

where in this basis (3.3.25) is written in the following way

$$K^Q = -2\left(-\frac{1}{4}\mathrm{Im}\,(\mathcal{F}_{\hat{K}\hat{L}})\left(N^{\hat{K}}-\bar{N}^{\hat{K}}\right)\left(N^{\hat{L}}-\bar{N}^{\hat{L}}\right)\right)\,, \tag{3.3.28}$$

note that $\mathcal{F}_{\hat{K}\hat{L}}$ is an homogeneous function of degree zero of $N^{\hat{K}}$. Also, K^Q obeys a no-scale type condition

$$K_{N^{\hat{K}}}K^{N^{\hat{K}}\bar{N}^{\hat{L}}}K_{\bar{N}^L} = 4\,. \tag{3.3.29}$$

Now we will focus on the Kähler moduli. From equations (3.3.3) and (3.3.7) we see that both J and $\hat{B}_2$ are odd and hence have to be expanded in a basis ω_a of harmonic (1,1)-forms

$$J = v^a\omega_a\ ,\ \hat{B}_2 = b^a\omega_a\ ,\ a=1,\ldots,h^{1,1}\,. \tag{3.3.30}$$

In the contrary to what we saw for type IIA (3.2.5) the four-dimensional two-firm B_2 gets projected out due to (3.3.7) and the fact that σ acts trivially on the flat dimensions. v^a and b^a are space-time scalars as in $\mathcal{N}=2$ they can be combined into complex coordinates

$$t^a = b^a + iv^a\ ,\ J_c = B_2 + iJ\,, \tag{3.3.31}$$

where we have also introduced the complexified Kähler form J_c. We see that in terms of the field variables, the same complex structure is chosen as in $\mathcal{N}=2$ but the dimension of the Kähler moduli space is truncated from $h^{1,1}$ to $h^{1,1}_{-}$. As we already stressed earlier the metric $G_{a\bar{b}}$ is a trivial truncation of the $\mathcal{N}=2$ special Kähler metric and therefore remains special Kähler. The Kähler potential is given by

$$K^K = -\log\left[\frac{i}{6}\mathcal{K}_{abc}\left(t^a-\bar{t}^a\right)\left(t^b-\bar{t}^b\right)\left(t^c-\bar{t}^c\right)\right]. \tag{3.3.32}$$

Moreover, K^K can be obtained from the prepotential $f(t) = -\frac{1}{6}\mathcal{K}_{abc}t^at^bt^c$. It is well-known that K^K satisfies the standard no-scale condition

$$K_{t^a}K^{t^a\bar{t}^b}K_{\bar{t}^b} = 3\,. \tag{3.3.33}$$

The effective action in $\mathcal{N}=1$ supergravity In $\mathcal{N}=1$ supergravity the action is expressed in terms of a Kähler potential and a superpotential and the holomophic gauge kinetic coupling functions f

$$S^4_{IIA} = -\int \frac{1}{2}R*\mathbf{1} + K_{I\bar{J}}dM^I\wedge *dM^{\bar{J}} + \frac{1}{2}\mathrm{Re}f_{\alpha\beta}F^\alpha\wedge *F^\beta + \frac{1}{2}\mathrm{Im}f_{\alpha\beta}F^\alpha\wedge F^\beta + V*\mathbf{1}\,, \tag{3.3.34}$$

where scalar potential is split into F-term and D-term parts $V = V_{\mathrm{F}} + V_{\mathrm{D}}$

$$V = e^K\left[K^{I\bar{K}}D_IWD_{\bar{J}}\bar{W} - 3\left|W\right|^2\right] + \frac{1}{2}\left(\mathrm{Re}f\right)^{-1\,\alpha\beta}D_\alpha D_\beta\,. \tag{3.3.35}$$

Here M^I collectively denote all complex scalars in the theory. The gauge-kinetic coupling function $f_{\alpha\beta}$ is given by

$$f_{\alpha\beta} = i\mathcal{K}_{\alpha\beta a}t^a\,. \tag{3.3.36}$$

After the orientifold projection, the moduli space still has the product structure

$$\tilde{\mathcal{M}}^K \times \tilde{\mathcal{M}}^Q\,. \tag{3.3.37}$$

The first factor is a subspace of the $\mathcal{N}=2$ moduli space $\mathcal{M}^K$ with dimension $h^{1,1}_{-}$ spanned by the complexified Kähler deformations t^a 3.3.31. The second factor is a subspace of the quaternionic manifold $\mathcal{M}^Q$ with dimension $h^{2,1}+1$ spanned by the hypermultiplet scalars: complex structure deformations z^K, the dilaton D and the scalars ξ^K arising from C_3.

3.3.2 Type IIB orientifolds with O3/O7 planes

We have seen in Table 3.3 that in the case of O3/O7 planes the involution σ acts on the Kähler form and the three-form Ω like

$$\sigma^*\Omega = -\Omega\ ,\ \sigma^*J = J\,. \tag{3.3.38}$$

As we did before, first of all we will determine the $\mathcal{O}$-invariant states. In the following table we will show how the fields in the NSNS and RR sector transform under the left-moving fermion number and the worldsheet parity operators.

	$(-1)^{F_L}$	Ω_p
$\hat{B}_2$	+	-
$\hat{g}$	+	+
$\hat{\phi}$	+	+
$\hat{C}_0$	-	-
$\hat{C}_2$	-	+
$\hat{C}_4$	-	-

Table 3.5: Summary of transformations under $(-1)^{F_L}$ and Ω_p of ten-dimensional type IIB NSNS and RR fields

Then, the invariant states behave under σ^* like

$$\sigma^*\hat{\phi} = \hat{\phi}\ ,\ \sigma^*\hat{g} = \hat{g}\ ,\ \sigma^*\hat{B}_2 = -\hat{B}_2\ ,\ \sigma^*\hat{C}_0 = \hat{C}_0\ ,\ \sigma^*\hat{C}_2 = -\hat{C}_2\ ,\ \sigma^*\hat{C}_4 = \hat{C}_4\,. \tag{3.3.39}$$

Now, we will focus on the expansion into harmonic forms (3.3.2). In this case we see that, due to (3.3.38) we obtain $h^{3,0}_+ = 0 = h^{0,3}_+$ while $h^{3,0}_- = 1 = h^{0,3}_-$. Since σ is holomorphic $h^{2,1}_\pm = h^{1,2}_\pm$. Also since the volume form (3.1.2) is even under σ^* we see that $h^{3,3}_- = 0 = h^{0,0}_-$ and $h^{3,3}_+ = 1 = h^{0,0}_+$. Finally, since σ preserves the orientation and the metric we see that $h^{1,1}_\pm = h^{2,2}_\pm$. Expanding the NSNS and RR sector into harmonic forms we see that

$$\hat{B}_2 = b^a\omega_a\ ,\ \hat{C}_2 = c^a\omega_a\ ,\ a = 1,\ldots,h^{1,1}_-\,, \tag{3.3.40}$$

$$\hat{C}_4 = D^A_2\wedge\omega_A + V^{\hat{K}}\wedge\alpha_{\hat{K}} + U_{\hat{K}}\wedge\beta^{\hat{K}} + \rho_a\tilde{\omega}^a\ ,\ \hat{K} = 1,\ldots,h^{1,2}_+\,, \tag{3.3.41}$$

where ω_a is the basis of $H^{1,1}_-$ and $\tilde{\omega}^a$ is its dual. In the other hand $\left(\alpha_{\hat{K}},\beta^{\hat{K}}\right)$ is a real and symplectic basis of H^3. As happened in Section 3.2.2 due to the self-duality of $\hat{F}_5$ we eliminate half of the degrees of freedom of $\hat{C}_4$. Now we will add the geometrical moduli and try to write it in terms of chiral multiplets.

Focusing on the complex structure moduli, we see that, in this case, the orientifold projection only makes survive $h^{2,1}_-$ complex structure deformations, $z^{\hat{K}}$. Thus the three-form Ω is expanded in the basis of H^3_-

$$\Omega = Z^{\hat{K}}\alpha_{\hat{K}} - \mathcal{F}_{\hat{K}}\beta^{\hat{K}}\ ,\ k = 0,\ldots,h^{2,1}_-\,. \tag{3.3.42}$$

The complex structure deformations $z^{\hat{K}}$ will still define a special Kähler manifold and the field space metric and Kähler potential will be given by (3.1.21).

The remaining fields will not be good Kähler coordinates. In order to overcome this problem, conventionally one defines the following fields

$$\tau = C_0 + ie^{-\phi}\ ,\ G^a = c^a - \tau b^a\,, \tag{3.3.43}$$

and

$$T_a = \frac{3i}{2}\rho_a + \frac{3}{4}\mathcal{K}_a + \frac{3i}{4\left(\tau-\bar{\tau}\right)}\mathcal{K}_{abc}G^b\left(G-\bar{G}\right)^c\,. \tag{3.3.44}$$

In terms of this coordinates we are able to write for the Kähler moduli and the dilaton the following Kähler potential

$$K^K = -\log\left[-i\left(\tau-\bar{\tau}\right)\right] - 2\log\left[\frac{1}{6}\mathcal{K}\left(\tau, T, G\right)\right], \tag{3.3.45}$$

where $\mathcal{K}$ in terms of v^A is given (3.1.12). In order to be written in the proper Kähler coordinates one should solve v^A in (3.3.44) in terms of τ, T and G

Finally, for completitute, we will give the Kähler coordinates for the Kähler moduli in compactifications where $h_-^{1,1} = 0$. In this case $G^a = 0$ and thus, we are able to write T in terms of the $\mathcal{N}=2$ Kähler coordinates

$$T_\alpha = b_\alpha + i\tau_\alpha\,, \tag{3.3.46}$$

where

$$\tau_a = \frac{3}{4}\mathcal{K}_a = \frac{1}{2}\mathcal{K}_{abc}v^b v^c\,. \tag{3.3.47}$$

And thus, in this case, the Kähler potential for the complex structure moduli is a function of $\left(T-\bar{T}\right)$.

With the former redefinitions we are able to write the low-energy effective action as

$$S^4_{IIB} = -\int \frac{1}{2}R*\mathbf{1} + K_{I\bar{J}}dM^I\wedge *dM^{\bar{J}} + \frac{1}{2}\mathrm{Re}f_{\alpha\beta}F^\alpha\wedge *F^\beta + \frac{1}{2}\mathrm{Im}f_{\alpha\beta}F^\alpha\wedge F^\beta + V*\mathbf{1}\,, \tag{3.3.48}$$

where the scalar potential is given by the well-known expression (3.3.35). Note that as in the former case we are able to write the moduli space as

$$\tilde{\mathcal{M}}^K \times \tilde{\mathcal{M}}^{\mathrm{cs}}\,. \tag{3.3.49}$$

3.4 Flux Compactifications and Moduli Stabilization

As we have seen in the former section, all scalar components of the chiral multiplets obtained in type II orientifold compactifications have a flat potential. This is because the superpotential at tree level is vanishing. In order to generate a superpotential for some of them, we will turn on background fluxes. These will be defined as the integral of the field strength F_p over a p-cycle and will be quantized due to Dirac quantization

$$\frac{1}{2\pi\alpha'}\int_{\Pi_n^i} F_p = n_i \in \mathbb{Z}\,. \tag{3.4.1}$$

These background fluxes have to be constant since $dF_p = 0 = d^\dagger F_p$. In general the presence of background fluxes will spontaneously break $\mathcal{N}=1$ supersymmetry. They also could be written in terms of harmonic forms ω_p^i which are Poincaré dual to the corresponding p-cycles. This means that we are able to write the field strengths as $F_p = n^i \omega_p^i$. Strickly speaking, due to the presence of orientifold planes and background fluxes we should add D-branes in order to cancel RR tadpoles. For simplicity, in this section we will consider these necessary D-branes stabilized on the top of the orientifold planes. In the following sections we will add D6 or D7-branes which will be taken into account in the $\mathcal{N}=1$ description.

Tadpole Condition The Bianchi identities for NS and RR fluxes are

$$dH = 0 \ , \ d\hat{F} - H \wedge \hat{F} = 0 \, , \tag{3.4.2}$$

and using Hodge duality we see that

$$d\left(*\hat{F}_p\right) + H \wedge *\hat{F}_{p+2} = 0 \, . \tag{3.4.3}$$

In order to preserve $\mathcal{N}=1$ supersymmetry we see that the former condition turns into

$$\left(d - H\wedge\right) F = 0 \ , \ \left(d - H\wedge\right)\left(e^A * F\right) = 0 \, . \tag{3.4.4}$$

This leads to the so-called no-go theorem, which states that Bianchi identities cannot be satisfied in setups were only background fluxes are turned on. This problem is solved by adding localized sources to (3.4.3) and thus obtaining

$$d\left(*\hat{F}_p\right) = H \wedge *\hat{F}_{p+2} + \left(2\pi\sqrt{\alpha'}\right)^{n-1} \rho_{8-n}^{\mathrm{loc}} \, , \tag{3.4.5}$$

where $\rho_{8-n}^{\mathrm{loc}}$ takes into account localized sources. We will derive the concrete expression (3.4.5) for type IIA and type IIB orientifolds.

We will see that, at tree-level, in the case of type IIA we will be able to generate a scalar potential for all the $\mathcal{N}=1$ moduli, while in type IIB we will be able only to generate potential for the complex structure moduli and the axio-dilaton. The Kähler moduli in this case will not have a scalar potential due to the no-scale structure. Afterwards we will discuss briefly how to stabilize moduli in both cases, where in type IIB, in general non-perturbative effects will be needed in order to stabilize the Kähler moduli.

3.4.1 Type IIA flux compactifications

In this case we turn on background fluxes of the NSNS and RR field strengths consistent with the orientifold projection as we have seen in Section 3.3.1

$$H_3 = q^\lambda \alpha_\lambda - p_k \beta^k \ , \ F_2 = -m_a \omega_a \ , \ F^4 = e_a \tilde{\omega}^a \ , \ F_0 = m_0 \, . \tag{3.4.6}$$

Note that m_0 is the mass parameter of massive type IIA. Also we will have $e_0 = \int F_6$. Note that $\left(q^\lambda, p_k\right)$ are $h^{2,1}+1$ real NS flux parameters and (e_a, m^a) are $2h_-^{1,1}$ real RR flux parameters This fluxes will source the Gukov-Vafa-Witten superpotential

$$W = W^Q\left(N, T\right) + W^K\left(T\right) \, , \tag{3.4.7}$$

where

$$
\begin{aligned}
W^Q &= \int_Y \Omega_c \wedge H_3 = -2N^k p_k - iT_\lambda q^\lambda \,, && (3.4.8)\\
W^K &= e_0 + \int_Y J_c \wedge F_4 - \frac{1}{2}\int_Y J_c \wedge J_c \wedge F_2 - \frac{1}{6} m_0 \int_Y J_c \wedge J_c \wedge J_c && (3.4.9)\\
&= e_0 + e_a t^a + \frac{1}{2}\mathcal{K}_{abc} m^a t^b t^c - \frac{1}{6} m^0 \mathcal{K}_{abc} t^a t^b t^c \,. && (3.4.10)
\end{aligned}
$$

This case is the contrary to what happens in type IIB (3.4.32). In type IIA, both types of moduli, Kähler and complex structure deformations appear in the superpotential suggesing the posibility that all moduli can be fixed in this setup.

Regarding tadpole cancellation, we see that the localized contributions to the tadpoles are O6 planes and D4-, D6- and D8-branes. Since there are not non-trivial one- and five-cycles in a Calabi-Yau D4- and D8-branes will not contribute to the tadpole. Thus the only contributions to the tadpole condition (3.4.5) are O6-planes and D6-branes which are electric sources of $\hat{F}_8$ and magnetic of $\hat{F}_2$. Thus, tadpole cancellation implies

$$
N_{D6} - 2N_{O6} + \frac{F_0}{2\pi\sqrt{\alpha'}} \int H_3 = 0 \,, \qquad (3.4.11)
$$

where N_{D6} and N_{O6} are the number of D6-branes and O6-planes respectively. Note that in general the charge of an Op-plane an a Dp-brane are related by $Q_{Op} = -2^{p-5} Q_{Dp}$.

As we can see, absent background fluxes, the RR charge induced by O6-planes must be cancelled by the presence of space-time filling D-branes. The simplest possibility[6] is to consider K stacks of D6-branes such that N_a D6-branes wrap the three-cycle Π_3^a and thus we can rewrite the former expression as

$$
\sum_{a=1}^{K} N_a [\Pi_3^a] \;=\; 4[\Pi_3^{O6}] \,, \qquad (3.4.12)
$$

which shoud be satisfied. Here Π_3^{O6} stands for the fixed point set of the isometric involution, σ, and the brackets denote the homology class of each three-cycle. By construction the whole set of D6-branes must be invariant under the orientifold action, so if Π_3^b is not left invariant by the action of σ there must be N_b D6-branes wrapping the three-cycle $\Pi_3^{b'} = \sigma(\Pi_3^b)$, with the index a in (3.4.12) running over both stacks of branes.

Moduli Stabilization

As we have seen, background fluxes will be able to generate a scalar potential for all moduli. With this at hand we will be able to stabilize all moduli in general, without the introduction of non-perturbative effects. Stabilizing moduli means finding a minimum for all the scalar fields that arise from the compactification $\partial_{n^i} V = 0$.

[6]See [89] for type IIA models which cancel tadpoles by also including coisotropic D8-branes.

Typically this condition will lead to non-supersymmetric vacuum configurations. One subset of this landscape corresponds to the vanishing of the F-term conditions $D_{N^i}W = 0$, this kind of solutions will give us, in general, supersymmetric AdS vacuum state as long as $W_0 \neq 0$.[7] We will discuss two ways to stabilize moduli in type IIA. They will differ, crucially, on the amount of H_3 flux, which stabilizes the complex structure moduli, turned on.

One of the main drawbacks of type IIA moduli stabilization is the absence of an F-term uplifting mechanism and thus, obtaining deSitter vacua is a difficult task. For issues about this kind of uplifting mechanisms in type IIA see [90].

Turning on all H_3 flux

In this section we will see a method to stabilize all Kähler and complex structure moduli in type IIA orientifold flux compactification. This method [91] relies on the fact that all H_3 fluxes (3.4.6) should be turned on. With this mechanism all Kähler moduli will be stabilized. Regarding the complex structure sector, all the saxionic components will be stabilized but only a linear combination of axions will be stabilized.

First of all we will focus on complex structure moduli. We see that canceling the F-terms impose the following conditions

$$\begin{aligned} D_{N^k}W &= p_k + 2ie^{2D}W\mathrm{Im}\,(C\mathcal{F}_k) = 0\,, & (3.4.13)\\ D_{T_\lambda}W &= q^\lambda + 2ie^{2D}W\mathrm{Im}\left(CZ^\lambda\right) = 0\,. & (3.4.14)\end{aligned}$$

Given the fact that the compensator field, C, and D are real definite, and looking at the imaginary part of the former equation we see that

$$q^\lambda\tilde{\xi}_\lambda - p_k\xi^k + \mathrm{Re}W^K = 0\,. \qquad (3.4.15)$$

Thus we see that only a linear combination of axions ξ^k, $\tilde{\xi}^k$ could be stabilized and, thus the remaining fields could not be stabilized using fluxes.[8] One possibility to stabilize those fields could be by the inclusion of Euclidean D2 branes [92]. Now, analyzing the real part of (3.4.14) one can see that $\mathrm{Re}W^K = 0$ is incompatible with non-zero H_3 flux. Also it is worthy to note that for any p_k, q^λ flux vanishing, the corresponding modulus associated with $\mathrm{Im}\mathcal{F}_k$, $\mathrm{Im}Z^\lambda$ has to vanish in order to satisfy (3.4.14). Thus assuming $\mathrm{Re}W^K \neq 0$ and $p_k, q^\lambda \neq 0$ for all k, λ we can obtain the following implicit relation which stabilizes all the complex structure moduli

$$e^{-K^{\mathrm{cs}/2}}\frac{p_{k_n}}{\mathrm{Im}\mathcal{F}_{k_n}} = \cdots = e^{-K^{\mathrm{cs}/2}}\frac{q^{\lambda_n}}{\mathrm{Im}Z^{\lambda_n}} = Q_0\,, \qquad (3.4.16)$$

where n runs for every complex structure moduli. These relations constitute $h^{2,1}$ real equations that, in general, will fix $h^{2,1}$ complex structure. Plugging this results

[7] W_0 denotes the vacuum expectation value of the superpotetial after moduli stabilization.

[8] In presence of D6-branes these axions could be stabilized through Stückelberg mechanism.

back into (3.4.14) we see that the dilaton is stabilized at

$$e^{-\phi} = 4\sqrt{2}e^{K^K/2}\frac{\mathrm{Im}W_0}{Q_0}\,. \tag{3.4.17}$$

Using the fact that $K^{\mathrm{cs}} = 4D$ thanks to the compensator field (3.3.13) we will be able to see that the vev of the superpotential once we stabilize the complex structure moduli is $iW = \frac{1}{2}\mathrm{Im}W^Q$. And thus, when the complex structure moduli satisfy their equations of motion, the vacuum expectation value of the superpotential could be written in terms of the Kähler moduli only

$$W\left(t_a, N^k, T_\lambda\right) = -i\mathrm{Im}W^K\left(t_a\right)\,. \tag{3.4.18}$$

Regarding the stabilization of the Kähler moduli sector, thanks to the relation (3.4.18) found, the Kähler moduli sector is decoupled from the complex structure sector, and thus the vanishing F-term conditions could read as

$$D_{t_a}W = \partial_{t_a}W^K - i\partial_{t_a}K^K\mathrm{Im}W^K = 0\,. \tag{3.4.19}$$

The first warning when we are stabilizing the Kähler sector is that if we switch off m_0 automatically in order to satisfy (3.4.19) all the other RR fluxes should vanish, i.e. $e_a = 0 = m_a$ or all the volumes have to be stabilized at $v_a = 0$. Thus, considering $m_0 \neq 0$ and looking at the imaginary part of (3.4.19) we see that

$$\mathrm{Im}\partial_{t_a}W^K = \mathcal{K}_{abc}v_b\left(m_c - m_0 b_c\right) = 0\,, \tag{3.4.20}$$

and one can see that all the axionic components of the Kähler moduli are stabilized at

$$b_c = \frac{m_c}{m_0}\,. \tag{3.4.21}$$

Now, considering the real part of (3.4.19) and plugging back (3.4.21) we obtain a system of $h^{1,1}$ cuadratic coupled equations.

$$3m_0^2\mathcal{K}_{abc}v_bv_c + 10m_0e_a + 5\mathcal{K}_{abc}m_bm_c = 0 \text{ where } a = 1,\dots,h^{1,1}\,. \tag{3.4.22}$$

Note that b and c are summed in the former expression. Note that, since we have $h^{1,1}$ equations generically all v_a will be stabilized. Finally, the value of W_0 in terms of the Kähler moduli, using (3.4.18), is given by

$$W_0 = \frac{2i}{15}m_0\mathcal{K}_{abc}v_a^0v_b^0v_c^0\,. \tag{3.4.23}$$

We have seen that, using this mechanism we are able to stabilize all the Kähler moduli, the saxionic components of the complex structure moduli and a linear combination of the axionic components at an AdS vacuum. For realizations on toroidal orientifolds see [93].

WEAKLY coupled scenario

This moduli stabilization mechanism is designed as the a dual version of the LARGE volume scenario in type IIA [94]. It consists on using the basis of complex structure moduli T_λ. This means that in this basis $k = 0$, and thus N^0 will be the dilaton, i.e. $N^0 = S$, and $\lambda = 1, \ldots, h^{2,1}$. Apart from this assumption this mechanism of moduli stabilization is based on the fact that all H^3 flux is turned off except the one that is sourcing the dilaton p_0. Also this setup needs the existence of α' and non-perturbative effects in order to stabilize all Kähler and complex structure moduli. The Kähler potential and superpotential in this setup is given by[9]

$$K = -\log 8\left(\mathcal{V} + \frac{1}{2}\varepsilon\right) - \log\left(S + \bar{S}\right) - \log\left(\mathcal{V}' + \frac{\xi'}{2}\right) , \qquad (3.4.24)$$

$$W = W^K - 2if_0\varepsilon - ip_0S + \sum_{\lambda}^{h^{2,1}-1} A_\lambda e^{-a_\lambda T_\lambda} . \qquad (3.4.25)$$

In this setup, ε corresponds to α' corrections which are mirror dual to corrections to the prepotential away from the large-complex structure point. Note that, in contrast to the type IIB case, where α' correction doesn't enter in the superpotential, in type IIA, at lowest order they enter like $-2ip_0\varepsilon$ as we see in (3.4.25). Also ξ' correspond to α'^3 corrections mirror dual to type IIB, where $\mathcal{V}' = \frac{1}{6}\mathcal{K}_{abc}q^a q^b q^c$. On the other hand, we can see that all complex structure moduli, T_λ, except one, will be stabilized via non-perturbative effects coming from euclidean D2-branes or gaugino condensation of D6 branes.

Focusing first of all on the Kähler moduli and dilaton sector one can see that all F-terms will vanish, as in the mirror IIB case where all complex structure moduli plus axio-dilaton are stabilized by ISD fluxes.

Regarding the complex structure sector, the procedure will be analogous as in the LARGE volume scenario and we leave the details for that section. In summary, one can consider the volume form of the complex structure moduli as the mirror dual of a "Swiss-Cheese" Calabi-Yau

$$\mathcal{V}' \sim \left(\left(U + \bar{U}\right)^{3/2} - \left(U_h + \bar{U}_h\right)^{3/2}\right) , \qquad (3.4.26)$$

where we have considered $\lambda = 2$ and we have defined $T_1 = U$ and $T_2 = U_h$. And thus, expanding the scalar potential in terms of $\frac{1}{\mathcal{V}'}$ and competing the contributions to the scalar potential coming from non-perturbative effects and α' corrections (given by ξ') one obtains a similar expression as in the LARGE volume case (3.4.53). Minimizing the scalar potential in terms of $\mathcal{V}'$ and U_h one obtains a non-supersymmetric AdS vacuum where the stabilization is similar to the one seen in (3.4.54). Note that in this scenario contributions coming from g_s loop corrections are neglected.

[9]Note that we have changed the conventions compared to Section 3.4.1 in order to keep the original expressions.

3.4.2 Type IIB flux compactifications

First of all, we define the three-form G_3 in terms of the field strengths and the axio-dilaton

$$G_3 = F_3 - \tau H_3 \,, \tag{3.4.27}$$

and expanding it in the real symplectic basis

$$G_3 = (e_K - i\tau n_K)\,\alpha_K - \left(m^L - i\tau \tilde{n}^L\right)\beta^L \,, \tag{3.4.28}$$

where e^K and m_L correspond to RR fluxes and n^K, $\tilde{n}_L$ correspond to NSNS fluxes. The presence of background fluxes and localized sources will modify the Calabi-Yau metric introducing a warp factor e^{-2A}

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} g_{i\bar{j}} dy^i d\bar{y}^{\bar{j}} \,. \tag{3.4.29}$$

The warp factor only could depend on internal components in order to not break Poincaré invariance. One of the conditions needed for the existence of this warped solution is that the G_3 flux has to be Imaginary Self Dual, i.e. $*G_3 = iG_3$[10]. Warping effects could be used to create hierarchies thanks to the redshift effect of warped throats were the matter is localized at the tip of the throat [96, 97]. Typically, in order to neglect warping effects, one should consider the large-radius limit where the warp factor will be $A \sim 1$ in most of the internal space.

Regarding tadpole cancellation condition (3.4.5) the contribution in the case that we are studying come from D3-branes and wraped D7-branes, since it has D3-brane charge.[11]

$$N_{D3} - \frac{1}{4} N_{O3} + \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3 = 0 \,. \tag{3.4.30}$$

Calabi-Yau orientifolds with D3/D7-branes admit an F-theory description on a elliptically fibered Calabi-Yau $\mathbf{X}_8$. In these cases the tadpole condition could be rewritten as

$$N_{D3} + \frac{1}{(2\pi)^4 \alpha'^2} \int H_3 \wedge F_3 = \frac{\xi(Z)}{24} \,, \tag{3.4.31}$$

where $\xi(Z)$ is the Euler-number of the corresponding four-fold. In this perpective D7-branes and O7-planes are geometrized and this is why its contribution to the tadpole appears through this topological number.

The Gukov-Vafa-Witten superpotential in this case is defined as

$$W = \int \Omega \wedge G_3 = (e_K - i\tau n_K)\, Z^K - \left(m^K - i\tau \tilde{n}^K\right) \mathcal{F}_K \,. \tag{3.4.32}$$

It is interesting to see that W only depends on the complex structure deformations and the axio-dilaton and not in the Kähler moduli. One way to overcome this problem is the addition of non-geometric fluxes coming from dualities from type IIA. The rigorous description of these fluxes is beyond of the scope of this text, but for more details see [98–100].

[10] For more details see [95]

[11] We don't consider contributions from D5-branes and NS5-branes to the tadpole, for a review see [86].

Moduli Stabilization

Achieving supersymmetric Minkowski vacuum

Not using non-perturbative effects First of all we will see he requirements for sypersymmetric solutions in four dimensions where ISD fluxes are turned. Supersymmetric moduli stabilization will by giving by vanishing the F-terms of all the moduli. Applying this conditions we see that

$$\begin{aligned} D_{z^K}W &= \partial_{z^K}W + K_{z^K}W = \int G_3 \wedge \xi^K = 0\,, & (3.4.33)\\ D_\tau W &= \partial_\tau W + K_\tau W \approx \int \bar{G}_3 \wedge \Omega = 0\,, & (3.4.34)\\ D_{T_A}W &= \partial_{T_A}W + K_{T_A}W = K_{T_A}W = 0\,. & (3.4.35)\end{aligned}$$

We see that from (3.4.33) $G^{1,2} = 0$, also from (3.4.34) $G^{3,0} = 0$ and finally (3.4.35) implies $G^{0,3} = 0$. Thus, we see that, in absence of non-perturbative terms, supersymmetric Minkowski vacuum is achieved by ISD G_3 with only (2,1)-components. Also, due to the fact that there are not non-trivial one-forms in a Calabi-Yau it implies that G_3 has to be primitive.

Stabilization using racetrack This scheme of supersymmetric moduli stabilization proposed in [101] is based on the inclusion of two non-perturbative terms which source the same Kähler modulus. In this case we consider ISD G_3-flux with (2,1) and (0,3) components. The basis setup is to consider the following superpotential

$$\begin{aligned} K &= -3\log\left(T+\bar{T}\right)\,, & (3.4.36)\\ W &= W_0 + Ae^{-aT} + Be^{-bT}\,. & (3.4.37)\end{aligned}$$

Assuming that, all complex structure moduli are stabilized through fluxes and the imaginary part of T is stabilized at the origin, one finds that

$$D_T W = 0 \to T_0 = \frac{1}{a-b}\left|\frac{aA}{bB}\right|\,, \tag{3.4.38}$$

which implies the following relation with W_0

$$W_0 = -A\left|\frac{aA}{bB}\right|^{\frac{a}{b-a}} - B\left|\frac{aA}{bB}\right|^{\frac{b}{b-a}}\,. \tag{3.4.39}$$

This means that all moduli are stabilized supersymmetrically at a Minkowski vacuum since the vev of the total superpotential (3.4.37) vanishes. This moduli stabilization scheme requires large fine-tuning of the coefficients in order to work.

KKLT scenario

We have seen that in type IIB Kähler moduli are not stabilized at tree level using fluxes. We will see that the Kähler moduli only will appear non-perturbatively in the superpotential. The KKLT scenario [102] in one of the most widespread ways to stabilize Kähler moduli in this kind of setups. To do that they rely on the addition to the flux superpotential of non-perturbative effects coming from Euclidean D3-branes [103] or gaugino condensation of D7-branes [104, 105]. Assuming that we have only one Kähler modulus we see that

$$W = W_0 + Ae^{-aT}\,, \tag{3.4.40}$$

where W_0 is treated as a constant and is the vacuum expectation value superpotential of the stabilized complex structure moduli and the axio-dilaton. Note that this superpotential is sourced by ISD (2,1) and (0,3) G_3-flux. In the other hand $a = \frac{2\pi}{N}$ where, for $N = 1$ we are considering Euclidean D3-branes and for $N > 1$ it will denote the rank group of the gaugino condensate. A in general depends on complex structure moduli and the open string sector. For the sake of simplicity we assume, in this section, that it is a constant. For an analysis of its implications on inflation when we treat it dynamically see [106].

At this level of approximation the scalar potential has two different minima, one corresponds to the decompactification limit $T \to \infty$ and the other one corresponds to canceling the F-term for the Kähler modulus

$$D_T W = -aAe^{-aT_{\text{AdS}}} - \frac{3}{T_{\text{AdS}} + \bar{T}_{\text{AdS}}}\left(W_0 + Ae^{-aT_{\text{AdS}}}\right) = 0\,, \tag{3.4.41}$$

and thus finding the supersymmetric AdS minimum

$$V_{\text{AdS}} = -3e^{K}\left|W_0\right|^2\,. \tag{3.4.42}$$

Note that in this case $\left(T + \bar{T}\right) = \mathcal{V}^{2/3}$, where $\mathcal{V}$ is the volume of the compactification manifold. And thus it is necessary that the Kähler modulus T to be stabilized at large values implying that W_0 has to be sufficiently small. The value of W_0 arises from all possible choices of integral fluxes and, its associated complex structure moduli stabilized at its vev. The necessary small values of W_0 could be achieved by landscape arguments. On the other hand, in order to justify the single-instanton approximation one should satisfy $aT_{\text{AdS}} \gg 1$

Finally, after obtaining the corresponding AdS vacuum state, it is necessary to uplift to a dS solution and thus, break supersymmetry. The first mechanism proposed was to include an $\overline{D3}$-brane at the tip of a warped throat. The warping will redshift the energy and, the authors argue that, one could be able to fine-tune it in order to achieve dS vacuum. We will show the scalar potential generated by the $\overline{D3}$-brane in the KKLT proposal [102] and the KKLMMT [107] one

$$V_{\text{up}}^{\text{KKLT}} = \frac{\Delta^2}{\left(T + \bar{T}\right)^3}\,,\; V_{\text{up}}^{\text{KKLMMT}} = \frac{\Delta^2}{\left(T + \bar{T}\right)^2}\,. \tag{3.4.43}$$

This mechanism could also be described by means of an F-term uplifting. It has been also described by F-term uplifting in $\mathcal{N} = 1$ supergravity for the KKLT scenario

[108] by means of the introduction of a Polonyi field[12] [109] or a O'Raifeartaigh model [110]. Recently, it has been described by the introduction of a nilpotent goldstino X [111, 112] (which satisfies $X^2 = 0$), in this last case the supergravity description will be

$$K^{(1)} = K^{\mathrm{cs}} - 3\log\left(T+\bar{T}\right) + X\bar{X}\ ,\ K^{(2)} = K^{\mathrm{cs}} - 3\log\left(T+\bar{T}-X\bar{X}\right), \tag{3.4.44}$$

$$W = W_0 + Ae^{-aT} + \Delta X\,. \tag{3.4.45}$$

And thus, the introduction of this new field will generate a non-vanishing F-term which uplifts the AdS vacuum state. Note that $K^{(1)}$ will generate the uplifting term proposed in the original KKLT article and $K^{(2)}$ will generate the one proposed by KKLMMT

$$V_{\mathrm{up}}^{(1)} \sim |F_X|^2 = \frac{\Delta^2}{\left(T+\bar{T}\right)^3}\ ,\ V_{\mathrm{up}}^{(2)} \sim |F_X|^2 = \frac{\Delta^2}{\left(T+\bar{T}\right)^2}\,. \tag{3.4.46}$$

The presence of the uplifting term will shift the Kähler modulus from its original supersymmetric minimum T_{AdS} given by

$$T_0 = T_{\mathrm{AdS}} + \frac{\Delta^2}{2a^2 T_{\mathrm{AdS}} W_0^2} + \mathcal{O}\left(\frac{1}{T_{\mathrm{AdS}}}\right)^2\,, \tag{3.4.47}$$

and thus the Kähler modulus will contribute to supersymmetry breaking.

Finally, it is worthy to mention that, since we are breaking supersymmetry the gravitino will become massive. In this case, the mass scale of the gravitino mass will be given by

$$m_{3/2} = e^{K/2} W \sim W_0\,, \tag{3.4.48}$$

and the mass scale of the Kähler modulus will be

$$m_T \sim 2aT_0 m_{3/2}\,. \tag{3.4.49}$$

We will describe in Part IV how to implement this scenario of moduli stabilization during inflation and its implications with backreaction.

LARGE volume scenario

Here we will focus on a different scenario to stabilize Kähler moduli. The existence of at least one blow-up mode resolving point-like singularities and a negative Euler number are necessary conditions to stabilize Kähler moduli at exponentially large volumes. The LARGE-volume scenario [113, 114] is formulated on the so-called Swiss-Cheese Calabi-Yau manifolds and, through the competence in the scalar potential of terms coming from α' corrections and non-perturbative effects, while neglecting g_s corrections, this scenario gives and exponentially large volume. Note

[12] Note that, in models using a Polonyi field, in order to perform the uplifting, the mass of this field will be $m_X = \frac{\sqrt{3}W_0}{2\Lambda} \gg m_{3/2}$ where the Kähler potential for this field is $K^X = X\bar{X} - \frac{\left(X\bar{X}\right)^2}{\Lambda^2}$

that, also the theory will be weakly-coupled in order to neglect corrections to g_s. For more details about LARGE-volume scenario moduli stabilization and string-loop corrections see [115].

In its simplest version it consists on two Kähler moduli, where the volume form could be written as

$$\mathcal{V} = \left(T + \bar{T}\right)^{3/2} - \left(T_h - \bar{T}_h\right)^{3/2} . \tag{3.4.50}$$

Where T controls the volume of a 'big' four-cycle and T_h controls the volume of a 'small' four-cycle called also 'hole'. This moduli stabilization mechanism relies on α' and non-perturbative corrections to stabilize the Kähler moduli in a non-supersymmetric AdS vacuum. In $\mathcal{N} = 1$ supergravity the setup is written as

$$K = -\log\left(\mathcal{V} + \xi\right) , \tag{3.4.51}$$
$$W = W_0 + Ae^{-aT_h} , \tag{3.4.52}$$

where $\xi = -\frac{\zeta(3)}{4(2\pi)^3 g_s^{3/2}}\chi$ where $\chi = 2\left(h^{1,1} - h^{2,1}\right)$ is the Euler number of the compactification manifold and g_s is the string coupling which is treated as a constant. This mechanism only works if $\xi > 0$ and thus, $\chi < 0$. Note that non-perturbative effects only will depend on blow-up modes T_h and not in the 'big' cycle.

Stabilizing $\mathrm{Im}T_h = \frac{\pi}{a}$ and focusing on the real parts of T_h and T we find that the scalar potential expanded for large volume $\mathcal{V}$ is given by

$$V \approx \frac{2\sqrt{2}a^2 A^2 \sqrt{T_h} e^{-2aT_2}}{3\mathcal{V}} - \frac{4aW_0 T_h e^{-aT_2}}{\mathcal{V}^2} + \frac{3\xi W_0^2}{2\mathcal{V}^2} + \mathcal{O}\left(\frac{1}{\mathcal{V}^4}\right) . \tag{3.4.53}$$

Note that W_0 in this scenario $|W_0| \sim \mathcal{O}(1)$ as opposed to the KKLT scenario. Also, it is worthy to mention that W_0 and A should have the same sign. We minimize the former expression with respect $\mathcal{V}$ and T_h and we find that their vevs at the non-supersymetric AdS are given by

$$T_h \approx \frac{\xi^2}{2} + \frac{1}{3a} , \, \mathcal{V} \approx \frac{3\sqrt{T_h} e^{aT_h} W_0}{\sqrt{2}aA} . \tag{3.4.54}$$

Thus, we see that the volume of the Calabi-Yau manifold depends exponentially on the vacuum expectation value of T_h. With this stabilization, we find that the value of the scalar potential is given by

$$V_{\mathrm{AdS}} \approx -\frac{W_0^2}{\mathcal{V}^3} . \tag{3.4.55}$$

Note that, in this case the nature of supersymmetry breaking is completely different as in the case of KKLT were we introduced and $\overline{D3}$-brane. And, thus the different volume power in the AdS vacuum compared to KKLT.

Afterwards, we should perform an uplifting mechanism in order to achieve deSitter vacuum. It could also be done by means of an F-term uplifing as in the former case. Typically the uplifting potential is similar to the one shown before, i.e. $V_{\mathrm{up}} \sim \frac{\Delta^2}{\mathcal{V}^2}$, and thus we will not explain further. Finally, note that the gravitino mass is, as in the former case

$$m_{3/2} \approx \frac{W_0}{\mathcal{V}}\,, \tag{3.4.56}$$

and the mass scale for the different Kähler moduli are

$$m_T \sim \frac{W_0}{\mathcal{V}^{3/2}}\,,\, m_{T_h} \sim \frac{W_0}{\mathcal{V}}\,. \tag{3.4.57}$$

We see that the lightest modulus is m_T. Moreover, the axionic component of T will be almost massless $m_{\mathrm{Im}T} \sim e^{-\mathcal{V}^{2/3}} M_{\mathrm{P}} \sim 0$. This light field is a generic prediction of the LARGE volume scenario which could play a role during reheating and could be related with dark radiation [116].

3.5 Type II orientifold compactifications with D-branes

Here, we will analyze how space-filling D-branes will enter into the action of type II orientifold compactifications while satisfying $\mathcal{N} = 1$ supersymmetry. This will establish calibration conditions on the cycles where the branes are wrapped. For simplicity, we will focus on the description of D6-branes in type IIA and D3-/D7-branes in type IIB.

Each space-filling Dp-brane contains a U(1) gauge theory in its worldvolue. It is straightforward to see that stacks of N coincident Dp-brane will give a non-abelian U(N) gauge theory. The low-energy description of a single Dp-brane, for its degrees of freedom and its couplings with the bulk NSNS fields, is given by the Dirac-Born-Infeld action

$$S_{DBI} = -\mu_p \int_{\mathcal{W}} d^{p+1}\xi\, e^{-\phi} \sqrt{-\det\left(P[E] - \frac{l_s^2}{2\pi} F\right)}\,, \tag{3.5.1}$$

where $E = e^{\phi/2} g + B_2$ and $P[\,]$ denotes the pullback on the worldvolume of the Dp-brane. The dynamics which describe the fluctuations of the worldvolume in the ambient manifold is encoded in the pullback. Finally, F denotes the field strength associated with the U(1) gauge theory living on the worldvolume. Note that μ_p is related with the tension of the Dp-brane. Since the D-brane could carry lower RR charges, which are spread over the worldvolume, they show as background fluxes of the U(1) gauge theory. Thus, in presence of this fluxes the field strength, F, is modified to

$$F = f + dA\,, \tag{3.5.2}$$

where f is the harmonic two-form of the worldvolume of the Dp-brane.

On the other hand, since Dp-branes, as we have seen, carry RR charges, they couple to RR fields on the bulk. These couplings are described by the Chern-Simons action which, for a single D-brane is given by

$$S_{CS} = \mu_p \int_{\mathcal{W}} \sum_p P\left[C^p\right] e^{\frac{l_s^2}{2\pi}F - P[B_2]}\,, \tag{3.5.3}$$

which is described as the power series of the exponential wedged with the RR fields of the bulk. Note that the only non-vanishing contributions to (3.5.1) and (3.5.3) are $p+1$ forms.

Once we have sketched the action for a sigle Dp-brane we will focus on calibration conditions for D6 and D7 branes and its embedding in the low-energy $\mathcal{N}=1$ type II orientifold theories which we have seen.

3.5.1 D6-branes on type IIA orientifold compactifications

The addition of a single $D6$-brane will impose constraints in order to preserve $\mathcal{N}=1$ supersymmetry in type IIA orientifold setups.

First of all, once we add a $D6$-brane wrapping a three-cycle, Π_3, on a Calabi-Yau manifold we have to add a mirror brane wrapping the corresponding mirror cycle $\Pi_3' = \sigma^* \Pi_3$. Now, we take the Poincaré dual 3-forms to these cycles π_3 and π_3' and we expand them in terms of harmonic three-forms in the real symplectic basis

$$\pi_3 = \pi_3^{\hat{K}} \alpha_{\hat{K}} + \pi_{3\,\hat{L}} \beta^{\hat{L}}\ ,\ \pi_3' = -\pi_3^{\hat{K}} \alpha_{\hat{K}} + \pi_{3\,\hat{L}} \beta^{\hat{L}}\,. \tag{3.5.4}$$

Since we are considering only a single D-brane, the scalars coming from dimensional reduction will describe the fluctuations of the brane in the normal direction to its worldvolume. The requirement of preserving $\mathcal{N}=1$ supersymemtry will impose constraints in the possible fluctuations of the brane. These constraints come from the fermionic fields introduced by the open strings ending on the D-brane which are not, in general, invariant under the supersymmetry generator of the theory. $\mathcal{N}=1$ supersymmetry will be preserved by the compensations of the fermion variation with the κ symmetry of the worldsheet action. Thus, in order to preserve supersymmetry the D6-brane has to be wrapping a sLag three-cycle $\hat{\Pi}_3$ calibrated by $e^{i\theta} i^* \Omega$ and a certain condition on the gauge flux. These conditions in the Einstein frame are

$$i^*\left(e^{\phi/2} J\right) = 0\ ,\ e^{\frac{1}{2}\left(K^Q - K^K\right)} i^* \Omega = e^{i\theta} e^{\frac{3}{4}\phi} d\mathrm{vol}_E|_{\hat{P}i_3}\ ,\ \mathcal{F} := \iota^* B_2 - \frac{l_s^2}{2\pi} F|_{\hat{\Pi}_3} = 0\,, \tag{3.5.5}$$

where $F_2|_{\hat{\Pi}_3}$ denotes the internal part of the U(1) field strength and K^Q and K^K are given in (3.3.25) and (3.3.32) respectively. Note that we have defined $\mathcal{F}$ as the worldvolume flux. In order to ensure that $D6$-branes don't break supersymmetry, they should be calibrated with the same phase, θ, as the O6-planes (3.3.4).

On the other hand, in order to preserve Lorentz invariance in the four-dimensional theory the background fluxes must have legs only in the internal components. Restricting to massless modes of the gauge field, which implies $dA|_{\hat{\Pi}_3} = 0$, we see that (3.5.2) turns to be

$$F_2|_{\hat{\Pi}_3} = f \to \frac{l_s^2}{2\pi} f \in H^2\left(\hat{\Pi}_3, \mathbb{Z}\right)\,, \tag{3.5.6}$$

where the right arrow corresponds to the condition imposed by Freed-Witten anomaly [117] cancellation in sLags three-cycles. Thus we see that the gauge fluxes on the worldvolume should be quantized. We can expand f in terms of harmonic two-forms as $f = n_{Fi}\rho^i$ where $n_{Fi} \in \mathbb{Z}$. Note that, the third condition in (3.5.5) forces us to cancel the gauge flux (3.5.6) with the pullback of B_2 on the internal components. Writing it in a basis of harmonic two-froms, ω_a, could be see as

$$\mathcal{F} = \left[\left(b^a - \frac{l_s^2}{2\pi} f^a\right)\iota^*\omega_a\right] = 0\,. \tag{3.5.7}$$

Note that the fluxes f^i associated to two-cycles of $\hat{\Pi}_3$ trivial in the ambient Calabi-Yau have to vanish. We have seen $\mathcal{N} = 1$ configurations in type IIA orientifolds require that the worldvolume of the D6-brane should be described by $\mathcal{W} = \mathbb{R}^{1,3} \times \hat{\Pi}_3$. The space of allowed deformations will depend, through the calibration conditions, on the bulk moduli and the total moduli space will not be a factorization like $\mathcal{M}_{\text{bulk}} \times \mathcal{M}_{\text{D6}}$. The total moduli space will be described by using techniques of relative cohomology and relative Hodge structures which are beyond the scope of this work.

But, in the limit of small fluctuations around the background value and small deformations of the brane, the moduli space could be approximated as a factorization between the brane and bulk moduli and this is the approach that we will follow. In order to obtain the description of the total moduli space in this approximation one should take a reference point of bulk moduli sapce J_0, Ω_0 and $\mathcal{F}_0$ and impose the calibration condition (3.5.5), which means that we are considering the brane fluctuations preserving the sLag condition. Using this approximation, the moduli space of the brane is a real smooth manifold of dimension $b_1\left(\hat{\Pi}_3\right)$. Thus, we can conclude that the moduli space of the D6-brane could be described locally by normal infinitesimal homotopic deformations of a reference special lagrangian three cycle $\hat{\Pi}_3^0$ to a sLag $\hat{\Pi}_3$. Due to McLean's theorem one can see that the deformations of the brane are in one-to-one correspondence with the basis of harmonic one-forms ζ. The complexified D6-deformations, Φ, contain Wilson lines and geometric deformations and, considering only one D6-brane, its defined in the following way

$$\Phi = \frac{l_s}{\pi}\left(A - \iota_{\varphi X} J_c|_{\hat{\Pi}_3}\right) = \Phi\zeta\,, \tag{3.5.8}$$

where $\zeta/2\pi l_s$ is the harmonic one form generating $H^1\left(\hat{\Pi}_3, \mathbb{Z}\right)$ and X a normal vector to the sLag three-cycle such that $e^{\pi/2} i_X J = \zeta$. Finally, note that A is the gauge field on the worldvolume of the D6-brane. Since it is embedded in the orientifold, in order to obtain $\mathcal{O}$-invariant states, the fluctuations should have even parity under σ. Finally, we have to note that since the Kähler form has odd parity it implies that the one-forms related with the fluctuation, ζ, should have odd parity under σ^*. With this definition at hand we will describe briefly the low-energy Kähler potential obtained from dimensional reduction of the DBI and CS action. The gauge-kinetic function will be modified due to the presence of the open string sector. It has been argued [118] that the Wilson lines only appear through its derivatives in the low-energy four-dimensional effective action [13] and therefore it should exhibit a shift symmetry in the Kähler potential. It was argued in [120] that using (3.5.8) one can redefine the complex structure moduli $N^{\hat{K}}$ and obtain the following manifestly shift symmetric Kähler potential

$$\begin{aligned} K^Q &= -2\log\left[\frac{1}{2i}\mathcal{F}_{\hat{K}\hat{L}}\left(N^{\hat{K}} - \bar{N}^{\hat{K}} + \frac{i}{4}\mathcal{Q}^{IJ\hat{K}}\left(\Phi_I - \bar{\Phi}_I\right)\left(\Phi_J - \bar{\Phi}_J\right)\right)\right. && (3.5.9)\\ &\times \left.\left(N^{\hat{L}} - \bar{N}^{\hat{L}} + \frac{i}{4}\mathcal{Q}^{IJ\hat{L}}\left(\Phi_I - \bar{\Phi}_I\right)\left(\Phi_J - \bar{\Phi}_J\right)\right)\right], && (3.5.10)\end{aligned}$$

where, in this case $\mathcal{Q}$ depends explicitly on the Kähler moduli. Also the authors argue that away from the limit of small fluctuations one can define the complexified D6-brane position modulus in the Hitchin's basis following

$$\Phi_\alpha = \frac{2}{l_s^2}\int_{\Gamma_\alpha}\tilde{F} - J_c\,, \tag{3.5.11}$$

where Γ_α is a two-chain which connects the Poincaré dual one-cycles corresponding to the one-forms of the reference sLag and the homotopically deformed sLag. $\tilde{F}$ is the extension of the worldvolume field strength to the two-chain. We reference the reader to that paper for technical details.

On the other hand, in presence of shift symmetry breaking effects which could come, for example, from g_s or α' corrections, one could expect a Kähler potential along the lines of that derived in [121, 122] which reproduces the kinetic terms seen before by redefining the complex structure moduli $N^{\hat{K}}$ as

$$N^{\hat{K}} = \xi^K + i\left[l^{\hat{K}} - \frac{i}{8}\mathcal{Q}^{IJ\hat{K}}\Phi_I\Phi_J\right], \tag{3.5.12}$$

where $\mathcal{Q}$ is related with the reference three-cycle which we are deforming and the worldvolume flux $\mathcal{F}$. Thus, rewriting (3.3.25) in terms of the new chiral field will modify its expression by the introduction of the open string modes.

$$K^Q = -2\log\left[\frac{1}{2i}\mathcal{F}_{\hat{K}\hat{L}}\left(N^{\hat{K}} - \bar{N}^{\hat{K}} + \frac{i}{4}\mathcal{Q}^{IJ\hat{K}}\Phi_I\bar{\Phi}_J\right)\times\left(N^{\hat{L}} - \bar{N}^{\hat{L}} + \frac{i}{4}\mathcal{Q}^{IJ\hat{L}}\Phi_I\bar{\Phi}_J\right)\right]. \tag{3.5.13}$$

It is straightforward to see that the Wilson line A will not enjoy a shift symmetry in the Kähler potential.

As a final remark it is worthy to mention that there is no constraint on the number of harmonic one-forms coming from tadpole cancellation, and thus it has nothing to do about the number of open string moduli. As we saw, the RR tadpole cancellation condition (3.4.12) is sensitive to the homology class of each of the three-cycles $\hat{\Pi}_3^a$ in $\mathbf{X}_6$ but it is not sensitive to the topology of each three-cycle itself. In

[13] This expectation is further sustained by the fact that D6-brane Wilson lines lift to integrals of the three-form A_3 over three-cycles in G_2 compactifications of M-theory, and that such scalars are absent in the corresponding Kähler potential [119], just like the scalars arising from C_3 are absent in (3.3.28).

particular, a priori it tells us nothing about the number of harmonic one-forms within each three-cycle, that is about $b_1(\hat{\Pi}_3^a)$. As mentioned above, such topological number indicates the number of complex open string moduli of an isolated D6-brane. More generally, it indicates the number of 4d chiral fields in the adjoint of the gauge group obtained from KK reduction of a stack of N_a D6-branes. For that reason, when building models of particle physics, the three-cycles Π_3^a describing the SM sector are chosen such that either $b_1(\Pi_3^a) = 0$ or else the adjoint fields are projected out by some orbifold action [123–126], and the same is often required for the remaining D6-branes of the model.

We will see in Part II applications with both setups to inflation.

3.5.2 D3-/D7-branes in type IIB orientifolds

In this section we will describe briefly how the spectrum is modified due to the inclusion of the open string sector coming from D3-/D7-branes. For the reader who needs more technical details see [127]. As we did in the previous case, first of all we will discuss the calibration conditions that the cycles has to satisfy in order to preserve the $\mathcal{N} = 1$ supersymmetry obtained in type IIB orientifolds wild O3-/O7-planes.

The calibration condition, again, will be related with the fact that adding Dp-branes will lead to fermionic fields introduced by the open strings ending on the D-brane. The D7-brane will wrap an internal four-cycle $\mathcal{S}$, and also we consider a mirror D7 wrapping a cycle $\mathcal{S}' = \sigma^* \mathcal{S}$. By convention, we introduce a four-cycle $\mathcal{S}_A = \frac{1}{2}(\mathcal{S} + \mathcal{S}')$, which is the union of both cycles. One can see straightforward that $\sigma^* \mathcal{S}_A = \mathcal{S}^A$. Thus the D7-brane that we will analyze will be the one wrapping $\mathcal{S}_\Lambda$ and, in this way describes both the D7-brane and its image. In order to preserve supersymmetry the worldvolume of the D7-brane has to satisfy the following callibration condition in absence of worldvolume flux

$$d^4\xi \sqrt{\det \hat{g}} = d\mathrm{vol}_{\mathcal{S}} = \frac{1}{2} J \wedge J \, . \tag{3.5.14}$$

One can see that the internal four cycle for the D7-brane needs to be calibrated with respect the Kähler form and then, in order to preserve supersymmetry, D7-branes have to wrap holomorphic four-cycles. In case of non-trivial worldvolume flux the callibration condition could be written as

$$\delta^4\xi \sqrt{\det\left(\hat{g} + \mathcal{F}^a i^* \omega_a\right)} = \frac{e^{-i\theta}}{2} \left(J + i\mathcal{F}^a i^* \omega_a\right) \wedge \Big(J + i\mathcal{F}^b i^* \omega_b\Big) \, , \tag{3.5.15}$$

and the BPS condition is given by

$$\mathcal{F}^{(2,0)} = 0 = \mathcal{F}^{(0,2)} \, , \, \mathcal{F} \wedge J = 0 \, . \tag{3.5.16}$$

Now, we will sketch the low-energy effective field theory coming from KK reduction of the DBI and CS terms. The open string sector, as in the case of type IIA, will enter in the Kähler potential as a redefinition of the geometrical moduli. We will consider here the case of D3-branes and D7-branes.

D3 branes

The inclusion of D3-branes add chiral matter fields, which we denote ζ_3 which parametrize the position of the D3-branes in the ambient Calabi-Yau manifold. The proper chiral Kähler coordinates in this case come from the redefinition of T_α as

$$\hat{T}_\alpha = T_\alpha + \frac{3i}{2}\mu_3 l^2 \left(\omega_a\right)_{i\bar{j}} \operatorname{Tr}\zeta^{3\,i}\left(\bar{\zeta}^{3\,\bar{j}} - \frac{i}{2}\bar{z}^{\bar{a}}\left(\bar{\chi}_{\bar{a}}\right)^{\bar{j}}_{l}\zeta^{3\,l}\right) , \tag{3.5.17}$$

where z^a are complex structure deformations and we have used T_a from (3.3.44). Thus we see that D3-branes will redefine the Kähler potential as

$$K = K^Q - \log\left[-i\left(\tau - \bar{\tau}\right)\right] - 2\log\left[\mathcal{K}\left(\tau, T, G, \zeta^3, z\right)\right] , \tag{3.5.18}$$

where, as we saw in Section 3.3.2, $\mathcal{K}$ is the same as in (3.1.11) where we have to substitute v^A in terms of the proper chiral Kähler variables using (3.5.21).

D7 branes

As we know the massless fields regarding the open string sector will be related with deformations of the D-brane, which are related with harmonic forms. In this case the massless fields which survive the orientifold projection are related with

$$\zeta \in H^0_+\left(S^\Lambda, NS^\Lambda\right) \ , \ a \in H^1_-\left(S^\Lambda, \mathcal{O}\right) , \tag{3.5.19}$$

where ζ parametrize the D7-moduli and belong to matter chiral multiplet related with the possible deformations of the internal four-cycle S^Λ. On the other hand a correspond to Wilson line moduli coming from the the worldvolume gauge field. As in the former case, taking as an approximation small fluctuations of the D7-brane we see that the appropriate chiral Kähler coordinates are

$$\begin{aligned} S &= \tau + \kappa_4^2\mu_7\mathcal{L}_{A\bar{B}}\zeta^A\bar{\zeta}^{\bar{B}} \ , \ G^a = c^a - \tau\mathcal{F}^a , & (3.5.20)\\ T_\alpha &= \frac{3i}{2}\left(\rho_\alpha - \frac{1}{2}\mathcal{K}_{abc}c^b\mathcal{F}^c\right) + \frac{3}{4}\mathcal{K}_\alpha + \frac{3i}{4\left(\tau - \bar{\tau}\right)}\mathcal{K}_{abc}G^b\left(G^c - \bar{G}^c\right) & (3.5.21)\\ &+ 3i\kappa_4^2\mu_7 l^2 f\left(a_I, \bar{a}_I\right) , \end{aligned}$$

where $\mathcal{L}_{A\bar{B}}$ are intersection numbers coming from the harmonic forms where ζ and α are defined (3.5.19). Also, $f\left(a_I, \bar{a}_I\right)$ is a function of the Wilson line moduli, we refer the reader to see [127–129] for different functional dependence on the D7-brane Wilson lines. Note that, in presence of D7-branes the dilaton τ is not anymore a good Kähler coordinate and thus the proper one is S.

Once we have obtained the proper chiral Kähler coordinates we are able to write the Kähler potential which will describe the $\mathcal{N} = 1$ low-energy approximation

$$K = K_{\text{cs}} - 2\log\left[\mathcal{K}\left(S, T, G, \zeta, a\right)\right] - \log\left[-i\left(S - \bar{S}\right) - 2i\mu_7\mathcal{L}_{I\bar{J}}\zeta^I\bar{\zeta}^{\bar{J}}\right] . \tag{3.5.22}$$

Note that we would be able to describe systems with D7 and D3 located at distant points in order to neglect interaction terms. To do that one just has to combine (3.5.17) and (3.5.21). And thus the Kähler potential will be modified accordingly.

Part II

Inflation in type IIA

4

D6-branes and axion monodromy inflation

In this chapter we will review the model of inflation proposed in [31, 130]. This model is a realization of F-term axion monodromy in type IIA with orientifolds, where the inflaton is either the B-field or a Wilson line. We will show the DBI analysis for the B-field and the supergravity description where our model could be understood as chaotic inflation with stabilizer fields [131, 132]. This model is based on the existence of a non-trivial two-cycle π_2 in the ambient Calabi-Yau which is Poincaré dual to the one-form describing the deformations of the D6-brane. This chapter will be structured as follows, first of all we will make a brief recap about the ingredients needed in our model, afterwards we will describe the cornerstone of this model, which is the existence of open-closed string bilinears. After that, we will perform de dimensional reduction of the DBI+CS action and describe two different scenarios of large-field inflation that arise in this setup. Finally, we will give the cosmological observables coming from the DBI analysis when we consider that the inflaton candidate is the B-field.

4.1 Needed ingredients

In this section we will do a brief recap about the needed background ingredients to build the proposed models of inflation. A more detailed discussion could be found in Chapter 3. As we have already mentioned, in this model we will consider type IIA orientifold flux compactifications on Calabi-Yau threefolds. As we have seen in section 3.3.1, these setups are characterized by the presence of space-filling O6-planes wraping sLag three-cycles. The moduli space will be made up by $\mathcal{O}$-invariant states which survive the orientifold projection (3.3.1). Calibration conditions in these setups impose (3.3.3) and (3.3.4). In the absence of background fluxes, the RR tadpoles induced (3.4.11) by the O6-plane can be cancelled by D6-branes wrapping suitable sLag three-cycles, leading to $\mathcal{N} = 1$ chiral compactifications to four-dimensional Minkowski [85, 133].

Regarding the closed string moduli, we have seen that the Kahler sector is described by the complexification of the Kahler form (3.3.31) and span a special Kahler submanifold in the ambient Calabi-Yau. The Kahler potential which describes it (3.3.32) enjoys a continuous shift symmetry. One can argue that perturbative α' corrections will not spoil such symmetry [94], while exponentially suppressed corrections arising from closed string worldsheet instantons are expected to break it to a discrete subgroup.[1]

The shift symmetry is also broken by the presence of background fluxes. More precisely, including RR background fluxes will generate a superpotential of the form

$$l_s W_K(T) \,=\, e_0 + e_a T^a + \frac{1}{2}\mathcal{K}_{abc} m^a T^b T^c - \frac{1}{6} m^0 \mathcal{K}_{abc} T^a T^b T^c \,, \tag{4.1.1}$$

where (e_0, e_a, m^a, m^0) are integer numbers that correspond to the RR flux quanta of (F_6, F_4, F_2, F_0) respectively, see [88] for their precise definition and [94] for how α' corrections modify the value of these flux parameters. One may generalize this superpotential by adding NS three-form fluxes and metric fluxes, after which a superpotential dependence on the dilaton and complex structure moduli will appear [93,134–136]. Notice that adding metric fluxes will take us to the realm of non-Kähler orientifold compactifications, whose effective theory via Kaluza-Klein reduction has not been derived in full generality. Nevertheless, one does not expect that adding such fluxes will modify the above Kähler potential (up to one-loop or warping effects that we are neglecting) and in particular its shift symmetries.

The same applies to the Kahler potential for the complex structure sector (3.3.19), where, as we have seen, the three-form Ω is complexified with the RR potential C_3. We have seen that these coordinates span a special Kahler submanifold, which in the real symplectic basis (3.3.11) where $\tilde{h} = h^{2,1}$ as (3.3.28). Note that, this Kahler potential also displays a shift symmetry for the scalars arising from C_3, which are axions as expected from general arguments [67]. In order to stabilize the fields N^K, we will use the approach seen in section 3.4.1, where it is necessary to introduce H_3 fluxes. We can also generalize the flux superpotential by adding metric fluxes and take into account into account the non-perturbative superpotential generated by D2-brane instantons

As we saw in section 3.5.1, including D6-brane moduli will redefine the complex structure moduli and thus, modify the Käler potential. In order to preserve $\mathcal{N} = 1$ supersymmetry in presence of D6-branes, they have to wrap special lagrangian three-cycles $\hat{\Pi}_3$ satisfying the corresponding callibration conditions (3.5.5). The D6-brane moduli space will be a mixture of geometric deformations of the sLag and Wilson lines whose complex dimension is $b_1(\Pi_\alpha)$. The complexified D6-brane deformation will be given by [137–139]

$$\Phi_{\rm D6} \,=\, \frac{l_s}{\pi}\left(A - \iota_{\varphi X} J_c|_{\hat{\Pi}_3}\right) \,=\, \frac{l_s}{\pi}(\xi^j - \lambda^j_i \varphi^i)\zeta_j \,=\, \Phi^j \zeta_j \,, \tag{4.1.2}$$

with $\zeta^j/2\pi l_s$ the harmonic one-forms generating $H^1(\hat{\Pi}_3, \mathbb{Z})$, and $X = \varphi^j X_j$ a normal vector to $\hat{\Pi}_3$ such that $e^{\phi/2}\iota_X J = e^{\phi/2}(X^m J_{mn})dx^n = \zeta$, which implies that $\iota_X J_c|_{\hat{\Pi}_3} =$

[1]We assume that g_s corrections to this Kähler potential are negligible in the weak coupling regime in which will be working.

$\lambda^j\zeta_j = (\eta^j + i)\zeta_j$ with $\eta^j \in \mathbb{R}$. Finally, $A = \xi^j\zeta_j$ describes the D6-brane Wilson line profile so ξ has period $1/l_s$ and Re Φ has period $1/\pi$.[2] As argued in [140], these open string fields may also enter into the non-perturbative superpotential generated by D2-brane Euclidean instantons.

Considering such open string modulus and performing a direct dimensional reduction of the D6-brane DBI action we have seen that the tree-level Kähler potential (3.3.28) is naively modified to

$$K_Q' \,=\, -2\log\left(\frac{1}{16i}\mathcal{F}_{KL}\left[N^K - \bar{N}^K - \frac{i}{8}Q^K(\Phi-\bar{\Phi})^2\right]\cdot\left[N^L - \bar{N}^L - \frac{i}{8}Q^L(\Phi-\bar{\Phi})^2\right]\right)\,, \tag{4.1.3}$$

in which the Wilson line shift symmetry is manifest.[3]

Finally, in addition to the flux and non-perturbative superpotentials, there will be a superpotential generated by worldsheet instantons, and that may affect both the Kähler and open string moduli of the compactification. On the one hand we will have closed string worldsheet instantons wrapping spheres of $\mathbf{X}_6$ and generating superpotential terms of the form exp (im_aT^a). These terms are suppressed by a factor $\exp(-A/\alpha')$, with A the string frame volume of a holomorphic two-cycle of $\mathbf{X}_6$, so in the supergravity large volume regime they will be subleading compared to the superpotential terms discussed previously. Nevertheless, they will also contribute to the scalar potential for Kähler moduli and in particular one expects that they generate a periodic sinusoidal-like potential for a B-field axion. On the other hand there may also be a superpotential generated by open string worldsheet instantons, see e.g. [118]. In general these will be disk instantons whose boundary lie on the non-trivial one-cycle of the D6-brane three-cycle Π_3. Such instantons will generate superpotential terms involving the D6-brane modulus Φ and the Kähler moduli T^a. Analogously to closed string instantons, disk instantons may generate sinusoidal-like potentials for D6-brane Wilson lines.

To summarise, we have seen that there may be three different kinds of axions in type IIA vacua: B-field axions, C_3 RR-axions and D6-brane Wilson lines, each of them developing different superpotential terms. B-field axions develop a tree-level polynomial superpotential that may be used to generate chaotic inflation upon the inclusion of RR background fluxes, while for C_3 axions this can be achieved by including NS and metric fluxes/torsion in cohomology.[4] Schematically we have that the different pieces of superpotentials arrange as

$$W_{\rm mod} \,=\, W_{\rm flux}(T,N) + W_{\rm D2}(N,\Phi) + W_{\rm WS}(\Phi,T)\,, \tag{4.1.4}$$

[2]In our conventions $\int_{\pi_2} F \in 2\pi\mathbb{Z}$ for every 2-cycle π_2, from where $A \sim A + l_s^{-1}\zeta$ for $\zeta/2\pi l_s \in H_1(\hat{\Pi}_3,\mathbb{Z})$.

[3]Considering a compactification where there exists a explicit symmetry breaking of the Wilson line one can argue the following Kähler potential claimed in [122] $K_Q = -2\log\left(\frac{1}{16i}\mathcal{F}_{KL}\left[N^K - \bar{N}^K + \frac{i}{4}Q^K\Phi\bar{\Phi}\right]\cdot\left[N^L - \bar{N}^L + \frac{i}{4}Q^L\Phi\bar{\Phi}\right]\right)$.

[4]For D6-brane Wilson lines one may also achieve quadratic superpotentials if one introduces torsional homology in the 3-cycle wrapped by the D6-brane [141]. This case, dubbed *massive Wilson lines* in [21], which will be considered in Part IV.

where W_{flux} is the superpotential generated by the closed string fluxes threading $\mathbf{X}_6$, W_{D2} is the superpotential generated by Euclidean D2-brane instantons and W_{WS} is the correction generated by worldsheet instantons.

Following the general philosophy of [21], we would like to build a model of large field inflation via a superpotential involving an axion and leading to chaotic inflation. In this sense it would seem that the inflaton should be one of the fields that enter W_{flux}. The challenge would then be to single out an axion which is much lighter than the rest of the moduli of the compactification, in order to decouple the latter from the inflationary potential. Such goal seems however quite difficult to achieve, as has been discussed in the setup of type IIB flux compactifications [70, 71, 142]. However, as we will discuss next there are further sources of polynomial superpotentials in type IIA vacua, which do not arise from background fluxes but rather from the presence of certain D-branes. As we will see, this will allow to develop a bilinear superpotential in which two of the above axions (namely B-field and Wilson line axions) are involved, and to build chaotic inflation scenarios for both of them.

4.2 Lifting axions using D6-branes

Once we have done a quick recap of all the ingredients needed, we will consider type IIA compactifications with at least one D6-brane wrapping a three-cycle Π_3^α with $b_1(\Pi_3^\alpha) = 1$. For simplicity, we will consider that such three-cycle is isolated from the rest, in the sense that it does not intersect the other three-cycles $\Pi_3^{a\neq\alpha}$ of the compactification, including its orientifold image. For this D6-brane to be supersymmetric it must satisfy the standard BPS conditions [85, 133]

$$J_c|_{\Pi_3^\alpha} - \frac{l_s^2}{2\pi}F \;=\; 0\,, \qquad (4.2.1)$$

$$\text{Im}\;\Omega|_{\Pi_3^\alpha} \;=\; 0\,, \qquad (4.2.2)$$

which require that Π_3^α is a special Lagrangian three-cycle and that the gauge invariant field strength $\mathcal{F} = B|_{\Pi_3^\alpha} - \frac{l_s^2}{2\pi}F$ vanishes on it. Since $b_1(\Pi_3^\alpha) = 1$, Π_3^a contains a harmonic one-form ζ and a Poincaré dual two-cycle π_2. It may then either happen that π_2 is homologically trivial or non-trivial in the ambient space $\mathbf{X}_6$. Our next step is to discuss what happens in each case

π_2 is trivial in the ambient Calabi-Yau If π_2 is trivial, then, any bulk closed two-form will integrate to zero over it. As a consequence the pull-back of the B-field on Π_3^α will be an exact one-form and so one can trivially satisfy the supersymmetry condition $\mathcal{F} = 0$ by switching on the appropriate field strength $F = dA$. When moving in the moduli space of B-fields the profile for such $B|_{\Pi_3^\alpha} = d\beta$ will change continuously, but the condition $\mathcal{F} = 0$ can always be satisfied by adjusting the profile for A. Hence the presence of such D6-brane does not constrain the moduli space of B-field axions.

π_2 is non-trivial in the ambient Calabi-Yau If, on the contrary, π_2 is non trivial in $\mathbf{X}_6$ (more precisely if $[\pi_2] \neq 0$ is an element of $H_2^-(\mathbf{X}_6, \mathbb{Z})$) an obstruction to changing the B-field will appear. Indeed, in that case there is a bulk harmonic two-form ω whose integral over π_2 is non-vanishing and we may in particular assume that $l_s^{-2} \int_{\pi_2} \omega = 1$. As before, switching on a B-field of the form $B = b\,\omega$ will disturb the D6-brane BPS condition (4.2.1), but now the pull-back of the B-field no longer is an exact two-form in the cohomology of Π_3^α, as $\omega|_{\Pi_3^\alpha}$ necessarily contains a harmonic piece that contributes to the integral over π_2. We may now add a field strength F to cancel out the B-field pull-back, but because the harmonic piece of F is quantised this is only possible whenever $b \in \mathbb{Z}$. As a result, when we move along this direction in the B-field moduli space we will generate a worldvolume flux $\mathcal{F} = b\,\rho$ (with ρ such that $l_s^{-2} \int_{\pi_2} \rho = 1$) and supersymmetry will be broken due to the presence of the D6-brane. Finally, because on general grounds (4.2.1) can be interpreted as an F-term condition in the effective four-dimensional theory, one expects that this effect can be understood in terms of a superpotential that lifts such B-field axion.

Open-closed string bilinears in type IIA The latter setup was analyzed in detail in [143] and it is the cornerstone of the inflationary models that we are going to present in this chapter. Under the mentioned assumptions the potential generated could be understood by means of the following superpotential

$$\Delta W_{\text{clas}}^{D6} = \int_{\Sigma_4} (J_c + F_2)^2 \ , \tag{4.2.3}$$

where Σ_4 is a four-chain connecting the reference sLag and a homotopic deformation. This superpotential arises by backreaction of the D6-brane before taking into accound worldsheet instantons. Assuming an infinitessimal deformation given by a normal vector X one could rewrite the former expression as

$$\Delta W_{\text{clas}}^{D6} = \int_{\Pi_\alpha} (J_c + F) \wedge (\iota_X J_c + A) \ . \tag{4.2.4}$$

Note that Δ implies the difference of superpotential between two D-brane positions. Now, we plug into this expression the definition in terms of harmonic forms of Φ (4.1.2) and J_c (3.3.31) and using the fact that the field Φ corresponds to a D6-brane deformation that preserves the BPS conditions (3.5.5) we arrive to

$$W_{\text{clas}}^{D6} = m_j^a \Phi^j T_a \,, \tag{4.2.5}$$

where $m_j^a = \int_{\Pi_\alpha} \omega_2^a \wedge \zeta_j$. Note that the superpotential is non-trivial if m_j^a is non-vanishing which immediately implies the existence of a two-cycle π_2 non-trivial in the ambient Calabi-Yau. As pointed before, this superpotential refers to the difference between two D-brane positions. This implies, without losing generality, that one can consider that at the origin the system describes a supersymmetric configuration and thus, we can safely remove the Δ on the former expression. For the cases which we are going to treat here we will consider only one D6-brane and we rewrite the superpotential as

$$W_{\text{inf}} \ = \ a\,\Phi\,T \,, \tag{4.2.6}$$

where a is a constant that will be fixed later, Φ represents the D6-brane modulus in (4.1.2), and T is a combination of Kähler moduli defined by

$$T \equiv l_s^{-2} \int_{\pi_2} J_c = \sum_a n_a T^a \,, \tag{4.2.7}$$

with $n_a = l_s^{-2} \int_{\pi_2} \omega_a \in \mathbb{Z}$. Hence as advanced, the presence of certain D6-branes supplies yet another source of superpotential for axion fields. Since the above discussion and the derivation of (4.2.6) also hold in the presence of background fluxes, (4.2.6) may be directly added to the expression (4.1.4). There is however a conceptual difference between (4.1.4) and (4.2.6), namely that the latter source of moduli lifting arises from a localised object. Hence in the same spirit of [40] one may use warping effects to lower the masses generated from W_{inf} as compared to those given by W_{mod}, as will be discussed in the next section.

Based on the latter and some further observations, in the next section we will propose two scenarios of chaotic inflation in which the inflaton mass arises form the superpotential (4.2.6). Since the supergravity description that involves W_{inf} is only valid at small values of the inflaton field, to perform the inflationary analysis at arbitrary field values it is necessary to derive the scalar potential microscopically and including α' corrections. This can be done for the B-field axion potential by analysing the DBI action of the D6-brane, as we do in the following.

4.2.1 DBI+CS dimensional reduction

As we have seen in section 3.5 the action for a single D6-brane is given by the Dirac-Born-Infeld (DBI) and Chern-Simons (CS) actions which, in this case, (3.5.1) and (3.5.3) are given by

$$S_{DBI} = -\mu_6 \int d^7\xi \, e^{-\phi} \sqrt{-\det\left(P[E] - \frac{l_s^2}{2\pi} F_2\right)} \,, \tag{4.2.8}$$

and

$$S_{CS} = \mu_6 \int P\left[C \wedge e^{-B}\right] e^{\frac{l_s^2}{2\pi} F} \,, \tag{4.2.9}$$

where

$$E = e^{\phi/2} g + B \quad , \quad \mu_6 = \frac{2\pi}{l_s^7} \quad , \quad C = C_7 + C_5 + C_3 + C_1 \,. \tag{4.2.10}$$

Now, we will consider that the D6 brane is wrapping $\mathbb{R}^{1,3} \times \Pi_3$, where Π_3 is a submanifold of the compact six-manifold $\mathbf{X}_6$ with $b_1(\Pi_3) = 1$ with non-trivial worldvolume flux $\mathcal{F}$. Before performing the dimensional reduction we apply the following four-dimensional Weyl rescaling

$$g_{\mu\nu} \to \frac{g_{\mu\nu}}{\hat{V}_{\mathbf{X}}/2} \,, \tag{4.2.11}$$

where $\hat{V}_{\mathbf{X}_6} = l_s^{-6} V_{\mathbf{X}_6}$ stands for the compactification volume of the covering space in string units. Applying dimensional reduction to (4.2.8) we obtain the following four-dimensional effective field theory

$$S_{4d} = -\int d^4x\, V_0 - \frac{1}{2}\int d^4x\, (\partial_\mu\varphi \;\; \partial_\mu\xi)\, \mathbf{M} \begin{pmatrix} \partial^\mu\varphi \\ \partial^\mu\xi \end{pmatrix}, \tag{4.2.12}$$

we refer the reader to [31] for more details about this computation. In the former expression we have neglected terms with more than two derivatives in four dimensions and kept only up to quadratic terms in the open string fields $(\varphi,\, \xi)$. Now we will focus on the first term of (4.2.12) which corresponds to the scalar potential

Scalar potential The first term in (4.2.12) corresponds to the contribution of the D6-brane to the vacuum energy of the compactification which is given by

$$V_0 \; = \; \frac{1}{2\pi\kappa_4^4 \hat{V}_{\mathbf{X}_6}^2} \int_{\Pi_3} d\hat{\text{vol}}_{\Pi_3} e^{\frac{3\phi}{4}} \tilde{Q} \sqrt{1 + \frac{1}{2e^{\phi}} \mathcal{F}_{ab}\mathcal{F}^{ab}}\,, \tag{4.2.13}$$

where $d\hat{\text{vol}}_{\Pi_3}$ is the volume form of Π_3 in string units, and $\kappa_4^2 = l_s^2/4\pi$. In addition $\tilde{Q}$ is a quadratic polynomial in $(\varphi,\, \xi)$ given in [31]. This polynomial, effectively in the case which we are going to treat could be replaced by the identity. The vacuum energy (4.2.13) will be partially canceled by the presence of 06-planes in the compactifications. Note that, in the case where the D6-brane is wrapping a sLag three-cycle, the vacuum energy will be totally canceled whenever $\mathcal{F} = 0$. Thus, we obtain the following scalar potential for the D6-brane.

$$\begin{aligned} V_{\text{D6}} \;\; &= \;\; \frac{g_s^{3/4}}{2\pi\kappa_4^4 \hat{V}_{\mathbf{X}_6}^2} \int_{\Pi_3} d\hat{\text{vol}}_{\Pi_3} \sqrt{1 + \frac{1}{2g_s} \mathcal{F}_{ab}\mathcal{F}^{ab}} - l_s^{-3} \text{Re}\ \Omega \\ &= \;\; \frac{g_s^{3/4}}{2\pi\kappa_4^4 \hat{V}_{\mathbf{X}_6}^2} \int_{\Pi_3} d\hat{\text{vol}}_{\Pi_3} \left(\sqrt{1 + g_s^{-1}\rho^2\, b^2} - 1 \right)\,, \end{aligned} \tag{4.2.14}$$

where for simplicity we have considered a constant dilaton. In the second line we have set $\mathcal{F} = b\,\rho$, with $b \in \mathbb{R}$, ρ a quantised two-form of Π_3, and $\rho^2 = \frac{1}{2}\rho_{ab}\rho^{ab}$ its squared norm.[5] Moreover, we have assumed that Π_3 is an sLag three-cycle and due to the calibration condition that Re $\Omega|_{\Pi_3} = d\text{vol}_{\Pi_3}$. In general we could consider that Π_3 is not a Lagrangian three-cycle, in this case the pull-back of the Kähler form on Π_3 is given by $e^{\phi/2} J|_{\Pi_3} = j\,\rho$, with $j \in \mathbb{R}$. In order to obtain the proper scalar potential we would need to take into account

$$d\text{vol}_{\Pi_3} \;\; = \;\; \frac{\text{Re}\ \Omega|_{\Pi_3}}{\sqrt{(\text{Re}\ \Omega|_{\Pi_3})^2}}\,, \tag{4.2.15}$$

[5]The precise profile of ρ will be determined by minimisation of the D6-brane potential, taking into account that because $\mathcal{F} = B - \sigma dA$ one can always add an arbitrary exact piece to it. In the small field limit ρ will be harmonic and such that $l_s^{-2}[\rho]$ generates $H^2(\Pi_3, \mathbb{Z})$. For large B-field values one can check that it will also develop an exact component whenever $d\rho^2 \neq 0$.

and the following identity

$$1 \;=\; (J|_{\Pi_3})^2 + (\text{Re }\Omega|_{\Pi_3})^2 + (\text{Im }\Omega|_{\Pi_3})^2 \,. \tag{4.2.16}$$

Thus, imposing the D-term condition Im $\Omega|_{\Pi_3} \equiv 0$ and using the former expressions we arrive to

$$V_{\text{D6}} \;=\; \frac{g_s^{3/4}}{2\pi\kappa_4^4 \hat{V}^2_{\mathbf{X}_6}} l_s^{-3} \int_{\Pi_3} \text{Re }\Omega \left(\sqrt{1 + \frac{\rho^2}{g_s\omega^2}\,(b^2 + j^2)} - 1 \right) \,, \tag{4.2.17}$$

where we have denoted $\omega^2 \equiv (\text{Re }\Omega|_{\Pi_3})^2$ in order to simplify the notation. This scalar potential directly depends on the complexified Kähler modulus T defined in (4.2.7), since applying the above definitions we have that

$$|T|^2 \;=\; b^2 + j^2 \,, \tag{4.2.18}$$

which contains all the dependence on the B-field axion b. Also we could have extra dependence on the saxionic component of the Kähler modulus j from ρ^2/ω^2 since the pullback of the metric into the three-cycle will depend, in general, on j.

Kinetic terms The last term in (4.2.12) contains the kinetic terms for the D6-brane fields φ and ξ, which include a transverse deformation for Π_3 and a Wilson line over its non-trivial one-cycle. In terms of the definition (4.1.2) we are able to write the complexified D6-brane field as $\Phi = \frac{l_s}{\pi}(\xi - \eta\varphi - i\varphi)$. Taking from [31] the explicit expression for the kinetic term matrix $\mathbf{M}$, which is valid for arbitrary values of $\mathcal{F}$ and $J|_{\Pi_3}$, we see that the entry $\mathbf{M}_{\xi\xi}$ is given by

$$\frac{1}{\pi\hat{V}_{\mathbf{X}_6}}\frac{1}{l_s^3}\int_{\Pi_3} d\text{vol}_{\Pi_3} e^{-\phi/4}\sqrt{W_{\mathcal{F}}}\left(\hat{g}^{ab} + \frac{\mathcal{F}^{ac}\mathcal{F}_c{}^b}{g_s W_{\mathcal{F}}}\right)\zeta_a\zeta_b, \quad , W_{\mathcal{F}} = 1 + \frac{1}{2g_s}\mathcal{F}_{ab}\mathcal{F}^{ab} \,, \tag{4.2.19}$$

where as before $\zeta/2\pi l_s$ is the quantised harmonic one-form of Π_3 and $\hat{g}^{ab}$ is the inverse of the induced metric. Therefore, for vanishing worldvolume flux the kinetic term is given by

$$\frac{1}{\pi\hat{V}_{\mathbf{X}_6}}\frac{1}{l_s^3}\int_{\Pi_3} d\text{vol}_{\Pi_3}\, e^{-\phi/4} g^{ab}\zeta_a\zeta_b \;=\; \frac{1}{\pi\hat{V}_{\mathbf{X}_6}}\frac{1}{l_s^3}\int_{\Pi_3} e^{-\phi/4}\zeta\wedge *_3\zeta \,. \tag{4.2.20}$$

On the other hand, for $\mathcal{F} = J|_{\Pi_3} = 0$ one could see that $\mathbf{M}_{\varphi\xi} = -\eta\,\mathbf{M}_{\xi\xi}$ and $\mathbf{M}_{\varphi\varphi} = (1+\eta^2)\,\mathbf{M}_{\xi\xi}$. In this limit we are therefore able to identify $\mathbf{M}$ with the kinetic term $K_{\Phi\bar{\Phi}}$ for the complex field Φ. In fact, we can derive the same kinetic term from the Kähler potentials discussed in section 3.5.1. Indeed, for this let us write (4.1.3) as $K_Q = -2\log\left(\frac{i}{4}\mathcal{F}_{KL}\text{Im } Z^K \text{Im } Z^L\right)$. Then it is easy to check that

$$K_{\Phi\bar{\Phi}} \;\equiv\; [\partial_\Phi\partial_{\bar{\Phi}}K_Q]_{\Phi=0} \;=\; -\frac{1}{2}\left.\frac{\mathcal{F}_{KL}Q^K\text{Im}\,Z^L}{\mathcal{F}_{KL}\text{Im}\,Z^K\text{Im}\,Z^L}\right|_{\Phi=0} \,, \tag{4.2.21}$$

where K_Q is given by (4.1.3). Using the fact that $e^{\phi/2}\iota_X \mathrm{Im}\,\Omega|_{\Pi_3} = -*_3\zeta$ and

$$i\mathcal{F}_{KL}\mathrm{Im}\,Z^L\beta^K = e^{-\phi/4}\mathrm{Im}\,\Omega \quad\Rightarrow\quad i\mathcal{F}_{KL}Q^K\mathrm{Im}\,Z^L = \frac{1}{l_s^3}\int_{\Pi_3} e^{-\phi/4}\iota_X\mathrm{Im}\,\Omega\wedge\zeta\,, \tag{4.2.22}$$

$$i\mathcal{F}_{KL}\mathrm{Im}\,Z^K\mathrm{Im}\,Z^L = 4e^{-\phi/2}\hat{V}_{\mathbf{X}_6}\,, \tag{4.2.23}$$

we recover (4.2.20) from (4.2.21).[6] We refer the reader to [120] and [122] for the underlying technical details regarding these identities. In the following we will use this explicit expression for the kinetic terms to show that, in supergravity low-energy limit, we can understand the scalar potential (4.2.17) as an F-term potential.

4.2.2 Superpotential description

As we have seen, the scalar potential (4.2.17) is non-trivial only when the pull-back two-form $J_c|_{\Pi_3}$ has a harmonic component in the homology of Π_3, and this is only possible when the two-cycle $\pi_2 \subset \Pi_3$ is non-trivial in the homology of $\mathbf{X}_6$. As we discussed before, this situation is when the uperpotential (4.2.6) is developed. We will describe the low-energy regime for small $|T|$ of (4.2.17) as an F-term induced scalar potential. Note, that in this regime we are able to expand the square root of (4.2.17) and thus obtaining

$$V_{\mathrm{D6}} \overset{|T|\ll 1}{=} \frac{g_s^{-1/4}}{4\pi\kappa_4^4\hat{V}_{\mathbf{X}_6}^2}|T|^2 l_s^{-3}\int_{\Pi_3}\rho\wedge *_3\rho\,, \tag{4.2.24}$$

assuming again constant dilaton. Now we would like to compare it with the usual F-term potential in $\mathcal{N}=1$ supergravity. Thus, we need to use that

$$e^K = \frac{g_s^{-1/2}}{8\hat{V}_{\mathbf{X}_6}^3}\,, \tag{4.2.25}$$

together with the inverse of the kinetic terms, which from (A.1.54) and the above read

$$K^{\Phi\bar{\Phi}}|_{\Phi=0} = 8\hat{V}_{\mathbf{X}_6}\, l_s^3\left(\int_{\Pi_3} e^{-\phi/4}\zeta\wedge *_3\zeta\right)^{-1} = \frac{2\hat{V}_{\mathbf{X}_6}g_s^{1/4}}{\pi^2}l_s^{-3}\int_{\Pi_3}\rho\wedge *_3\rho\,, \tag{4.2.26}$$

where we used that ρ and $*_3\zeta$ are proportional in the string frame and that $\int_{\Pi_3}\rho\wedge\zeta = 2\pi l_s^3$. Therefore, in this limit we can write (4.2.24) as

$$V_{\mathrm{D6}} \overset{|T|\ll 1}{=} \frac{1}{\kappa_4^2}e^K K^{\Phi\bar{\Phi}}|\partial_\Phi W_{\mathrm{inf}}|^2\,, \tag{4.2.27}$$

after fixing the value of a introduced in (4.2.6) to

$$a = \frac{2\pi}{l_s}\,. \tag{4.2.28}$$

[6] More precisely, we have that the four-dimensional kinetic terms are $S_{4d}^{kin} = -\frac{1}{\kappa_4^2}\int K_{\Phi\bar{\Phi}}\, d\Phi\wedge *d\bar{\Phi}$ so in the small field limit we have $K_{\Phi\bar{\Phi}} = \frac{\pi}{8}\mathbf{M}_{\xi\xi}$. To connect to the notation of [122] one should replace $\mathrm{Im}\,Z^K \to l^K$.

This means that we can understand the excess energy of the D6-brane as an F-term induced potential in an $\mathcal{N} = 1$ Minkowski vacuum, in the same spirit as in [144]. Note, however, that the scalar potential that arises from applying the supergravity formula to (4.2.6) has yet another term proportional to $|\partial_T W_{\text{inf}}|^2 = |a|^2|\Phi|^2$ which will stabilize the D6-brane field, and in particular the D6-brane Wilson line. The microscopic origin of this second term can only be detected by taking into account global aspects of backreaction of the D6-branes in the model, as discussed in [143]. Instead of that, we will give in the following section an alternative derivation for the scalar potential of Φ based on the coupling of an axion and a four-form and thus recovering the Kaloper-Sorbo lagrangian.

4.2.3 Obtaining the Kaloper-Sorbo lagrangian

As we have seen in section 1.2.2 the Kaloper-Sorbo formalism [15,16] provides a four-dimensional framework in which non-renormalizable higher-dimensional operators are under control in a UV completed large-field inflationary model based on axions. It could be understood as an axion-monodromy model of inflation. Also, as we saw in section 2.3.2, F-term axion monodromy models embed naturally this framework, where the four-dimensional description given by Kaloper-Sorbo was recovered from dimensional reduction of string theory compactifications in [21]. In this section we will show explicitly how the Kaloper-Sorbo lagrangian could be achieved in our model.

In the computations done in the last section we have implicitly ignored the fact that the DBI action does not depend on the pull-back $B|_{\Pi_3}$ but rather on the gauge invariant worldvolume flux $\mathcal{F} = B|_{\Pi_3} - \frac{l_s^2}{2\pi}F$ where F is the field strength associated to the U(1) theory living on the worldvolume of the D6-brane. As we saw in (3.5.2) and (3.5.6) this field strength could be decomposed as

$$\frac{l_s^2}{2\pi}F = \frac{l_s^2}{2\pi}dA + n_F\rho, \qquad n_F = \mathbb{Z}\,. \tag{4.2.29}$$

In the small B-field limit, the role of dA is to remove any exact piece that $B|_{\Pi_3}$ has, so that $\mathcal{F}$ is a harmonic two-form of Π_3. The role of the second component of (4.2.29) is to shift the value of the B-field axion b by an integer n_F. Taking this into account one finds that in the expressions (4.2.17) and (4.2.24) one should replace $b^2 \to (b - n_F)^2$. Or in other words that instead of (4.2.27) we should have

$$V_{\text{D6}} \overset{|T|\ll 1}{=} \frac{1}{\kappa_4^2}e^K K^{\Phi\bar{\Phi}}|D_\Phi W_{\text{inf}} - a\, n_F|^2\,, \tag{4.2.30}$$

which has its minimum at $b = n_F$. Since n_F can take any possible integer value, we actually have a multi-branched potential, which recovers the periodicity of the axion moduli space broken by the superpotential. Indeed, for quantized values of the B-field axion we can go back to zero energy by changing the integral of F, which is interpreted as a change of potential branch. The same structure is obtained in the

DBI potential (4.2.17), which contains the α' corrections to the supergravity scalar potential.

This sort of multi-branched structure for supergravity potentials has been recently studied in [20], where it was argued that it is generally obtained when coupling four-dimensional four-forms to axions or polynomials thereof. The simplest possibility for such coupling is of the form

$$\int_{X_4} -\frac{Z}{2}F_4' \wedge *F_4' - \frac{1}{2}f_{\hat{\xi}}^2 d\hat{\xi} \wedge *d\hat{\xi} + \sqrt{Z} f_{\hat{\xi}} \mu\, \hat{\xi} F_4' \,, \tag{4.2.31}$$

where $\hat{\xi}$ is a dimensionless axion of period one given by $\hat{\xi} = l_s\, \xi$ and F_4' a four-form in four dimensions whose kinetic term is Z. All mass dimensions are fixed by $[\sqrt{Z}] = [f_{\hat{\xi}}] = [\mu] = L^{-1}$.

Following [145] one may express this Lagrangian in terms of a shifted four-form $\tilde{F}_4$, which we then integrate out. The resulting Lagrangian contains a scalar potential of the form

$$V \,=\, \frac{1}{2}\left(\sqrt{Z}c + \mu f_{\hat{\xi}}\hat{\xi}\right)^2 \,, \tag{4.2.32}$$

where c is an integration constant quantized in terms of the four dimensional domain wall charge as [146]

$$c \,=\, \frac{e}{Z}n \qquad n \in \mathbb{Z} \,. \tag{4.2.33}$$

Finally, the discrete symmetry of this theory imposes the relation $|e| \,=\, \mu f_{\hat{\xi}}\sqrt{Z}$, where $f_{\hat{\xi}}$ is the axion decay constant, and so this allows to rewrite the potential as

$$V \,=\, \frac{1}{2}\mu^2 f_{\hat{\xi}}^2 (n+\hat{\xi})^2 \,=\, \frac{1}{2}\frac{e^2}{Z}(n+\hat{\xi})^2 \,. \tag{4.2.34}$$

We see that the former potential has the same multi-branced structure as (4.2.30).

Note that this setup has the same ingredients as in [21], namely some B-field b and Wilson line $\hat{\xi} \,=\, \pi \mathrm{Re}\,\Phi$ axions with a superpotential generating a mass for them. Therefore one would also expect to recover a multi-branched potential whose discrete symmetry is still preserved once that α' corrections have been taken into account, as we have already obtained for the case of b. Although the scalar potential for the Wilson line $\hat{\xi}$ is invisible to the DBI analysis done in the last section, we are able to recover a Kaloper-Sorbo lagrangian from the D6-brane CS action

$$\frac{\mu_6 l_s^2}{2\pi}\int_{X_4\times\Pi_3} C_5 \wedge F \,=\, \frac{1}{l_s^6}\int_{X_4}\hat{\xi}F_4' \cdot \int_{\Pi_3}\zeta \wedge \omega \,=\, \frac{2\pi}{l_s^3}\int_{X_4}\hat{\xi}F_4' \,, \tag{4.2.35}$$

where $F_4' = dC_3'$ and we have decomposed the RR potential C_5 and D6-brane gauge potential A as

$$A \,=\, l_s^{-1}\hat{\xi}\zeta \qquad , C_5 \,=\, C_3' \wedge \omega \qquad , \omega \,=\, n_a \omega_a \,, \tag{4.2.36}$$

and used that $\int_{\Pi_3}\zeta \wedge \omega = \int_{\Pi_3}\zeta \wedge \rho \,=\, 2\pi l_s^3$. Finally, a term of the form $-\frac{1}{2}\int_{X_4} ZF_4' \wedge *F_4'$ will arise from the dimensional reduction of the 10d kinetic term $\int (dC_5)^2$ in the ten-dimensional type IIA supergravity Lagrangian (3.2.1). Thus, we recover the full Kaloper-Sorbo Lagrangian (4.2.31), with

$$Z = \frac{1}{4\kappa_4^2} g_s^{1/2} \hat{V}_{\mathbf{X}_6}^3 K_{T\bar{T}} = \frac{1}{32\kappa_4^2} e^{-K} K_{T\bar{T}} \,, \tag{4.2.37}$$

where we have used (A.1.56). Using this expression for Z and (4.2.35) we see that the scalar potential felt by the Wilson line at small field values is given by

$$V = \frac{1}{\kappa_4^2} e^K K^{T\bar{T}} \frac{a^2}{\pi^2} \left(\pi \mathrm{Re}\, \Phi - n\right)^2 \quad \rightarrow \quad V = \frac{1}{\kappa_4^2} e^K K^{T\bar{T}} |D_T W_{\mathrm{inf}} - l_s^{-1} 2n|^2 \,, \tag{4.2.38}$$

with a again given by (4.2.28). Note that we provided an independent derivation of this value. We have extended the potential to include the saxion dependence, which can be included directly or by applying the approach in [147]. Here n labels each of the branches of the potential, and the $n = 0$ branch is directly described by the F-term generated potential applied to (4.2.6). As usual, transition between these branches is possible via domain wall nucleations. Since the four-dimensional three-form that these domain walls couple to arises from the dimensional reduction of the RR potential C_5, these domain walls must correspond to D4-branes wrapping the non-trivial two-cycle π_2 of the D6-brane that is also non-trivial in the bulk. Microscopically such domain walls shift the value of the internal RR flux $F_4 = dC_3$ along the four-form of $\mathbf{X}_6$ Poincaré dual to π_2. As a result, in the system at hand an internal large gauge transformation on the D6-brane implies a discrete shift in the Wilson line $\pi\mathrm{Re}\,\Phi \rightarrow \pi\mathrm{Re}\,\Phi + k$ and a compensating discrete shift of F_4 in the Poincaré dual of $[\pi_2]$.

4.3 Large-field inflation with stabilizer fields in type IIA

In the former section we have seen the scalar potential obtained for the complexified D6-brane position Φ and T with the specific topology described. This scalar potential should be completed with all the other closed string moduli sourced by background fluxes. In order to describe a consistent theory of inflation we should be able to stabilize all the closed string sector at a higher scale compared to the Hubble scale. Now we will discuss the interplay between the inflationary potential and the potential for the closed string sector. The strategy that we will follow here is to consider the low-energy $\mathcal{N} = 1$ theory where the inflationary superpotential will be given by (4.2.6). The full superpotential will be given by

$$W = W_{\mathrm{mod}} + W_{\mathrm{inf}} \,, \tag{4.3.1}$$

with W_{mod} given by (4.1.4) and W_{inf} by (4.2.6). Such supergravity framework allows to see if a hierarchy of mass scales can be obtained between the inflaton candidate and all the other moduli, and how taking the inflaton away from its minimum affects the stabilization of heavier scalars.

If one succeeds in decoupling the inflaton sector from the rest of the compactification moduli, then a natural question is whether one can recover a four-dimensional $\mathcal{N} = 1$ supergravity model of chaotic inflation with stabilizer fields like the ones shown in [131, 132, 148–150]. Those models given in supergravity are based on a bilinear superpotential like (4.2.6), as well as on a particular sort of Kähler potentials that allow to give masses above the Hubble scale to all the scalars in Φ and T except the inflaton. One could analyze these questions in the context of type IIA compactifications with the superpotential (4.3.1) where two different scenarios of large-field inflation arise naturally. First of all we will analyze the case where inflation could be driven by the B-field axion which is complexified in T. Afterwards we will consider the case where the inflaton candidate could be the D6-brane Wilson line which is complexified in Φ

4.3.1 Inflating with the B-field

In this section we will consider as the inflaton candidate the B-field axion which is $\mathrm{Re}\, T$

$$b = l_s^{-2} \int_{\pi_2} B = \sum_a n_a b^a \,, \tag{4.3.2}$$

which was analyzed in [31, 130]. First of all we will consider that T is a linear combination of Kähler moduli that does not appear in the flux superpotential W_{mod}. In this kind of models, the mass scale of the closed string sector will adquire a mass above the Hubble scale from W_{mod} while the fields that appear in the inflationary sector will be stabilized in a lower scale. In this case one could integrate out all the massive moduli and keep an effective field theory for the complex fields T and Φ, whose dynamics will be dictated by an effective potential $V^{\mathrm{eff}}(T, \bar{T}, \Phi, \bar{\Phi})$ obtained after freezing all the other moduli. We refer the reader to Part IV for a detailed computation of backreaction effects. To this potential corresponds an effective Kähler and superpotentials $K^{\mathrm{eff}}(T, \bar{T}, \Phi, \bar{\Phi})$ and $W^{\mathrm{eff}}(T, \Phi)$, and whenever $\partial_T W_{\mathrm{mod}} = \partial_\Phi W_{\mathrm{mod}} = 0$ one would expect that the latter is of the form $W^{\mathrm{eff}} = W_{\mathrm{inf}} + W_0$. Finally, if we impose that $|W_0|$ vanishes or it is very small [7], the effective supergravity model falls into the category considered in [131, 132, 148–150], with the field T containing the inflaton and Φ being a stabiliser field.

Let us in particular consider the analysis of [132] for general Kähler potentials. There it is shown that if $K^{\mathrm{eff}}(T, \bar{T}, \Phi, \bar{\Phi})$ is invariant under the following transformations

$$T \to \bar{T} \tag{4.3.3a}$$
$$T \to T + c, \quad c \in \mathbb{R} \tag{4.3.3b}$$
$$\Phi \to -\Phi \tag{4.3.3c}$$

[7]For means of simplicity one could consider a constant uplifting term in order to achieve dS vacua.

then the supergravity scalar potential $V^{\text{eff}}(T,\bar{T},\Phi,\bar{\Phi})$ is extremized following the inflationary trajectory

$$\Phi \;=\; \text{Im}\,T \;=\; 0\,, \tag{4.3.4}$$

with respect to Φ, $\bar{\Phi}$ and Im T. In our setup, we are able to analyze straightforwardly these symmetries in the full type IIA Kähler potential $K = K_K + K_Q$, assuming that if present in K they will also be there in K^{eff}. On the one hand, it is then easy to check that the last two conditions in (4.3.3) are automatically satisfied. On the other hand, the first one is only satisfied if the intersection numbers $\mathcal{K}_{abc}$ in (3.1.12) are chosen so that K_K only depends on $(T-\bar{T})^2$, something that we will impose in the following.

Apart from checking that (4.3.4) is an extremum of $V^{\text{eff}}(T,\bar{T},\Phi,\bar{\Phi})$ we should also verify that the orthogonal directions are non-tachyonic and in particular whether the masses of the fields Im T and Φ are above the Hubble scale. Following [132] one could do that by analyzing the quartic derivatives of the effective Kähler potential and, thus, finding some stability bounds for the inflationary trajectory. Rather than doing so, we will carry the analysis of such stability bounds directly in terms of the effective potential $V^{\text{eff}}(T,\bar{T},\Phi,\bar{\Phi})$ of this scenario, which we are going to analyze.

Inflaton potential from a two-step approach

Following the results given in [31, 130] one could argue to perform moduli stabilization using a two-step procedure.[8] Illustratively, we will show here the two-step process which will be valid as long as one neglects backreaction of the closed string sector.

First of all one considers type IIA flux compactifications with no D6-branes. The dynamics of the closed string sector will be given by the superpotential W_{mod} as in (4.1.4) but with $\Phi = 0$ and by a Kähler potential which is the sum of (3.1.12) and (3.3.28). One should assume that W_{mod} does not depend on T and that K_K depends on it via $(\text{Im}\,T)^2$. Under this assumptions one should stabilize the closed string sector canceling the F-terms for T^a and N^K with a very small or vanishing value $|W^0_{\text{mod}}|$ for $|W_{\text{mod}}|$ at the locus where the closed string moduli are stabilized. This first step should stabilize all closed string besides T at a mass scale above the Hubble scale.

The second step consists on adding the D6-brane that generates the superpotential W_{inf} which shifts the superpotential to (4.3.1). Finally, as we have see, adding the D6-brane will modify K_Q to (4.1.3).[9] All these changes will modify the expression of the scalar potential, which one can analyze around the trajectory (4.3.4). In particular, the F-terms for the complex structure moduli now read

$$D_{N^K}W \;=\; D_{N^K}W_{\text{mod}} + K_{N^K}W_{\text{inf}}\,, \tag{4.3.5}$$

[8] As we will analyze in detail in Chapter 7, this procedure will not be valid once we take into account backreaction effects.

[9] For compactifications with explicit shift symmetry breaking of Φ one could aso consider (3.5.13)

with $[D_{N^K}W_{\rm mod}]_{\Phi=0} = 0$ from the first step. For the Kähler moduli other than T we have

$$D_{T^\alpha}W \;=\; D_{T^\alpha}W_{\rm mod} + K_{T^\alpha}W_{\rm inf} \;=\; K_{T^\alpha}W_{\rm inf} + \ldots\,, \tag{4.3.6}$$

where in the dots contain terms beyond linear order in Im T, Φ or $\bar\Phi$. We may now plug these expressions into the four-dimensional supergravity scalar potential

$$V \;=\; \kappa_4^{-2}e^K\left(K^{\alpha\bar\beta}D_\alpha W D_{\bar\beta}\bar W - 3|W|^2\right) \qquad \alpha,\beta \;=\; N^K, T^a, \Phi\,, \tag{4.3.7}$$

in order to derive an effective potential $V(T,\bar T,\Phi,\bar\Phi)$ around the locus Im $T = \Phi = \bar\Phi = 0$ up to terms of quadratic order in Im T, Φ, $\bar\Phi$. For a detailed computation we refer the reader to Appendix A. The dependence of $W_{\rm mod}$ on Φ will come typically through D-brane and worldsheet instantons and thus we can consider it negligible. With this at hand, using this procedure, one could find that

$$V = \kappa_4^{-2}e^K\left(K^{\Phi\bar\Phi}\,|\partial_\Phi W_{\rm inf}|^2 + K^{T\bar T}|\partial_T W_{\rm inf} + 2TW_2^0|^2 + 4|a|^2(\text{Re }T)^2(\text{Re }\Phi)^2\right) + \mathcal{O}(W^0_{\rm mod})\,, \tag{4.3.8}$$

where terms of order $|W^0_{\rm mod}|$ are neglected by using the assumption that it stabilized at a small value. As discussed in Appendix A the inflationary trajectory

$$\text{Traj} = \{\text{Re}\,T \neq 0,\, \text{Im}\,T = 0,\, \Phi = 0\}\,, \tag{4.3.9}$$

is stable, in the former potential, in the sense that it is a minimum of the non-inflationary directions. Indeed we have that

$$\partial_{\text{Im}\,T}V|_{\rm Traj} \;=\; \partial_\Phi V|_{\rm Traj} \;=\; \partial_{\bar\Phi}V|_{\rm Traj} \;=\; 0\,, \tag{4.3.10}$$

and that the masses for the canonically normalized saxionic component and the stabilizer field are given by

$$m^2_{\rm saxion}\,|_{\rm Traj} \simeq 6H^2 \quad , \quad m^2_{s_1}\,|_{\rm Traj} \simeq 12H^2 \quad , \quad m^2_{s_2}\,|_{\rm Traj} \simeq 6H^2\,, \tag{4.3.11}$$

where s_1 and s_2 denote the axionic and saxionic components of the stabilizer field, Φ, and H is the Hubble scale. On the trajectory one would find that

$$V|_{\rm Traj} \;=\; \frac{e^K}{2\kappa_4^4}\frac{K^{\Phi\bar\Phi}}{K_{T\bar T}}|a|^2\,\phi_b^2\,, \tag{4.3.12}$$

where $\phi_b = \kappa_4\sqrt{2K_{T\bar T}}\,b$ is the canonically normalized inflaton. This quadratic potential matches the one obtained from (4.2.27) for small values of the field T. We will analyze in detail in section 4.4 the scalar potential for large values of the B-field which is given by the DBI action. In that case we should replace the former expression by (4.2.17)

4.3.2 Inflating with a Wilson line

Following the same reasoning followed in the former section one could also consider the inflaton candidate as the Wilson line ξ complexified in the brane position modulus of the D6-brane Φ, as defined in (4.1.2). Note that the Kähler potential satisfies the following symmetries

$$\Phi \to \bar{\Phi} \tag{4.3.13a}$$
$$\Phi \to \Phi + c, \quad c \in \mathbb{R} \tag{4.3.13b}$$
$$T \to -T \tag{4.3.13c}$$

in some compactifications. Thus, one could consider that the inflationary trajectory would be given by

$$\text{Traj} = \{\text{Re }\Phi \neq 0 \ , \text{Im }\Phi = 0 \ , T = 0\} \ , \tag{4.3.14}$$

and with all the remaining closed string moduli $\{N^K, T^\alpha\}$ stabilized at its supersymmetric point and thus considering W_{mod} as a constant. As done before, one could apply the same two-step procedure and obtain the same scalar potential as before (4.3.8). Note that, in this particular case, since the inflaton candidate comes from the open-string sector one could use the any of the schemes of moduli stabilization for type IIA reviewed in Section 3.4.1. We refer the reader to [91, 93, 94] for more details.

A subtlety in this case is that the closed string moduli has to be stabilized at some locus which should be compatible with $T = 0$ and also satisfy (4.3.13) in the Kähler potential, otherwise the D6-brane which is introduced in the second step cannot be BPS. A concrete scenario where all closed-string moduli are stabilized, following this prescription, could be seen in Appendix B. One can see that this sort of condition is however easy to satisfy in concrete examples by appropriate choices of background fluxes, and then one recovers a superpotential of the form

$$W_{\text{mod}} \;=\; W_1 + W_2 T^2 + \dots \ , \tag{4.3.15}$$

where $\partial_T W_1 = \partial_T W_2 = 0$ and the dots contain higher polynomials in T. Finally, imposing

$$\partial_{\text{Im}\,T} V|_{\text{Traj}} \;=\; \partial_\Phi V|_{\text{Traj}} \;=\; \partial_{\bar{\Phi}} V|_{\text{Traj}} \;=\; 0\,. \tag{4.3.16}$$

One would obtain the same hierarchy obtained in the former section and the inflationary potential is given by

$$V|_{\text{Traj}} \;=\; \frac{e^K}{2\kappa_4^4}\frac{K^{T\bar{T}}}{K_{\Phi\bar{\Phi}}}|a|^2\,\theta^2\,, \tag{4.3.17}$$

where θ is the canonically normalized Wilson line. This reproduces the quadratic potential obtained in the previous section either via supergravity or axion-four-form Lagrangian techniques.

Compared to the case of the B-field shown in the last section, a technical disadvantage of this scenario is that it is not known how to compute the Planck suppressed corrections that may modify the scalar potential for large values of the inflaton. This is because the potential that the Wilson line suffers is due to back-reaction of the D6-brane into the supergravity background, and so the D6-brane action is insensitive to it [143]. Hence, even if like in [83] the inflaton is an open string field, in order to find the scalar potential for large inflaton values would imply computing the relevant α' corrections to the supergravity Lagrangian, which is beyond the scope of this text. Notice however that because the potential arises from an axion-four-form Lagrangian one would argue that these corrections cannot be arbitrary, and that the corrected potential and kinetic terms should be expressed as powers of the initial potential itself [15, 16]. It however remains to be seen whether such corrections will lead to a flattening of the scalar potential for large values of the inflaton field and allow this scenario to be compatible with experimental data.

4.3.3 Generating mass hierarchies

One of the key assumptions of this section is the fact that all scalar fields beyond the inflaton and the stabilizer are stabilized through fluxes and thus gain a mass via W_{mod} which is much higher than the Hubble scale, so that we can neglect their dynamics during inflation up to a good approximation.[10] In particular, one would like that all those heavy closed string moduli gain a mass of at least one order of magnitude above the Hubble scale at the supersymmetric vacuum and two above the inflaton mass. In the supergravity models of chaotic inflation [131, 132, 148–150] this can, in principle, be done by tuning the parameter a in the inflationary superpotential (4.2.6) to a small value, which allows to have an inflaton parametrically lighter than any field entering W_{mod}. In the string constructions considered here this is however not possible, for reasons that we now explain.

Could a be fine-tuned?

For simplicity let us consider a type IIA compactification where the dependence of W_{mod} on Kähler moduli is contained in (4.1.1). Let us then add the superpotential term (4.2.6) that we can write as

$$W_{\text{inf}} = a\Phi\, n_a T^a\,, \tag{4.3.18}$$

with $n_a \in \mathbb{Z}$ as defined below (4.2.7). Notice that the full superpotential then satisfies

$$W \supset W_K(e_0, e_a, m^a, m^0) + W_{\text{inf}} = W_K(e_0, e_a + a l_s \Phi n_a, m^a, m^0)\,, \tag{4.3.19}$$

or in other words, that adding W_{inf} can be absorbed into a redefinition of the flux superpotential integer parameter e_a. As a consequence we have that the superpotential

[10] We will review this approximation in Part IV.

is invariant under the simultaneous shift

$$\pi \mathrm{Re}\,\Phi \;\to\; \pi \mathrm{Re}\,\Phi + \frac{2\pi k}{a l_s} \quad , \quad e_a \;\to\; e_a - 2k n_a\,, \tag{4.3.20}$$

where $k \in \mathbb{Z}$ so that e_a is shifted by an even integer number and flux quantisation around O-planes is left unaffected [151].[11] This discrete shift symmetry is reminiscent of the one encountered in the branched-potential (4.2.38), with now the branches being labeled by the RR four-form quanta e_a. Notice that this makes precise the intuitive picture developed below eq.(4.2.38), where it was concluded that an integer shift of the Wilson line $\pi\mathrm{Re}\,\Phi \to \pi\mathrm{Re}\,\Phi + k$ must be compensated by a corresponding shift in the RR four-form flux, and more precisely along the Poincaré dual of the two-cycle π_2 within the D6-brane, which corresponds to the shift $e_a \to e_a - 2kn_a$ described above. Because this discrete Wilson line shift is a large gauge transformation, the invariance must not only be manifest at the level of the scalar potential, but also at the level of the superpotential, and this is why we can detect it via the above reasoning. Finally, the Wilson line shift in (4.3.20) corresponds to an integer period of the Wilson line only if

$$a = \frac{2\pi}{l_s}\,, \tag{4.3.21}$$

as obtained independently via the expressions (4.2.27) and (4.2.38). We however now see that the fact that a is comparable to the other coefficients in the flux superpotential is not an accident of the model, but that instead is related to the discrete symmetry underlying the multi-branched potential, the same one that it is invoked in [15, 16] and in F-term axion monodromy models [21] in order to protect the scalar potential against dangerous transplanckian corrections.

Choices to build mass hierarchies

Due to the fact that the coefficient a in the bilinear superpotential cannot be tuned to a small value, in order to generate a sufficient mass hierarchy with respect the closed string sector one has, in principle, two options

i) Make the coefficients in W_{mod} large.

For instance, one may scale the flux quanta in (4.1.1) by a large integer number, in the spirit of [70, 152]. This strategy has several drawbacks, first of all this fluxes contribute to the RR tadpoles which means that there is an upper bound. On the other hand, typically larger fluxes will imply to set the scale of the closed string sector above the KK scale and this losing control over our theory. For these reasons we will not consider this strategy in order to achieve a parametrically large hierarchy,

[11] We are assuming that g.c.d.$(n_a) = 1$, which is typically the case for irreducible two-cycles like π_2.

ii) Create hierarchies via the kinetic terms.

Notice that in both of the scenarios described above the physical inflaton mass is suppressed by the open string kinetic term $K_{\Phi\bar{\Phi}}$, as can be seen from (4.3.12) and (4.3.17). Hence, if we construct a setup in which

$$K_{\Phi\bar{\Phi}} \gg K_{\alpha\bar{\beta}} \qquad \alpha, \beta = N^K, T^a \,, \tag{4.3.22}$$

then we will typically generate a hierarchy of masses between the inflaton sector and the fields in $W_{\rm mod}$.

Looking at eq.(4.2.21) and comparing to the kinetic terms for the closed string moduli, we see that (4.3.22) will be easily satisfied with respect to the complex structure moduli in the limit of large complex structure. The hierarchy is not so clear with respect to the Kähler moduli, and in general the answer will depend on the value at which closed string moduli are stabilised.

However, taking into account that Φ is a field localised at the D6-brane worldvolume, one may consider using warping effects in order to generate a hierarchy with the closed string kinetic terms. Indeed, let us consider a type IIA flux compactification with Ansatz

$$ds^2 = Z^{-1/2} g^{4\rm d}_{\mu\nu} dx^\mu dx^\nu + ds^2_{\mathbf{X}_6} \,, \tag{4.3.23}$$

where the warp factor Z only depends on the internal coordinates of $\mathbf{X}_6$. Such backgrounds may develop regions of strong warping, like those analysed in [153], where $Z \gg 1$. If we now place the D6-brane generating the superpotential $W_{\rm inf}$ in such region, the kinetic terms for the D6-brane field Φ will be enhanced with respect to those of the closed string moduli, since the latter come from bulk integrals that are typically insensitive to warping effects. Following similar computations to those in [154], in simple cases one obtains an enhancement for $K_{\Phi\bar{\Phi}}$ which can be encoded in the rescaling of the form

$$Q^K \rightarrow Z^p_{\rm D6} Q^K \,, \tag{4.3.24}$$

where $Z_{\rm D6}$ is an approximate value of the warp factor at the region where the D6-brane is located, and the value of the parameter $p \in [0,1]$ depends on how the warping enters $ds^2_{\mathbf{X}_6}$ and on the specific D6-brane embedding.[12] In any case this enhancement via warping will contribute to increase the value of the open string kinetic terms, hence decreasing the mass of the inflaton system with respect to those moduli affected by $W_{\rm mod}$.

This effect of warping that lowers the inflaton mass can be understood intuitively in the scenario of section 4.3.1 where the inflaton is the B-field. Indeed, in that case the inflaton potential is generated because the pull-back of the B-field induces D4-brane charge and tension on the worldvolume of the D6-brane, and this breaks supersymmetry. Placing the D6-brane in a region of strong warping will warp down such induced tension, flattening the potential and lowering the inflaton mass. In this sense, this mechanism for lowering the inflaton mass is analogous to the one

[12] In terms of a mirror D7-brane without worldvolume fluxes, the case $p = 1$ corresponds to a position modulus and the case $p = 0$ to a Wilson line modulus [154].

used in [40], with our D6-brane replaced by a NS5-brane and the induced D4-brane tension with that of a D3-brane. It is however important to notice two differences with the setup in [40]. First in our case the induced charge is non-conserved (simply because in generic compactifications there are no non-torsional one-cycles that a D4-brane can wrap) hence no anti-brane is needed and the caveats raised in [155] do not apply. Second, as usual in models of F-term axion monodromy the system is supersymmetric at the minimum of the potential [21], and in fact admits an effective supergravity description in the small field regime which we have worked out in the previous section. As a result in this regime the effect of warping should be understood in terms of four-dimensional supergravity quantities. As we have seen above the coefficient in the superpotential $W_{\rm inf}$ are fixed by the discrete symmetry underlying the system, and therefore the only quantity that the warping can affect is the Kähler potential and more precisely the open string kinetic terms.

Scale dependence of the model

In order to illustrate the above discussion let us see how the kinetic terms and masses for the inflaton system and the moduli in $W_{\rm mod}$ depend on the scales of the compactification. As usual the relation between the four-dimensional Planck mass and the string scale is given by

$$M_{\rm pl}^2 = \frac{2\pi \hat{V}^E_{\mathbf{X}_6}}{l_s^2}\,, \tag{4.3.25}$$

where $\hat{V}^E_{\mathbf{X}_6}$ stands for the the compactification volume in string units and in the Einstein frame.[13] After performing the four-dimensional Weyl rescaling

$$g_{\mu\nu} \to \frac{g_{\mu\nu}}{\hat{V}^E_{\mathbf{X}_6}/2}\,, \tag{4.3.26}$$

made in [122] the compactification volume dependence in $M_{\rm pl}^2$ disappears and is encoded in the four-dimensional metric. Therefore, in order to measure mass scales in Planck units we need to compare write them in terms of the mass scale $\kappa_4^{-1} = \sqrt{4\pi}l_s^{-1}$ that has appeared in several instances in the previous sections.

To evaluate the typical value of the kinetic terms we will show the typical lengths of the compactification and of the D6-brane internal worldvolume as

$$\hat{L}_{\mathbf{X}_6} = \left(\hat{V}^E_{\mathbf{X}_6}\right)^{1/6} \quad , \quad \hat{L}_{\Pi_3} = \left(\hat{V}^E_{\Pi_3}\right)^{1/3}\,, \tag{4.3.27}$$

respectively. Then it is easy to see that the Kähler metrics for the open string and Kähler moduli at the minimum of the potential scale as

$$K_{\Phi\bar{\Phi}} \sim \frac{\pi^2}{2} Z^p_{\rm D6} g_s^{-1/4} \frac{\hat{L}_{\Pi_3}}{\hat{L}^6_{\mathbf{X}_6}}\,, \tag{4.3.28}$$

[13] This quantity is simply denoted by $\hat{V}_{\mathbf{X}_6}$ in the rest of the chapter, but here we make the superscript explicit in order to distinguish it from the volume measured in the string frame.

$$K_{T\bar{T}} \sim \frac{1}{2} g_s^{-1} \hat{L}_{\mathbf{X}_6}^{-4}\,, \tag{4.3.29}$$

respectively. At this point the inflaton potential is correctly described by four-dimensional supergravity and so we can extract the inflaton mass for our two scenarios from either eq.(4.3.12) or eq.(4.3.17). In both cases we find that the inflaton mass is given by

$$\kappa_4^2\, m_{\rm inf}^2 \;=\; e^K \left(K_{\Phi\bar{\Phi}} K_{T\bar{T}}\right)^{-1} |\tilde{a}|^2 \;\sim\; \frac{1}{2\pi} \frac{g_s^{3/4} Z_{\rm D6}^{-p}}{\hat{L}_{\mathbf{X}_6}^{8} \hat{L}_{\Pi_3}}\,, \tag{4.3.30}$$

where $\tilde{a} = a\kappa_4$. On the other hand the typical mass of a Kähler modulus that appears in (4.1.1) will be

$$\kappa_4^2 m_{T^\alpha}^2 \;=\; e^K \left(K_{T\bar{T}}\right)^{-2} \frac{(2n)^2}{4\pi} \;\sim\; \frac{n^2}{2\pi} \frac{g_s^{3/2}}{\hat{L}_{\mathbf{X}_6}^{10}}\,, \tag{4.3.31}$$

where $2n \in 2\mathbb{Z}$ is the relevant quantum of RR flux. The quotient of both masses is then

$$\frac{m_{T^\alpha}^2}{m_{\rm inf}^2} \;\sim\; n^2 Z_{\rm D6}^p g_s^{3/4} \frac{\hat{L}_{\Pi_3}}{\hat{L}_{\mathbf{X}_6}^2}\,.\,. \tag{4.3.32}$$

In order to see if this dependence of the compactification scales can give an appropriate hierarchy of scales let us consider the following values

$$\hat{V}_{\mathbf{X}_6}^{\rm st} \sim 10^3 \quad , \quad \hat{V}_{\Pi_3}^{\rm st} \sim 10 \quad , \quad g_s^2 \sim 0.1\,, \tag{4.3.33}$$

where now all the volumes are measured in string units and in the string frame. In terms of the Einstein frame we have that

$$\hat{L}_{\mathbf{X}_6} \sim 10^{15/24} \quad , \quad \hat{L}_{\Pi_3} \sim 10^{11/24} \quad , \quad g_s^2 \sim 0.1\,, \tag{4.3.34}$$

and so plugging these values in the expressions that we have seen we find that the inflaton mass in Planck units is given by

$$\kappa_4\, m_{\rm inf} \;\sim\; Z_{\rm D6}^{-p/2} 10^{-35/10}\,. \tag{4.3.35}$$

Hence one recovers the standard value of $m_{\rm inf} \sim 10^{13} GeV$ by considering a warp factor of the order $Z_{\rm D6}^p \sim 10^3$. Finally, plugging the values (4.3.34) into (4.3.32) we find

$$m_{T^\alpha}^2 \sim 10^{-1} n^2 Z_{\rm D6}^p\, m_{\rm inf}^2 \qquad \Rightarrow \qquad m_{T^\alpha} \sim 10 n\, m_{\rm inf}\,, \tag{4.3.36}$$

where we have plugged the above value for the warp factor. Thus, we see that setting the flux quanta of the order $n \sim 10$ or higher we find an acceptable hierarchy between the masses induced by the flux superpotential and that of the inflaton candidate.

4.4 Cosmological observables from the DBI

As we have seen before, the case weher the inflaton candidate is the B-field shown in section 4.3.1 has an advantage with respect the Wilson line scenario. This is because we are able to compute the scalar potential coming from the DBI action, which was obtained in section 4.2.

From the supergravity effective potential (4.3.8) and evaluating it at $\Phi = 0$ then we have that

$$V = \frac{\pi}{\kappa_4^4} e^K K^{\Phi\bar{\Phi}} |T|^2 = \frac{1}{\kappa_4^4} \frac{\pi g_s^{-1/2}}{8(\hat{V}_{\mathbf{X}_6}^{E,0})^3} \frac{K^{\Phi\bar{\Phi}}\,(b^2+j^2)}{1-2K^0_{T\bar{T}} j^2}\,, \tag{4.4.1}$$

where $\hat{V}_{\mathbf{X}_6}^{E,0}$ is the compactification volume in the Einstein frame and $K^0_{T\bar{T}}$ the the kinetic terms for the complex field T evaluated at $j = 0$. As in (4.2.18) b stands for the axionic component and j for its saxionic partner. At large values of $|T|$ this potential is replaced by one obtained from the DBI action, namely the square-root potential of eq.(4.2.17). In general, evaluating of such potential will depend on the specific geometry of the three-cycle Π_3 wrapped by the D6-brane. Let us however take the simplifying assumption that the quantity ρ^2/ω^2 inside the square bracket is constant over Π_3 and independent of j. In that case the potential can be approximated by

$$V_{\rm D6} \simeq \frac{1}{\kappa_4^4} \frac{g_s^{3/4}\hat{V}^0_{\Pi_3}}{2\pi(\hat{V}^0_{\mathbf{X}_6})^2} \frac{1}{1-2K^0_{T\bar{T}} j^2} \left(\sqrt{1+\frac{\pi^2 K^{\Phi\bar{\Phi}}}{2g_s^{5/4}\hat{V}^0_{\mathbf{X}_6}\hat{V}^0_{\Pi_3}}\,(b^2+j^2)} - 1 \right)\,, \tag{4.4.2}$$

which clearly reduces to (4.4.1) for small values of $|T|$. Notice that in this limit the kinetic terms for b and j are not canonical but given by

$$K_{T\bar{T}} = K^0_{T\bar{T}} \cdot \frac{1+2K^0_{T\bar{T}} j^2}{(1-2K^0_{T\bar{T}} j^2)^2}\,. \tag{4.4.3}$$

Due to the fact that this kinetic term arises from a bulk integral computed at an arbitrary point of the Kähler moduli space, we will assume that it will not receive corrections for large values of the inflaton. Therefore the only effects of α' corrections to the inflationary dynamics appears through the square-root behavior of the potential (4.4.2).

Note that the corrected potential (4.4.2) only includes the dependence of one of the two complex fields (T, Φ) of the inflationary sector. Ideally one would like to have a corrected potential for both of the complex fields in order to analyze the stability of the inflationary trajectory (4.3.9). Nevertheless, by the analysis of the previous section and Appendix A we have seen that the inflaton b and its saxionic partner j are the two lightest fields of the system in the supergravity limit. If we assume that such hierarchy of scales is still valid at large field values we may set $\Phi = 0$ and then recover the potential (4.4.2). In the following we will take such approach and analyse the dynamics for the fields b and j from (4.4.2). In fact, in section 4.4.2 we will see that this α'-corrected potential exactly reproduces the saxion mass estimate obtained in (4.3.11). Therefore along the inflationary trajectory it makes sense to set $j = 0$ and study the single field inflationary potential for b, as we will do in the following.

4.4.1 Slow roll parameters for large inflaton vevs

Along the inflationary trajectory (4.3.9) the α' corrected inflationary potential for the B-field b can be taken directly from (4.2.14) by taking Π_3 to be an special Lagrangian. By making the simplifying assumption that ρ^2 is constant along the three-cycle Π_3 (or equivalently that $\rho \wedge *_3\rho$ is harmonic on Π_3) we recover a potential of the form[14]

$$V \simeq \gamma \left(\sqrt{1 + \delta \left(\frac{\phi_b}{M_{\rm pl}} \right)^2} - 1 \right) M_{\rm pl}^4 \,, \tag{4.4.4}$$

where $\phi_b = M_{\rm pl}\sqrt{2K^0_{T\bar{T}}}\, b$ is the canonically normalised B-field in the scenario of section 4.3.1. Alternatively one may take the limit $j \to 0$ in (4.4.2). In both cases one obtains that the dimensionless parameters β and γ are given by

$$\gamma \sim \frac{1}{2\pi} g_s^{3/4} \frac{\hat{V}^{E,0}_{\Pi_3}}{(\hat{V}^{E,0}_{\mathbf{X}_6})^2} \sim 10^{-7} \,, \tag{4.4.5}$$

$$\delta^{-1} \sim \frac{4}{\pi^2} g_s^{5/4} K_{\Phi\bar{\Phi}} K^0_{T\bar{T}} \hat{V}^{E,0}_{\Pi_3} \hat{V}^{E,0}_{\mathbf{X}_6} \sim 10^2 \,, \tag{4.4.6}$$

where we have estimated the value of these parameters by plugging the values (4.3.34) as well as $Z^p_{\rm D6} \sim 10^3$ used in the previous section. As these values may slightly vary from one model to another, in particular β due to the approximations that we have taken, let us take a phenomenological approach and analyse the potential (4.4.4) for the parameter range

$$\delta \sim 10^{-1} - 10^{-3} \quad , \quad \sqrt{\delta\gamma} \sim 10^{-4} - 10^{-5} \,. \tag{4.4.7}$$

Given this single field inflationary potential one may compute the cosmological parameters associated to the range (4.4.7). In particular one finds that slow-roll inflation typically occurs for $1.4M_{\rm pl} < \phi_b < 13 - 15M_{\rm pl}$ for 60 efolds, and for $1.4M_{\rm pl} < \phi_b < 12 - 14M_{\rm pl}$ for 50 efolds, the precise upper limit ϕ_{b*} depending on the value of δ. Since the $b \sim K^{1/2}_{T\bar{T}}\phi_b$ we find that the number of periods that the axion must undertake is of order 10^2.

In general, cosmological parameters of the model are mostly sensitive to the value of δ, which interpolates between a model of quadratic chaotic inflation ($\delta \sim 10^{-3}$) and linear chaotic inflation ($\delta \sim 10^{-1}$).

[14] Interestingly, such potential form is also recovered in one of the single field limit cases of [83] after the fields have been canonically normalised. See [156] for more details.

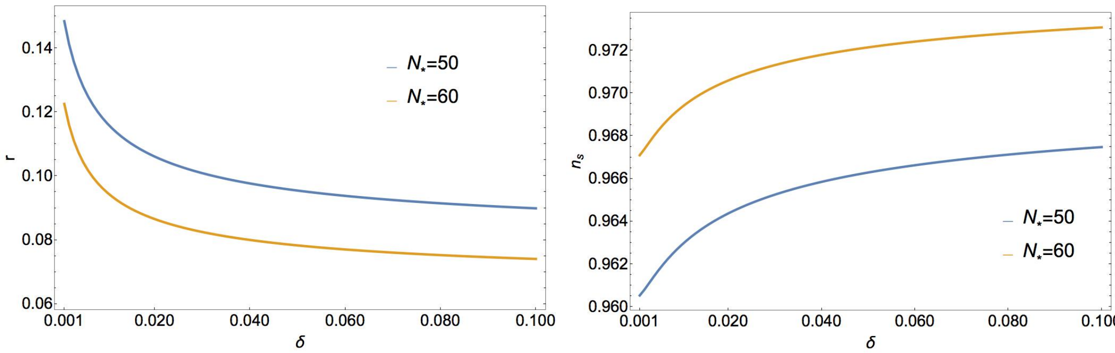


Figure 4.1: Tensor-to-scalar ratio (left) and spectral index (right) in terms of δ.

In figure 4.1 we display the tensor-to-scalar ratio and the spectral index in terms of the parameter δ, for the number of efolds $N_* = 50$ (blue line) and $N_* = 60$ (red line). Their behaviour can be understood in terms of an interpolation from quadratic to linear inflation as we increase the value of δ. Such interpolation is also illustrated by plotting one cosmological parameter in terms of the other and superimposing the result on the plot recently given by the Planck collaboration [75], as we do in figure 4.2.[15]

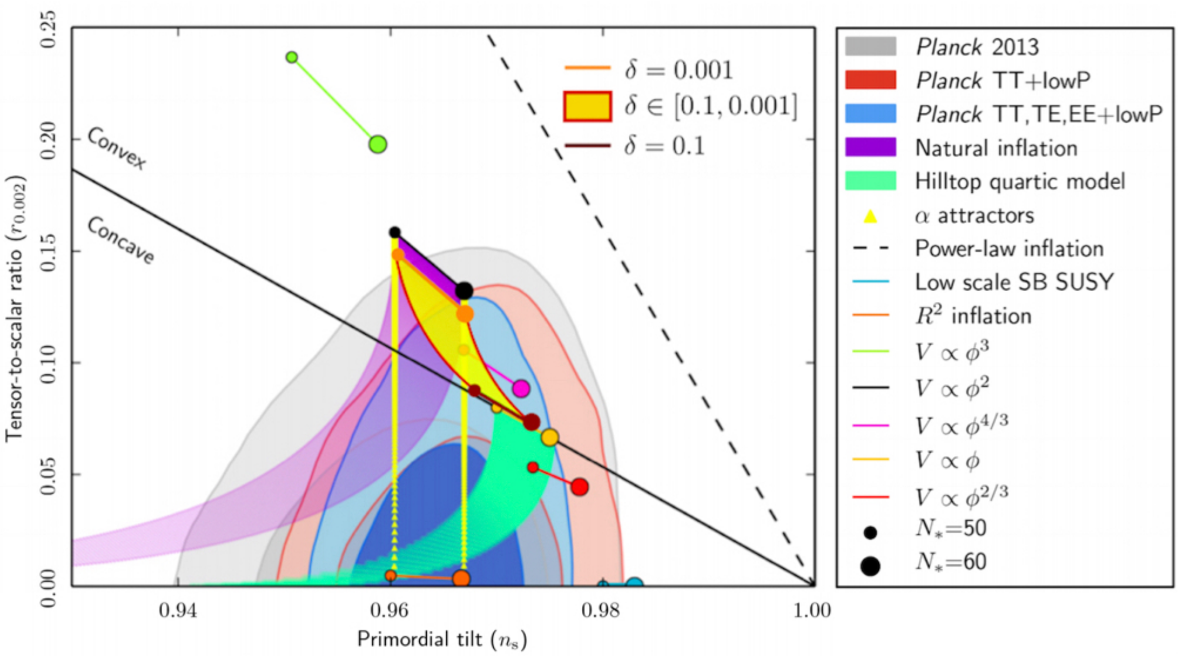


Figure 4.2: Primordial tilt n_s vs tensor-to-scalar ratio r superimposed by the plot given by Planck Collaboration (2015) [75]. The yellow area shows the region of parameters covered by the potential (4.4.4) for the parameter range $\delta \sim 10^{-1} - 10^{-3}$.

[15]This interpolation is also recovered in the context of field theory in [157], up to UV completion effects.

4.4.2 Stability bounds on the DBI potential

Given the α'-corrected potential (4.4.2) we may revisit the computation that, in the supergravity approximation, led us to the estimate (4.3.11) for the mass of the saxion j along the inflationary trajectory. For this it is useful to rewrite the potential (4.4.2) as

$$V \simeq \frac{\gamma}{1 - 2K^0_{T\bar{T}} j^2} \left(\sqrt{1 + \lambda\,(b^2 + j^2)} - 1 \right) M_{\rm pl}^4 \,, \tag{4.4.8}$$

where γ is given in (4.4.5) and

$$\lambda = 2K^0_{T\bar{T}}\,\delta \,. \tag{4.4.9}$$

Now, we will repeat the computation below eq.(A.1.34) for the current potential. As in there we have that

$$m^2_{\rm saxion}\,|_{\rm Traj} = \frac{1}{2K_{T\bar{T}}} \partial_j^2 V\,|_{\rm Traj} \,, \tag{4.4.10}$$

with the trajectory given by (4.3.9) which implies $j = 0$. We then find

$$\begin{aligned} m^2_{\rm saxion}\,|_{\rm Traj} &= \gamma \left(\frac{\lambda/(2K^0_{T\bar{T}})}{\sqrt{1 + \lambda\, b^2}} + 2\left[\sqrt{1 + \lambda\, b^2} - 1 \right] \right) && (4.4.11) \\ &= \gamma \left[\sqrt{1 + \lambda\, b^2} - 1 \right] \left(\frac{2}{1 + \frac{1}{\sqrt{1 + \lambda\, b^2}}}\,\epsilon + 2 \right) && (4.4.12) \\ &= 3H^2 \left(\frac{2}{1 + \frac{1}{\sqrt{1 + \lambda\, b^2}}}\,\epsilon + 2 \right) \,, && (4.4.13) \end{aligned}$$

where

$$\epsilon = \frac{1}{4K^0_{T\bar{T}}} \left(\frac{b\,\lambda}{\sqrt{1 + \lambda b^2}(\sqrt{1 + \lambda b^2} - 1)} \right)^2 \,. \tag{4.4.14}$$

During inflation $\epsilon \ll 1$ and so we can neglect the piece proportional to it. Then we obtain

$$m^2_{\rm saxion}\,|_{\rm Traj} \simeq 6H^2 \,, \tag{4.4.15}$$

in agreement with the supergravity result (A.1.38) and thus we see that this model, naturally, predicts single field inflation.

Part III

Inflation in type IIB

5
Flux-flattening in axion monodromy inflation

In this chapter, we would like to point out a new source of flattening that we dub as *flux flattening* [158]. This source of flattening is only visible for sufficiently large field ranges and hence it is not captured in the supergravity limit. Therefore, it represents an additional source of flattening to the effects seen in the supergravity literature.

We will analyze flux flattening in the context of type IIB/F-theory flux compactifications with mobile D7-branes [32, 33, 83], where this effect is easily described. Indeed, it is well known that in the presence of three-form background fluxes D7-branes experience a potential as we displace their position moduli from the vacuum. At small field values, such potential only depends on certain flux components, namely those that induce a non-supersymmetric B-field on the D7-brane worldvolume [159, 160]. However, at large field values all background fluxes will contribute to the D7-brane energy, as one can see through direct evaluation of the DBI+CS action. Moreover, the kinetic term of a given position modulus will also depend on all these fluxes, resulting in an inflaton dependent kinetic term that will flatten the potential. The latter effect was already observed in [83] for a particular choice of background fluxes allowed by an orbifold projection. There, the growth of the kinetic terms with large inflaton values matched that of the potential, resulting in flattening to a linear potential. As we will show, once all background fluxes are taken into account the growth of the D7-brane position kinetic term will always be larger than that of its potential, thus inducing larger flattening effects than those observed in [83]. The functional dependence of the scalar potential that arises in this more general case has moreover a richer structure and interesting phenomenological features.

5.1 D7-branes antipasti

As we mentioned above, for the description of flux flattening we will focus on scenarios of large-field inflation where the inflaton candidate will be D7-brane position moduli lifted by the presence of background fluxes. The potential generated for such moduli can be easily computed by means of four-dimensional supergravity for small inflaton vevs but, as shown in [32], in the regime of interest for inflation this approximation fails and one should compute the potential directly from the D7-brane action. This large-field computation was carried out in [83] for the restricted set of ISD background fluxes that respect the orbifold symmetry of the Higgs-otic setup, and generalized in [161] to include IASD fluxes respecting the same symmetry. In this section we will give, briefly, the background needed in order to get a deeper insight of the flux flattening mechanism.

Needed Ingredients For the sake of simplicity we will focus in this section on type IIB toroidal flux compactifications with O3/O7-planes (for more details see Section 3.4.2) where the internal manifold could be described by $\mathbf{T}^4/\mathbb{Z}_2 \times \mathbf{T}^2$. We will consider the presence of ISD background fluxes, G_3, with only (0,3) components since they will be the ones that will generate non-supersymmetric worldvolume flux, $\mathcal{F}$, on the D7-brane. Note that, due to the presence of background fluxes, we have to deal with warped Calabi-Yau compactifications. We define G_3 as

$$G_3 = G_{\bar{1}\bar{2}\bar{3}}\, d\bar{z}_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3 \,, \tag{5.1.1}$$

where $G_{\bar{1}\bar{2}\bar{3}}$ is approximated to be constant.

On the other hand we consider that the space-filling D7-brane is wrapping $\mathbf{T}^4/\mathbb{Z}_2$ and we identify the normal coordinate to the D7-brane position modulus via $z_3 = \sigma\Phi$. The position moduli of D7-branes are sensitive to the presence of these fluxes due to the pullback of the B-field on their worldvolume. Since the D7-brane is describing a periodic motion on $\mathbf{T}^2$ the presence of the G_3-flux will be the source of the monodromy. In a neighborhood of the D7-brane we integrate the relation $dB_2 = -\frac{\mathrm{Im}G_3}{\mathrm{Im}\tau}$ obtaining the following components for the B-field

$$B_{12} = -\frac{g_s}{2i}\,(2\pi\alpha')\,\bar{G}_{\bar{1}\bar{2}\bar{3}}\Phi dz^1 \wedge dz^2 \;,\; B_{\bar{1}\bar{2}} = -\frac{g_s}{2i}\,(2\pi\alpha')\,G_{\bar{1}\bar{2}\bar{3}}\bar{\Phi} d\bar{z}^1 \wedge d\bar{z}^2 \,. \tag{5.1.2}$$

This expression, straightforwardly, shows that the worldvolume flux $\mathcal{F} = B_2 - (2\pi\alpha')\,F$ will be position-dependent. As we have seen in Section 3.4.2 supersymmetry will be achieved when $\mathcal{F}^{(0,2)} = 0$. This condition, in absence of magnetic fluxes on the D7-brane, will be achieved whenever

$$B_{\bar{1}\bar{2}} = 0 \rightarrow \Phi = 0 \vee G_{\bar{1}\bar{2}\bar{3}} = 0 \,. \tag{5.1.3}$$

In order to obtain the dimensional reduction of the DBI (3.5.1) and CS (3.5.3) action, one also will need the values of the RR potentials and fluxes that enter in the CS action. Since we are going to show a more general computation in the next section we will leave the details there. The dimensional reduction, at weak coupling limit, of the DBI+CS will give us the following scalar potential

$$V_{\mathrm{DBI}} \approx \mu_7 \left(2\pi\alpha'\right) \int_{\mathbb{R}^{1,3}\times\mathcal{S}} \frac{1}{2}\mathcal{F}\wedge *\mathcal{F} \approx \mu_7 \left(2\pi\alpha'\right) \left|\bar{G}_{\bar{1}\bar{2}\bar{3}}\Phi\right|^2 , \tag{5.1.4}$$

and the kinetic term $g\left(\Phi\right) D_\mu \Phi D^\mu \bar{\Phi}$

$$g\left(\Phi\right) = 1 + \frac{1}{2}\left|\bar{G}_{\bar{1}\bar{2}\bar{3}}\Phi\right|^2 . \tag{5.1.5}$$

We see that the scalar potential coming from the DBI gives us naturally a quadratic potential for the inflaton candidate. However, once we take into account the canonically normalization for the inflaton field one see that for large values of Φ the scalar potential is flattened. But this flattening is rather limited due to the fact that the kinetic term could not grow more than the scalar potential. In fact, this flattening will is limited to linear inflation as noted in [83]. As a final remark, we would like to note that this flattening effect is similar to the one obtained in other axion monodromy models like [40, 80].

Recovering Kaloper-Sorbo lagrangian The Kaloper-Sorbo lagrangian (1.2.31) could be easily recovered from the DBI dimensional reduction of the former system. Here we will show, briefly, how the coupling between the inflaton candidate and a four-form arises naturally. The details of a similar computation could be found in [20, 83]. The DBI+CS action contain a term

$$\mu_7 \left(2\pi\alpha'\right) \int_{\mathbb{R}^{1,3}\times\mathcal{S}} \frac{1}{2}\mathcal{F}\wedge *\mathcal{F} \subset S_{DBI} , \tag{5.1.6}$$

from where the coupling between the inflaton candidate and the four-form, needed to reproduce the Kaloper-Sorbo lagrangian, could be argued from

$$\int_{\mathcal{S}} B_2 \wedge F_6 = \frac{1}{2} g_s \left(2\pi\alpha'\right) \left(F_4 \bar{G}_{\bar{1}\bar{2}\bar{3}}\Phi + \bar{F}_4 G_{\bar{1}\bar{2}\bar{3}}\bar{\Phi}\right) \int_{\mathcal{S}} \omega_2 \wedge \bar{\omega}_2 , \tag{5.1.7}$$

where we have expanded $F_6 = iF_4 \wedge \bar{\omega}_2 - i\bar{F}_4 \wedge \omega_2$ and ω is a (2,0)-form associated to the position modulus of the D7-brane. Integrating out the four-form one obtains the typical multibranched scalar potential shown in former sections

$$V \sim \mu_7 \left(2\pi\alpha'\right) \left|\lambda - \frac{1}{2} g_s \left(2\pi\alpha'\right) \bar{G}_{\bar{1}\bar{2}\bar{3}}\Phi\right|^2 , \tag{5.1.8}$$

where λ is a complex number which comes from the quantized magnitic fluxes on the worldvolume of the D7-brane transformed to the complex basis where the position of the D7-brane is defined.

Supergravity description The supergravity description of the model shown above could be described following [32] by the $\mathcal{N}=1$ lagrangian shown below

$$K = K^K + K^{\rm cx}\left(U^i, S, \left(\Phi - \bar{\Phi}\right)\right) , \qquad (5.1.9)$$

$$W = W_{\rm flux}\left(U^i, S\right) + W_K\left(T^i\right) + W_{\rm inf}\left(\Phi^2\right) , \qquad (5.1.10)$$

where U^i and S denote complex structure moduli and the axio-dilaton respectively and T^i denote the Kähler moduli sector. Note that W_K is the superpotential coming from non-perturbative effects which will stabilize the Kähler moduli. As a final remark, the inflaton candidate will be the axionic component of Φ, which as we can see does not appear in the Kähler potential. For the reader interested in more details about the supergravity description we refer to Section 5.3 and Chapter 8.

5.2 D7-branes and flux flattening

In the following we would like to generalize the results argued in the former section and the computation done in [83] by including the most generic set of ISD background fluxes that will appear in general compactifications with mobile D7-branes like in [32], and to consider varying dilaton and warp factors. As we will see, while the effect of these extra fluxes does not appear in the scalar potential for small D7-brane displacements (and it is therefore invisible in the supergravity approximation) it produces an important flattening in the scalar potential for sufficiently large values of the D7-brane position modulus.

5.2.1 Needed ingredients

As mentioned in the former section we will consider type IIB/F-theory flux compactification with a 10d Einstein frame metric of the form

$$ds^2_{10} = Z^{-1/2}(y)dx^\mu dx_\mu + Z^{1/2}(y)\hat{g}_{mn}(y)dy^m dy^n , \qquad (5.2.1)$$

where $\hat{g}$ is an F-theory three-fold metric on the internal space, with Kähler form $\hat{J}$ and holomorphic $(3,0)$-form $\Omega_0 = g_s^{1/2}\hat{\Omega}$, and Z is the warping. As in [95], on top of this background there is a set of D7-branes sourcing a holomorphic axio-dilaton $\tau = C_0 + ig_s^{-1}$, D3-branes sourcing Z and the self-dual RR flux F_5, and an imaginary-self-dual (ISD) three-form flux background $G_3 = F_3 - \tau H_3$.

Let us now look at a neighbourhood of a D7-brane wrapping a four-cycle $\mathcal{S}$, and introduce local coordinates (z_1, z_2, z_3) such that the D7-brane is localized in the z_3-plane. In such a region we consider an ISD primitive three-form flux G_3 of the form

$$G_3 = S_{\bar{1}\bar{1}}\, d\bar{z}_1 \wedge dz_2 \wedge dz_3 + S_{\bar{2}\bar{2}}\, dz_1 \wedge d\bar{z}_2 \wedge dz_3 + S_{\bar{3}\bar{3}}\, dz_1 \wedge dz_2 \wedge d\bar{z}_3 + G_{\bar{1}\bar{2}\bar{3}}\, d\bar{z}_1 \wedge d\bar{z}_2 \wedge d\bar{z}_3 , \qquad (5.2.2)$$

where $S_{\bar{k}\bar{k}}$ and $G_{\bar{1}\bar{2}\bar{3}}$ are approximated to be constant. As we have seen in last section, the presence of the G_3 flux will translate into dynamics of the D7-brane by the pullback of the B-field on its worldvolume. In particular, in the proximity of the D7-brane we can integrate the relation $dB_2 = -\mathrm{Im}\, G_3/\mathrm{Im}\tau$, obtaining

$$B_2 = -\frac{g_s}{2i}\Big[S_{\bar{1}\bar{1}}\, z_3\, d\bar{z}_1 \wedge dz_2 + S_{\bar{2}\bar{2}}\, z_3\, dz_1 \wedge d\bar{z}_2 + S_{\bar{3}\bar{3}}\, \bar{z}_3\, dz_1 \wedge dz_2 + G_{\bar{1}\bar{2}\bar{3}}\, \bar{z}_3\, d\bar{z}_1 \wedge d\bar{z}_2 - \text{h.c.}\Big]\,, \tag{5.2.3}$$

where, as before, we identify the D7-brane with the brane position modulus via $z_3 = \sigma\Phi$, with $\sigma = 2\pi\alpha' = l_s^2/2\pi$. This implies that the pullback of the B-field on the worldvolume of the D7-brane, and therefore $\mathcal{F} = B_2 - \sigma F$, will depend on its location. Since supersymmetry is achieved when $\mathcal{F}^{(0,2)} = 0$ on the D7-brane, we see that the flux components $S_{\bar{3}\bar{3}}$ and $G_{\bar{1}\bar{2}\bar{3}}$ will naturally stabilize the brane position modulus at loci where this condition is met, which for vanishing magnetic fluxes on the worldvolume of the D7-brane is attained at $B^{(0,2)} = 0$ or equivalently $z_3 = 0$.

In addition to the form of the G_3 flux we will need the values of the RR fluxes and potentials that enter the D7-brane Chern-Simons action. In particular we will need the following set of relations

$$dC_6 - H_3 \wedge C_4 = -g_s *_{10}\, \mathrm{Re}\, G_3 \; = \; -Z^{-1} d\mathrm{vol}_{\mathbb{R}^{1,3}} \wedge H_3, \tag{5.2.4}$$

$$dC_8 - H_3 \wedge C_6 = g_s^2 *_{10}\, \mathrm{Re}\, d\tau \; = \; -\frac{1}{2} d\left(g_s\, d\mathrm{vol}_{\mathbb{R}^{1,3}} \wedge \hat{J} \wedge \hat{J}\right), \tag{5.2.5}$$

that can be obtained from the equations of motion. Finally we have that

$$\tilde{F}_5 \; = \; dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3 \; = \; (1 + *_{10}) d\chi_4\,, \tag{5.2.6}$$

where

$$\chi_4 = \chi\, d\mathrm{vol}_{\mathbb{R}^{1,3}}\,, \qquad d\chi \; = \; dZ^{-1}. \tag{5.2.7}$$

With this at hand we proceed to compute the scalar potential felt by a D7-brane.

5.2.2 The DBI+CS computation

In this section we will perform the dimensional reduction of the DBI and CS actions which control the dynamics of a single D7-brane

$$S_{DBI} = -\mu_7 \int d^8\xi \; g_s^{-1} \sqrt{-\det(P[E_{MN} + \sigma F_{MN})}\,, \tag{5.2.8}$$

$$S_{CS} = \mu_7 \int P\left[\sum_n C_{2n} \wedge e^{-B_2}\right] \wedge e^{\sigma F}\,, \tag{5.2.9}$$

where $P[\;]$ denotes the pull-back on the worldvolume of the D7-brane and

$$E_{MN} = g_s^{1/2} G_{MN} - B_{MN}\,, \qquad \mu_7 = (2\pi)^{-3} \sigma^{-4}\,, \tag{5.2.10}$$

where G is the 10d Einstein frame metric.

Dimensional reduction of the CS action. In order to evaluate the CS action of the D7-brane, first of all, we consider how this action changes between two different D7-brane locations. That is, we consider a reference four-cycle $\mathcal{S}_0$ and take a homotopic deformation $\mathcal{S}$. Since both four-cycles lie in the same homology class there is a five-chain Σ_5 such that $\partial\Sigma_5 = \mathcal{S} - \mathcal{S}_0$, and we have that

$$\begin{aligned}\Delta S_{CS} &= \mu_7 \int_{\mathbb{R}^{1,3}\times\Sigma_5} P\left[d\left(\sum_n C_{2n}\wedge e^{-B_2}\right)\right] \\ &= \mu_7 \int_{\mathbb{R}^{1,3}\times\Sigma_5} (dC_8 - H_3\wedge C_6) - B_2\wedge(dC_6 - H_3\wedge C_4) + \frac{1}{2}\tilde{F}_5\wedge B_2\wedge B_2 + \dots \\ &= \frac{\mu_7}{2}\int_{\mathbb{R}^{1,3}} d\text{vol}_{\mathbb{R}^{1,3}} \int_{\Sigma_5} d\left(Z^{-1}B_2\wedge B_2 - g_s\hat{J}\wedge\hat{J}\right),\end{aligned} \tag{5.2.11}$$

where for simplicity we have turned off the gauge worldvolume flux F, and in the second line we have neglected terms that do not contribute to the chain integral. If in addition we assume that at $\mathcal{S}_0$ the pull-back of B_2 vanishes and the volume contribution cancels with that of the remaining 7-branes we obtain that

$$S_{CS} = \frac{1}{2}\mu_7 \int_{\mathbb{R}^{1,3}} d\text{vol}_{\mathbb{R}^{1,3}} \int_{\mathcal{S}} \left(Z^{-1}B_2\wedge B_2 - g_s\hat{J}\wedge\hat{J}\right). \tag{5.2.12}$$

Dimensional reduction of the DBI action. To dimensionally reduce the DBI action we may follow a procedure similar to the one outlined in [83]. We arrive at the result

$$S_{DBI} = -\mu_7 \int_{\mathbb{R}^{1,3}\times\mathcal{S}} d^8\xi\, g_s \sqrt{\det(g_{ab}) f(\mathcal{F})}\left[1 + 2Z\sigma^2\partial_\mu\Phi\partial^\mu\overline{\Phi} + \frac{1}{2}g_s^{-1}Z\sigma^2 F_{\mu\nu}F^{\mu\nu}\right], \tag{5.2.13}$$

where by Φ we denote the complexified brane position modulus. The function $f(\mathcal{F})$ appearing in (5.2.13) is defined as

$$f(\mathcal{F}) = 1 + \epsilon\,\mathcal{F}^2 + \frac{1}{4}\epsilon^2(\mathcal{F}\wedge\mathcal{F})^2, \tag{5.2.14}$$

where $\epsilon = Z^{-1}g_s^{-1}$ and the contractions are made with the unwarped metric $\hat{g}_{ab}$ of $\mathcal{S}$. Note that, since we are considering more general fluxes than the case appearing in [83], the function $f(\mathcal{F})$ is not a perfect square. Retaining only terms quadratic in derivatives we obtain the following terms from the DBI action

$$S_{DBI} = \mu_7 \int_{\mathbb{R}^{1,3}} d\text{vol}_{\mathbb{R}^{1,3}} \int_{\mathcal{S}} \frac{g_s}{2}\hat{J}\wedge\hat{J}\sqrt{f(\mathcal{F})}\left[1 + Z\sigma^2\partial_\mu\Phi\partial^\mu\overline{\Phi} + \dots\right], \tag{5.2.15}$$

where we have used that the pull-back of $-\frac{1}{2}\hat{J}\wedge\hat{J}$ is the volume form of a holomorphic four-cycle like $\mathcal{S}$, and where the dots include higher derivative terms as well as terms involving the gauge field on the D7-brane.

The brane position modulus effective action. Let us summarize the 4d effective action controlling the dynamics of the brane position modulus. Adding up the DBI and CS contribution we obtain

$$S_\Phi = -\int_{\mathbb{R}^{1,3}} d\mathrm{vol}_{\mathbb{R}^{1,3}} \Big[g(\mathcal{F}) \partial_\mu \Phi \partial^\mu \overline{\Phi} + V(\mathcal{F}) \Big] , \tag{5.2.16}$$

where

$$g(\mathcal{F}) = \frac{1}{(2\pi)^3 \sigma^2} \int_{\mathcal{S}} g_s Z \sqrt{f(\mathcal{F})}\, d\hat{\mathrm{vol}}_{\mathcal{S}} , \tag{5.2.17}$$

$$V(\mathcal{F}) = \mu_7 \int_{\mathcal{S}} g_s \left[\sqrt{f(\mathcal{F})} - 1 \right] d\hat{\mathrm{vol}}_{\mathcal{S}} - \frac{1}{2} Z^{-1} \mathcal{F} \wedge \mathcal{F} , \tag{5.2.18}$$

and $d\hat{\mathrm{vol}}_{\mathcal{S}}$ is the unwarped volume form of the D7-brane four-cycle. We may now perform the 4d Weyl rescaling

$$g_{\mu\nu} \;\to\; \frac{g_{\mu\nu}}{\mathrm{Vol}_{\mathbf{X}_6}} . \tag{5.2.19}$$

with $\mathrm{Vol}_{\mathbf{X}_6}$ is the volume of the compactification manifold $\mathbf{X}_6$ in units of $l_s = 2\pi\sqrt{\alpha'}$. After that, mass scales in Planck units should be measured in terms of $\kappa_4^{-1} = \sqrt{4\pi} l_s^{-1}$ and the above quantities read

$$g(\mathcal{F}) = \frac{1}{2\pi \mathrm{Vol}_{\mathbf{X}_6}} \frac{1}{l_s^4} \int_{\mathcal{S}} g_s Z \sqrt{f(\mathcal{F})}\, d\hat{\mathrm{vol}}_{\mathcal{S}} , \tag{5.2.20}$$

$$\kappa_4^4 V(\mathcal{F}) = \frac{1}{8\pi \mathrm{Vol}^2_{\mathbf{X}_6}} \frac{1}{l_s^4} \int_{\mathcal{S}} g_s \left[\sqrt{f(\mathcal{F})} - 1 \right] d\hat{\mathrm{vol}}_{\mathcal{S}} - \frac{1}{2} Z^{-1} \mathcal{F} \wedge \mathcal{F} , \tag{5.2.21}$$

Notice that if $\mathcal{F}$ is a self-dual or anti-self-dual two-form in $\mathcal{S}$ then

$$\mathcal{F} \wedge \mathcal{F} = \pm \mathcal{F}^2 d\hat{\mathrm{vol}}_{\mathcal{S}} \quad \Rightarrow \quad f(\mathcal{F}) \;=\; \left(1 + \frac{1}{2} \epsilon \mathcal{F}^2 \right)^2 . \tag{5.2.22}$$

and so in the former case the potential vanishes while in the latter we have

$$\kappa_4^4 V(\mathcal{F}) = \frac{1}{8\pi \mathrm{Vol}^2_{\mathbf{X}_6}} \frac{1}{l_s^4} \int_{\mathcal{S}} Z^{-1} \mathcal{F}^2\, d\hat{\mathrm{vol}}_{\mathcal{S}} . \tag{5.2.23}$$

as obtained in [160]. The kinetic term and potential depend on Φ through eq.(5.2.3) and the identification $z_3 = \sigma\Phi$. To make this dependence more explicit we will turn off the worldvolume flux F and introduce a new normalization for the brane position modulus

$$\Phi \;\to\; \left(\frac{\tilde{\mathcal{V}}_{\mathcal{S}_0}}{2\pi \mathrm{Vol}_{\mathbf{X}_6}} \right)^{-1/2} \Phi , \tag{5.2.24}$$

where

$$\tilde{\mathcal{V}}_{\mathcal{S}} \;=\; \frac{1}{l_s^4} \int_{\mathcal{S}} g_s Z\, d\hat{\mathrm{vol}}_{\mathcal{S}} . \tag{5.2.25}$$

and $\mathcal{S}_0$ is the reference four-cycle where $P[B_2]$ vanishes, hence the minimum of the potential that corresponds to $\Phi = 0$. Note that, with this choice of normalization Φ has canonical kinetic terms at its minimum. After this redefinition we find that the kinetic term and potential are given by

$$g(\Phi) = \frac{1}{\tilde{\mathcal{V}}_{\mathcal{S}_0}}\frac{1}{l_s^4}\int_{\mathcal{S}} g_s Z\left[1+\hat{\epsilon}\,(\mathcal{G}+\mathcal{H})+\frac{1}{4}\hat{\epsilon}^2\,(\mathcal{G}-\mathcal{H})^2\right]^{\frac{1}{2}} d\hat{\text{vol}}_{\mathcal{S}}\,, \tag{5.2.26}$$

$$\kappa_4^4\, V(\Phi) = \frac{1}{8\pi \text{Vol}^2_{\mathbf{X}_6}}\frac{1}{l_s^4}\int_{\mathcal{S}} g_s \left(\left[1+\hat{\epsilon}\,(\mathcal{G}+\mathcal{H})+\frac{1}{4}\hat{\epsilon}^2\,(\mathcal{G}-\mathcal{H})^2\right]^{\frac{1}{2}}+\frac{1}{2}\hat{\epsilon}\,\mathcal{G}-\frac{1}{2}\hat{\epsilon}\,\mathcal{H}-1\right) d\hat{\text{vol}}_{\mathcal{S}}\,, \tag{5.2.27}$$

where we have defined

$$\hat{\epsilon} = g_s\frac{2\pi\text{Vol}_{\mathbf{X}_6}}{4Z\tilde{\mathcal{V}}_{\mathcal{S}_0}}\,. \tag{5.2.28}$$

and $\mathcal{H}$ and $\mathcal{G}$ stand for the self-dual and anti-self-dual components of $P[B_2]$, respectively. Given (5.2.3) they read

$$\mathcal{G} = |\overline{G}_{\bar{1}\bar{2}\bar{3}}\Phi - S_{\bar{3}\bar{3}}\overline{\Phi}|^2\,, \quad \mathcal{H} = |S_{\bar{2}\bar{2}}\Phi - \overline{S}_{\bar{1}\bar{1}}\overline{\Phi}|^2\,. \tag{5.2.29}$$

In order to compare with the results in [83] one should consider that g_s and Z are constant.[1] Then $\tilde{\mathcal{V}}_{\mathcal{S}} = \tilde{\mathcal{V}}_{\mathcal{S}_0}$ for any $\mathcal{S}$ and so these expressions reduce to

$$g(\Phi) = \left[1+\hat{\epsilon}\,(\mathcal{G}+\mathcal{H})+\frac{1}{4}\hat{\epsilon}^2\,(\mathcal{G}-\mathcal{H})^2\right]^{\frac{1}{2}}\,, \tag{5.2.30}$$

$$\kappa_4^4\, V(\Phi) = \frac{\tilde{\mathcal{V}}_{\mathcal{S}}}{8\pi\text{Vol}^2_{\mathbf{X}_6}Z}\left(\left[1+\hat{\epsilon}\,(\mathcal{G}+\mathcal{H})+\frac{1}{4}\hat{\epsilon}^2\,(\mathcal{G}-\mathcal{H})^2\right]^{\frac{1}{2}}+\frac{1}{2}\hat{\epsilon}\,\mathcal{G}-\frac{1}{2}\hat{\epsilon}\,\mathcal{H}-1\right)\,. \tag{5.2.31}$$

Note that if we set $\mathcal{H} = 0$ we recover the results in [83].[2] On the contrary, if $\mathcal{H} \neq 0$ we have that $[g(\Phi)]^2$ no longer is a perfect square and that g and V depend on quite different functions of Φ.

Finally, in order to analyze the potential it is convenient to move to a different parametrization for the brane position modulus. Specifically we may switch to polar coordinates in the plane normal to the D7-brane location and define

$$\rho^2 = \Phi\overline{\Phi}\,\kappa_4^{-2}\,, \tag{5.2.32a}$$
$$A = 2|G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}}|/(|G_{\bar{1}\bar{2}\bar{3}}|^2+|S_{\bar{3}\bar{3}}|^2)\,, \tag{5.2.32b}$$
$$\tilde{A} = 2|S_{\bar{1}\bar{1}}S_{\bar{2}\bar{2}}|/(|S_{\bar{1}\bar{1}}|^2+|S_{\bar{2}\bar{2}}|^2)\,, \tag{5.2.32c}$$
$$\theta = 2\text{Arg}\,\Phi - \text{Arg}\,G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}}\,, \tag{5.2.32d}$$
$$\zeta = \text{Arg}\,G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}} - \text{Arg}\,S_{\bar{1}\bar{1}}S_{\bar{2}\bar{2}}. \tag{5.2.32e}$$

[1] Despite this simplification it could still happen that g_s does depend on Φ, which would complicate the functional dependence of $g(\Phi)$ and $V(\Phi)$. The effect of flux flattening discussed below would nevertheless still remain.

[2] Also, tuning off all supersymmetric components of the induced B-field on the worldvolume of the D7-brane, i.e. $S_{\bar{i}\bar{i}} = 0$, where $i = 1, \ldots, 3$ we recover the results shown in Section 5.2.

The quantities $\mathcal{G}$ and $\mathcal{H}$ then simplify with this notation and become

$$\mathcal{G} = \kappa_4^2(|G_{\bar{1}\bar{2}\bar{3}}|^2 + |S_{\bar{3}\bar{3}}|^2)\Big[1 - A\cos\theta\Big]\rho^2\,, \quad \mathcal{H} = \kappa_4^2(|S_{\bar{1}\bar{1}}|^2 + |S_{\bar{2}\bar{2}}|^2)\Big[1 - \tilde{A}\cos(\theta+\zeta)\Big]\rho^2\,. \tag{5.2.33}$$

5.2.3 Potential asymptotics and flux flattening

Now, we will focus on the analysis of the asymptotic behavior of the above scalar potential. In order to compare with the large-field linear behavior found in [83] we again consider the simplified version (5.2.31), and for convenience we define the following quantities

$$\tilde{G} = \hat{\epsilon}\,(|G_{\bar{1}\bar{2}\bar{3}}|^2 + |S_{\bar{3}\bar{3}}|^2)\kappa_4^2\,, \quad \Upsilon = \frac{|S_{\bar{1}\bar{1}}|^2 + |S_{\bar{2}\bar{2}}|^2}{|G_{\bar{1}\bar{2}\bar{3}}|^2 + |S_{\bar{3}\bar{3}}|^2}\,. \tag{5.2.34}$$

The important parameter in the upcoming analysis will be Υ, which measures the strength of supersymmetric components of the B-field induced on the D7-brane vs the non supersymmetric ones, and it will parametrically control the flattening of the scalar potential. To gain an intuition over the asymptotics of the scalar potential we will consider regions in the parameter space where we effectively achieve single field inflation, as one of the components of Φ is much heavier than the other one. As we will see in section 5.3 and also pointed out in [161], this limit seems favored when embedding our D7-brane system in a setup with full moduli stabilization. These cases admit an unified description and the shape of the potential will depend on two parameters, one the aforementioned Υ and the other which we choose to call $\hat{G}$ to be defined for each case. The cases we look into are the following two:

- **Single field I.** Here we take $A = \tilde{A} = 0$ so that the angular variable θ disappears from the potential. The inflaton is identified with the radial variable $\rho = \sqrt{\Phi\overline{\Phi}}\kappa_4^{-1}$ and in this case $\hat{G} = \tilde{G}$.

- **Single field II.** Here we take $A = \tilde{A} \simeq 1$ and $\zeta = 0$. Now the inflaton is the real part of $\Phi' = e^{-i\gamma/2}\Phi$ where $\gamma = \mathrm{Arg}(G_{\bar{1}\bar{2}\bar{3}}S_{\bar{3}\bar{3}})$. Due to the fact that A is very close to 1 the imaginary part of Φ' will have a much higher mass as compared to the real part. Therefore considering trajectories where the inflaton is Re Φ' is the inflaton and Im Φ' is frozen at the origin is a good approximation and the model becomes a single field model to all effects. In this case $\hat{G} = (1-A)\tilde{G}$.[3]

Both cases have in common that along the trajectories described it occurs that $\mathcal{G} = \hat{G}\rho^2$ and $\mathcal{H} = \Upsilon\hat{G}\rho^2$, where ρ is identified with the inflaton field. Therefore the

[3]In the limiting case where $A = \tilde{A} = 1$ and $\zeta = 0$, Re Φ' becomes a flat direction and one could see Im Φ' as driving single field inflaton, as considered in [83]. In that case one should take $\hat{G} = \tilde{G}$.

potential is identical in both and we can discuss its asymptotic shape at the same time. The scalar potential we obtain is

$$\frac{V(\rho)}{V_0} = \sqrt{1 + \hat{G}(\Upsilon + 1)\rho^2 + \frac{1}{4}\hat{G}^2(\Upsilon - 1)^2\rho^4} + \frac{1}{2}\hat{G}(1 - \Upsilon)\rho^2 - 1\,, \tag{5.2.35}$$

where $\kappa_4^4 V_0 = \tilde{\mathcal{V}}_{\mathcal{S}}(8\pi \mathrm{Vol}^2_{\mathbf{X}_6} Z)^{-1}$. We can easily analyze the asymptotic behavior of the scalar potential for $\rho \to \infty$. The result turns out to heavily depend on the value of Υ

$$\lim_{\rho\to\infty} \frac{V(\rho)}{V_0} = \begin{cases} \hat{G}(1-\Upsilon)\rho^2\,, & 0 \leqslant \Upsilon < 1\,, \\ \sqrt{2\hat{G}}\,\rho & \Upsilon = 1\,, \\ \dfrac{2}{\Upsilon - 1} - \dfrac{4\Upsilon}{\hat{G}\rho^2(\Upsilon-1)^3}\,, & \Upsilon > 1\,. \end{cases} \tag{5.2.36}$$

We see therefore that if $\Upsilon > 1$ – namely when the strength of the self-dual B-field components is larger than the anti-self-dual ones – the potential will approach a constant value as ρ draws nearer to infinity. The resulting potential in this regime exhibits a plateau-like shape and inflationary models constructed using this scalar potential will have a much lower value of tensor-to-scalar ratio as opposed to the usual power-law like potentials. So far we have discussed the effect of flattening in the scalar potential, however as already noted in [83] additional flattening in the scalar potential will appear when considering the effect of the non trivial kinetic terms. To obtain the canonically normalized inflaton field $\hat{\rho}$ it is necessary to solve the integral equation

$$\hat{\rho} = \int^{\rho} g^{1/2}(\rho')d\rho'\,, \tag{5.2.37}$$

and invert the relation between $\hat{\rho}$ and ρ. Given the complexity of the kinetic terms we find it possible to attain canonical normalization only numerically. Nevertheless we can gain some intuition looking at large values of the inflaton field where the kinetic terms drastically simplify

$$\lim_{\rho\to\infty} K_{\rho\rho} = \begin{cases} \dfrac{1}{2}\hat{G}\,|\Upsilon - 1|\,\rho^2 & \Upsilon \neq 1\,, \\ \sqrt{2\hat{G}}\,\rho & \Upsilon = 1\,, \end{cases} \tag{5.2.38}$$

which yields the following potential for large values of the inflaton field in terms of the canonically normalized field

$$\lim_{\hat{\rho}\to\infty} \frac{V(\hat{\rho})}{V_0} = \begin{cases} \sqrt{8\hat{G}}\,\dfrac{(1-\Upsilon)}{\sqrt{|\Upsilon - 1|}}\,\hat{\rho}\,, & 0 \leqslant \Upsilon < 1\,, \\ \left(\dfrac{9}{2}\right)^{\frac{1}{3}} \hat{G}^{\frac{1}{3}}\,\hat{\rho}^{\frac{2}{3}} & \Upsilon = 1\,, \\ \dfrac{2}{\Upsilon - 1} - \dfrac{\sqrt{2|\Upsilon - 1|}\,\Upsilon}{\sqrt{\hat{G}}\,\hat{\rho}\,(\Upsilon - 1)^3}\,, & \Upsilon > 1\,. \end{cases} \tag{5.2.39}$$

We chose to plot the form of the scalar potential for the canonically normalized inflaton field $\hat{\rho}$ for different values of Υ in figure 5.1 to show more explicitly the flattening effect in the scalar potential.

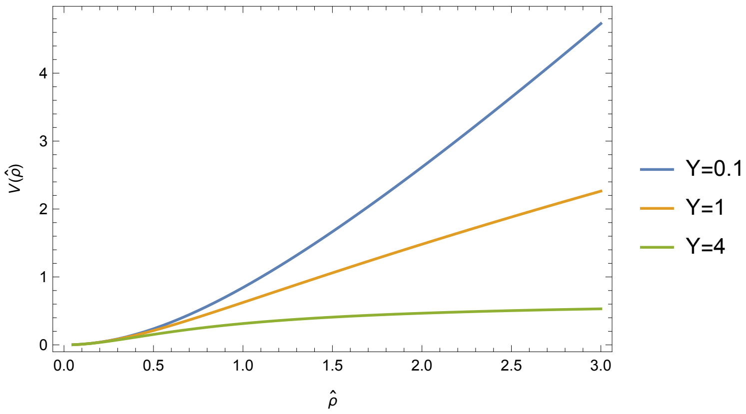


Figure 5.1: The single field scalar potential for the canonically normalized inflaton $\hat{\rho}$ for different values of Υ keeping fixed $\hat{G} = 1$.

Let us stress that this strong flattening effect will be absent in the supergravity discussion that we will carry in the next section, which will able to capture the inflaton scalar potential only in the regime of small values for ρ. Nevertheless, such a supergravity analysis will allow us to draw up an estimate for the typical values of the parameter in the DBI potential, as we discuss in the following.

5.2.4 Estimating the scales of the model

Let us briefly discuss a DBI potential compatible with the compactification scheme discussed in section 5.3, and which considers the interplay of the D7-brane position modulus with the closed string moduli of the compactification. In particular, in subsection 5.3.4 we will argue that a simple way to reproduce a scalar mass spectrum compatible with large field inflation and moduli stabilization is by having one of the two components of the complex field Φ much lighter than the other one. Therefore, we will recover a single field inflation model with a potential of the kind discussed above, and the details from the compactification will translate into some specific values for the parameters V_0, $\hat{G}$ and Υ. In the following we would like to consider those typical values for V_0, $\hat{G}$ and Υ that are compatible with a realistic scalar mass spectrum and the moduli stabilization scheme discussed in section 5.3.4, in order to obtain a constrained range of cosmological observables in the next subsection.

First, we have that for small values of ρ the potential becomes

$$V(\rho) \;=\; V_0\,\hat{G}\rho^2 + \dots\,, \tag{5.2.40}$$

with

$$\kappa_4^4 V_0 \sim \frac{g_s \mathrm{Vol}_{\mathcal{S}}}{8\pi \mathrm{Vol}^2_{\mathbf{X}_6}} \sim 4\times\left(10^{-6} - 10^{-5}\right)\,, \tag{5.2.41}$$

where we have taken $g_s\mathrm{Vol}_{\mathcal{S}} \sim 1-10$ and $\mathrm{Vol}^2_{\mathbf{X}_6} \sim 10^4$, the latter being a typical value compatible with the hierarchy of mass scales discussed in subsection 5.3.4, see e.g. footnote 11. Comparing with the estimated mass for the inflaton near the vacuum we have that

$$\kappa_4^4 V(\rho) \simeq 4\times 10^{-11}\rho^2 \quad \Rightarrow \quad \hat{G} \sim 10^{-6} - 10^{-5}\,. \tag{5.2.42}$$

Moreover, we have that Υ is the quotient between two different kind of fluxes. On the one hand $G_{\bar{1}\bar{2}\bar{3}}$ and $S_{\bar{3}\bar{3}}$ are fluxes that enter the inflaton scalar potential even at small field. On the other hand, $S_{\bar{1}\bar{1}}$ and $S_{\bar{2}\bar{2}}$ will be fluxes to which the D7-brane will be insensitive near the vacuum. However, these fluxes will be sensed by the complex structure moduli, to which they will give masses. Hence, unless Υ is constrained by some specific feature of the compactification,[4] one may estimate $\Upsilon^{1/2}$ as the quotient between the typical complex structure moduli mass (that is, the flux scale) and the mass of a D7-brane modulus. If we now focus on the single field scenario considered in section 5.3.4, which corresponds to the single field case II discussed above, and look at the mass relations found in section 5.3.4, we have that $\Upsilon^{1/2}$ is roughly the quotient between the flux scale and the mass of the heaviest component of the D7-brane modulus, namely Im Φ'. In other words we have that

$$\Upsilon \sim \frac{m^2_{\text{flux}}}{m^2_{\mathrm{Im}\Phi'}} \sim \frac{N^2}{\kappa_4^2|W_0|^2} \sim 10^2 - 10^3\,, \tag{5.2.43}$$

where $N \in \mathbb{Z}$ is the typical value of flux quanta, which we have taken around $N^2 \sim 1-10$. Finally, W_0 is as defined in subsection 5.3.4, from where we have taken the typical value $\kappa_4 W_0 \sim 0.1$.

Given this large value of Υ and the small value of $\hat{G}$, we may approximate (5.2.35) by

$$\frac{V(\rho)}{V_0} = \frac{\hat{G}\rho^2}{1+\frac{1}{2}\hat{G}(\Upsilon-1)\rho^2} + \dots\,, \tag{5.2.44}$$

so asymptotically

$$V(\rho) \quad \xrightarrow{\rho\to\infty} \quad 2V_0\Upsilon^{-1} \sim (10^{-9}-10^{-7})\,\kappa_4^{-4}\,, \tag{5.2.45}$$

which is intriguingly close to the scale of large-field inflation $V^{1/4}_{\text{inf},*} = (10r)^{1/4} 1.88 \times 10^{16}$GeV [4]. This asymptotic constant value will not be changed by the field-dependent inflaton kinetic term, which for this choice of parameters can be approximated to be

$$g(\rho) = 1 + \frac{1}{2}\hat{G}(\Upsilon-1)\rho^2 + \dots\,. \tag{5.2.46}$$

Using (5.2.37) we have that the canonically normalized field is given by

$$\hat{\rho} = \frac{\rho}{2}\sqrt{1+\frac{1}{2}\hat{G}(\Upsilon-1)\rho^2} + \frac{\sinh^{-1}\left(\sqrt{\frac{1}{2}\hat{G}(\Upsilon-1)\rho^2}\right)}{\sqrt{2\hat{G}(\Upsilon-1)}}\,. \tag{5.2.47}$$

[4]More precisely, Υ could be constrained to vanish by an orbifold symmetry like in [83] or by the fact that $h^{1,1}(\mathcal{S}) = 1$, see the discussion in section 5.3.1.

Hence, in the region where $\hat{G}(\Upsilon-1)\rho^2 \ll 2$ we have that $\hat{\rho} \simeq \rho$ and that (5.2.44) is a quadratic potential, and in the large field limit we have that $\hat{\rho} \simeq \sqrt{\frac{1}{8}\hat{G}(\Upsilon-1)}\rho^2$ and that the potential asymptotes to the constant value (5.2.45). In any event notice that for this range of parameters the potential can be written as

$$V(\hat{\rho}) = \hat{V}_0 \cdot \hat{V}(\hat{\rho}) \,, \tag{5.2.48}$$

where $\hat{V}_0 = 2V_0/(\Upsilon-1)$ and $\hat{V}$ is a monotonic function that only depends on the parameter $\hat{\Upsilon} = \hat{G}(\Upsilon-1)$, such that $\hat{V} \simeq \frac{1}{2}\hat{\Upsilon}\rho^2$ at small field and asymptotes to 1 for $\hat{\rho} \to \infty$. In figure 5.2 we plot $\hat{V}$ for some typical values of this parameter, within the range $\hat{\Upsilon} \sim 10^{-4} - 10^{-2}$.

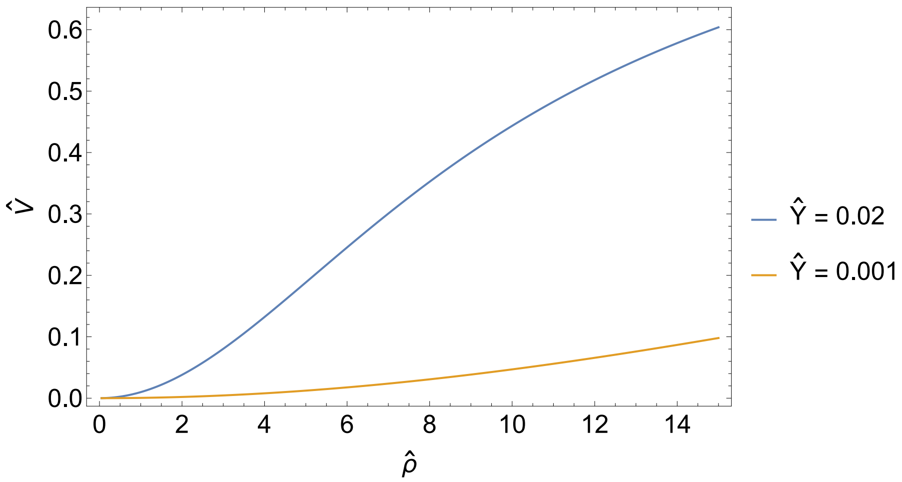


Figure 5.2: Scalar potential $\hat{V}$ for the canonical field $\hat{\rho}$ for two different values of $\hat{\Upsilon}$.

5.2.5 Cosmological observables

Let us now analyze in some detail the cosmological observables that can be derived from the potential discussed above. In the single field scheme of subsection 5.3.4 one finds that the distortion effect coming from the stabilization of other moduli is sufficiently suppressed, and therefore the DBI+CS potential discussed in this section is a good approximation during the field ranges where inflation occurs.[5] Therefore, in the following we will focus on the single field scalar potential (5.2.48) and derive the phenomenological features of this model. We will see that even in this concrete case there is a rich phenomenology allowing for the possibility of having a moderately low tensor-to-scalar ratio. One may also analyse the features of single field D7-brane potential for other choices of parameters that may occur in different setups, as we do in appendix C.

[5]More precisely, we find negligible backreaction effects from the heavy component of Φ and Kähler moduli in the 4d supergravity model describing a mobile D7-brane, and we expect the same conclusion to apply to the flux-flattened DBI potential.

As it usually happens for single field inflation to obtain the main cosmological observables, the spectral index n_s and the tensor-to-scalar ratio r, it is sufficient to obtain the slow-roll parameters η and ϵ. For a single scalar field ϕ with non-canonical kinetic terms the slow-roll parameters are

$$\epsilon = \frac{M_P^2}{2} G^{\phi\phi} \left(\frac{D_\phi V}{V}\right)^2 , \tag{5.2.49}$$

$$\eta = M_P^2 G^{\phi\phi} \frac{D_\phi D_\phi V}{V} . \tag{5.2.50}$$

where $G^{\phi\phi}$ is the inverse of the target space metric and derivatives are covariant derivatives with the connection derived from the metric $G_{\phi\phi}$. Knowledge of the slow-roll parameters is sufficient to compute cosmological observables: we copy here the well-known relations

$$n_s = 1 + 2\eta_* - 6\epsilon_* , \tag{5.2.51}$$

$$r = 16\epsilon_* , \tag{5.2.52}$$

where η_* and ϵ_* are the values of η and ϵ at the beginning of inflation.

Since an overall factor V_0 drops out in the computation of ϵ and η, in the single field limit there are only two relevant parameters in the D7-brane potential, namely $\hat{G}$ and Υ. Moreover, after we add the input from the moduli stabilization scheme of section 5.3.4 the potential simplifies to (5.2.48) whose only relevant parameter is $\hat{\Upsilon} \equiv (\Upsilon - 1)\hat{G}$, with typical range $10^{-4} \leqslant \hat{\Upsilon} \leqslant 10^{-2}$. We have scanned over this range of $\hat{\Upsilon}$ showing how the cosmological observables evolve when this parameter is varied, displaying the results in figure 5.3. We find that the typical range for these cosmological observables is $n_s \simeq 0.96 - 0.97$ and $r \simeq 0.04 - 0.14$. In figure 5.4 we have superimposed the precise region in the $n_s - r$ plane over the Planck collaboration results [4].

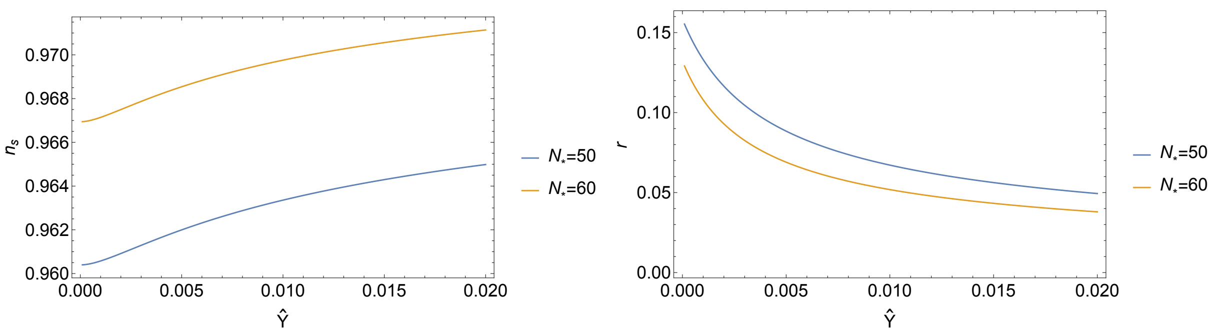


Figure 5.3: Spectral index and tensor-to-scalar ratio in terms of $10^{-4} \leqslant \hat{\Upsilon} \leqslant 10^{-2}$ for $N_* = 50$ and $N_* = 60$ e-folds.

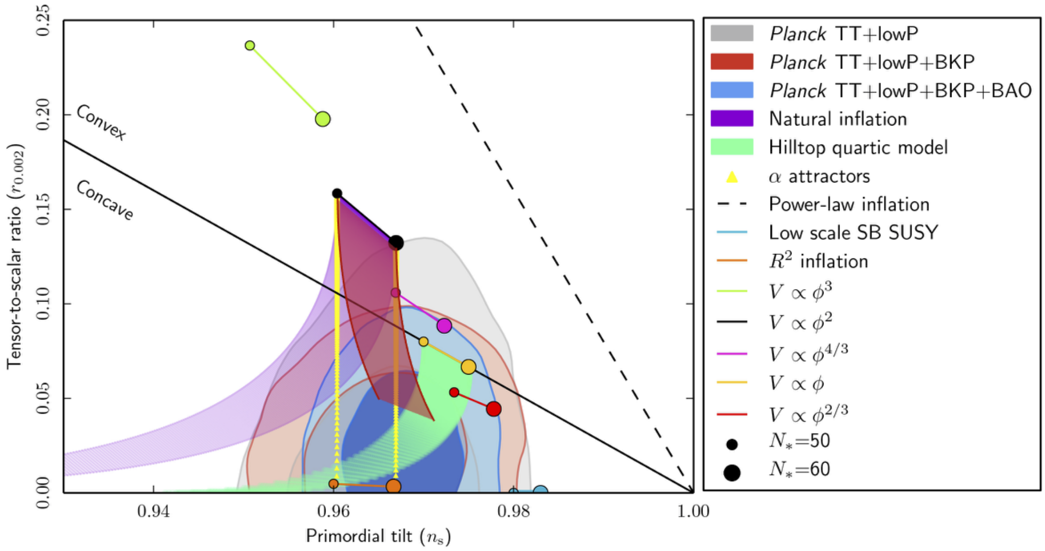


Figure 5.4: Spectral index n_s vs tensor-to-scalar ratio r superimposed over the plot given by the Planck collaboration [4] for the single field model with $10^{-4} \leqslant \hat{\Upsilon} \leqslant 10^{-2}$.

5.3 Embedding into type IIB/F-theory

Let us now consider how to construct compactifications in which the above flux-flattened 7-brane scalar potential drives large-field inflation. One important ingredient when building models of large field inflation is to provide a configuration in which the inflaton candidate is allowed to perform trans-Planckian excursions. In the case of D7-brane position moduli, this requires using the framework of F-term axion monodromy [21], and in particular D7-branes with periodic directions in their moduli space, as already pointed out in [32, 33, 83, 162]. We will discuss the general features of these constructions and the relation to the D7-brane potential discussed in the previous section, paying special attention to the case of D7-branes on $\mathbf{T}^4/\mathbb{Z}_2 \times \mathbf{T}^2$ and its F-theory lift to $\mathbf{K3} \times \mathbf{K3}$ [32, 162–167]. This simple embedding not only contains the main features of an inflationary model of mobile D7-branes, but it is also well-understood in terms of the Kähler and superpotential that describe the full four-dimensional scalar potential at small field values. The latter will be crucial to understand how to generate mass hierarchies between the inflaton sector and the rest of the scalars of the compactification and, ultimately, to embed the 7-brane scalar potential into a consistent framework of moduli stabilization, along the lines of [161].

5.3.1 Periodic 7-branes and model building

Let us consider type IIB string theory compactified in a Calabi-Yau orientifold $\mathbf{X}_6$, and a D7-brane wrapping a holomorphic four-cycle $\mathcal{S}$ in it. The moduli space of such four-cycle will depend on its topology, and in particular on the Hodge number $h^{2,0}(\mathcal{S})$

that gives the complex dimension of holomorphic deformations of $\mathcal{S}$. As we are interested in mobile D7-branes, we will assume that $h^{2,0}(\mathcal{S}) > 0$. The infinitesimal holomorphic deformations of $\mathcal{S}$ are given by a set of normal holomorphic vectors $\{X^i\}$ such that

$$\iota_{X^i}\Omega|_{\mathcal{S}} = \tilde{\alpha}_i \,, \tag{5.3.1}$$

where Ω is the holomorphic three-form in $\mathbf{X}_6$ and $\tilde{\alpha}_i$ is a basis of (2,0)-forms in $\mathcal{S}$. We may choose the X^i such that the $\tilde{\alpha}_i$ have a constant norm, and integrate the infinitesimal deformations to define D7-brane position coordinates in terms of the chain integrals

$$\Phi^i = \frac{1}{l_s^5}\int_{\Sigma_5} \Omega \wedge \alpha^i \,, \tag{5.3.2}$$

where Σ_5 is a five-chain connecting the initial four-cycle $\mathcal{S}$ to a homotopic divisor $\mathcal{S}'$, and α^i is a dual basis of (0,2)-forms such that $\int_{\mathcal{S}} \tilde{\alpha}_i \wedge \alpha^j = \delta_i^j$, extended to Σ_5. Finally, we will assume that there are one or more periodic directions in the moduli space of $\mathcal{S}$, and dub a D7-brane wrapping such a four-cycle as a periodic D7-brane.[6]

Let us now consider the presence of background three-form fluxes F_3 and H_3 threading $\mathbf{X}_6$. In order to cancel the Freed-Witten anomaly [10, 117], we must require that the pull-back of H_3 on $\mathcal{S}$ vanishes in cohomology. Such a condition is trivially satisfied whenever $h^{1,0}(\mathcal{S}) = 0$, but in general we may have that $H_3|_{\mathcal{S}}$ does not vanish identically. For simplicity let us first assume that H_3 is transverse to $\mathcal{S}$ and so $H_3|_{\mathcal{S}} = 0$, as implicitly taken in the computation of the previous section, namely in (5.2.2). Then the gauge invariant worldvolume flux $\mathcal{F} = \sigma F - B$ is closed, and can always be taken to be harmonic in $\mathcal{S}$ as this choice minimizes the energy of the D7-brane. Finally, let us assume that the embedding of $\mathcal{S}$ is such that at this locus the D7-brane is BPS. In practice this means that $\mathcal{F}$, if non-vanishing, is a primitive (1,1)-form of $\mathcal{S}$. We may now consider deforming $\mathcal{S}$ along one of its periodic directions. Here there are several possibilities depending on the topology of $\mathcal{S}$.

Considering $\mathbf{h^{1,1}}(\mathcal{S}) = \mathbf{1}$ Then there is only one harmonic (1,1)-form on $\mathcal{S}$, which is necessarily its Kähler form and therefore non-primitive. Using the assumption that $H_3|_{\mathcal{S}_4} = 0$ and that the D7-brane is BPS, this means that $\mathcal{F}$ must vanish on $\mathcal{S}$. Now, as the D7-brane moves in its moduli space, a non-vanishing B-field and hence a flux $\mathcal{F}$ will be induced in its worldvolume. Because H is primitive in $\mathbf{X}_6$ the induced B-field will be primitive in $\mathcal{S}$ [160], and so $\mathcal{F}$ can only be a harmonic $(2,0)+(0,2)$-form. As a result $\mathcal{F}$ will be anti-self-dual, the function $f(\mathcal{F})$ will be a perfect square as in (5.2.22) and we will recover a potential of the form (5.2.23). Therefore, under the above conditions we obtain a setup similar to that in [83], with the differences that we only have one D7-brane and no orbifold projection is present. Moreover, the potential $V(\mathcal{F})$ and kinetic function $g(\mathcal{F})$ do not need to be quadratic

[6] One particular example could be a D7-brane wrapping a **K3** submanifold fibered over a Riemann surface. As we will see below, this condition of periodicity can be relaxed in the more general context of F-theory compactifications.

in Φ, as the induced B-field is such that

$$B^{(0,2)} = c_i \, \alpha^i \,, \tag{5.3.3}$$

with c_i more general than a linear function of Φ and $\overline{\Phi}$. What such a B-field needs to satisfy is that, upon closing a loop in the moduli space of the D7-brane, the change in B should be quantized. Hence this variation can be compensated in $\mathcal{F}$ by a discrete change in F and the multi-branched structure of axion-monodromy models arises. Due to that, along a closed loop c^i will depend on the D7-brane position as a superposition of a linear plus a periodic function, a dependence that will be translated into the function $f(\mathcal{F})$.

Considering $\mathbf{h^{1,1}}(\mathcal{S}) > \mathbf{1}$ Let us now consider the case where $h^{1,1}(\mathcal{S}) > 1$, while still assuming that $H_3|_{\mathcal{S}} = 0$ along its moduli space. Then the induced B-field will be harmonic but it may have both anti-self-dual $(2,0)+(0,2)$ and self-dual (1,1)-primitive components, depending on the components of $\iota_X \mathrm{Im}\, G_3|_{\mathcal{S}}$. The former will contribute to the kinetic term and potential as the quantity $\mathcal{G}$ in (5.2.26) and (5.2.27), while the latter will contribute as $\mathcal{H}$. Again, these quantities need not be the square of a linear function of Φ and $\overline{\Phi}$ as in the previous section, but rather of a linear plus a periodic function along each periodic coordinate of the D7-brane, giving a quadratic potential with modulations. In any event the potential and kinetic term will be of this form and so the effect of flux flattening will occur for large values of Φ, specially when the induced B-field has an amount of self-dual component which is comparable or bigger than that of the anti-self-dual component.

Finally, let us consider the case where $H_3|_{\mathcal{S}} \neq 0$. Then, even at its BPS locus, the D7-brane will have a non-closed, co-exact induced B-field component $B^{\rm co}$ that solves $dB^{\rm co} = H_3|_{\mathcal{S}}$. Now, in order to minimize the D7-brane energy, the system can always develop an exact piece for F, $F^{\rm ex} = da$ such that $\mathcal{F} - \mathcal{F}^{\rm h} = \sigma F^{\rm ex} - B^{\rm co}$ is self-dual, independently of what the harmonic component $\mathcal{F}^{\rm h}$ of the worldvolume flux is. As a result, this non-closed B-field will contribute to the D7-brane potential and kinetic term as $\mathcal{H}$ in (5.2.26) and (5.2.27), inducing the effect of flux-flattening even in the case where $h^{1,1}(\mathcal{S}) = 1$. Notice however that this self-dual, non-harmonic component of $\mathcal{F}$ is by definition periodic upon completing a loop in the D7-brane position space, so in order to induce a parametrically large flux flattening we need to consider the case where $h^{1,1}(\mathcal{S}) > 1$.

Part of this dynamics will be captured by the 4d effective action of the compactification. In particular in the absence of fluxes we have that the Kähler potential capturing the 4d axio-dilaton S, the complex structure moduli and D7-brane kinetic terms has the form [127, 168, 169]

$$K = -\log\left[-\frac{i}{l_s^6}\int_{\mathbf{X}_6} \Omega \wedge \overline{\Omega}\right] - \log\left[-i(S - \overline{S} + \mathcal{C}(\Phi, \overline{\Phi}))\right] \,, \tag{5.3.4}$$

where $\mathcal{C}$ is a real function of the D7-brane position and the complex structure moduli. Clearly, $\mathcal{C}$ must respect the periodicity of the moduli space of periodic D7-branes

[162]. This will manifest as discrete shift symmetries that should be respected even when one-loop [170–174] and warping effects [129, 175, 176] are taken into account.

When including background and worldvolume fluxes a potential will be generated for the dilaton, complex structure and D7-brane position moduli. For small values of these fields such potential will be captured by the effective superpotential [144, 169, 177]

$$W = W_{\text{GVW}} + W_{D7} = \frac{1}{l_s^6}\int_{\mathbf{X}_6} G_3 \wedge \Omega + \frac{1}{l_s^5}\int_{\Sigma_5} \Omega \wedge \mathcal{F}\,, \tag{5.3.5}$$

where Σ_5 is defined as in (5.3.1).

Finally, we may also understand this effective theory from the perspective of F-theory, where all the above moduli become complex structure moduli of the Calabi-Yau fourfold $\mathbf{Y}_8$. In this case it is straightforward to write Kähler potential and superpotential for these moduli as [95, 177, 178]

$$K = -\log\left[\frac{1}{l_M^8}\int_{\mathbf{Y}_8} \Omega_4 \wedge \overline{\Omega}_4\right]\,, \tag{5.3.6}$$

$$W = \frac{1}{l_M^8}\int_{\mathbf{Y}_8} G_4 \wedge \Omega_4\,. \tag{5.3.7}$$

As we will discuss below, this description allows to generalize the setup with a periodic D7-branes to more general compactifications in which models of F-term axion monodromy can also be constructed.

5.3.2 A simple K3 × K3 embedding

As pointed out in [32], one simple case where periodic D7-branes are realized is in type IIB string theory compactified in an orientifold of $\mathbf{T}^4/\mathbb{Z}_2 \times \mathbf{T}^2$, which is the orbifold limit of the $\mathbf{K3} \times \mathbf{T}^2$ orientifold. This compactification space is constructed by first considering the orbifold $\mathbf{T}^4/\mathbb{Z}_2 \times \mathbf{T}^2$, with the $\mathbb{Z}_2$ action generated by θ : $(z_1, z_2, z_3) \to (-z_1, -z_2, z_3)$, and with the coordinate z_i spanning the i-th torus. One then mods out by the orientifold action $\Omega\mathcal{R}(-1)^{F_L}$ with $\mathcal{R}$: $(z_1, z_2, z_3) \to (-z_1, -z_2, -z_3)$, which introduces a total of 64 O3-plane located at the fixed loci of $\mathcal{R}$ as well as 4 orientifold O7-planes located at the fixed loci of $\mathcal{R}\cdot\theta$. In the case where no exotic O3-planes are present, the condition of cancellation of D3-brane tadpoles is

$$N_{D3} + \frac{1}{2l_s^4}\int_{\mathbf{X}_6} H_3 \wedge F_3 = 16\,, \tag{5.3.8}$$

where $l_s^2 = 2\pi\sigma$. Here the closed string fluxes F_3, H_3 are constant and obey the following quantization conditions[7]

$$\frac{1}{l_s^2}\int_{\gamma_3} F_3 \in 2\mathbb{Z}\,, \quad \frac{1}{l_s^2}\int_{\gamma_3} H_3 \in 2\mathbb{Z}\,, \tag{5.3.9}$$

[7] Flux quanta should be multiples of 2 in the particular orbifold we are considering, see [151].

for all $\gamma_3 \in H_3(X, \mathbb{Z})$. Finally, cancellation of D7-brane tadpoles is ensured by introducing 16 D7-branes wrapping $\mathbf{T}^4/\mathbb{Z}_2$ and being point-like in the transverse coordinates of $\mathbf{T}^2$, which is parametrized by the complex position field Φ. Any of these D7-branes is then a periodic D7-brane with one complex modulus and two periodic directions.

One nice feature of this system is that it admits a simple embedding in a F-theory compactification on a Calabi-Yau fourfold $\mathbf{Y}_8$ given by $\mathbf{K3} \times \widetilde{\mathbf{K3}}$. Indeed, if $\widetilde{\mathbf{K3}}$ is elliptically fibered upon taking the weak coupling limit we obtain a type IIB compactification on $\mathbf{K3} \times \mathbf{T}^2$ with 16 D7-branes located at points on the torus and 4 O7-planes. Our initial setup may be easily recovered upon taking the limit in complex structure moduli space where the $\mathbf{K3}$ becomes the orbifold $\mathbf{T}^4/\mathbb{Z}_2$. The F-theory description has the advantage of describing on the same ground closed and open string moduli. Note that in this setup the cancellation of D3-brane tadpole translates to

$$N_{D3} + \frac{1}{2l_M^6}\int_{\mathbf{Y}_8} G_4 \wedge G_4 = \frac{\chi(Y)}{24}\,, \tag{5.3.10}$$

where l_M is the M-theory Planck length and in the case at hand $\chi(Y) = 24^2$. In this case the closed string flux G_4 will be quantized as[8]

$$\frac{1}{l_M^3}\int_{\gamma_4} G_4 \in \mathbb{Z}\,, \tag{5.3.11}$$

for all $\gamma_4 \in H_4(\mathbf{Y}_8, \mathbb{Z})$.

This F-theory description also has the advantage that provides a simple description of the 4d $\mathcal{N} = 1$ effective action for small field values, and in particular explicit expressions for the tree-level Kähler and superpotentials (5.3.6) and (5.3.7), see e.g. [162,165]. Since the holomorphic 4-form decomposes into the wedge product of the holomorphic 2-forms of each $\mathbf{K3}$ surface as $\Omega_4 = \Omega_2 \wedge \widetilde{\Omega}_2$, to express the Kähler potential it is convenient to introduce the period vectors Π and $\widetilde{\Pi}$, respectively defined as the integrals of Ω_2 and $\widetilde{\Omega}_2$ over a basis of integral 2-cycles. The periods of each $\mathbf{K3}$ may be written as [32,162,165–167]

$$\Pi = \frac{1}{2}\begin{pmatrix} 1 \\ C^2 - \tau_1\tau_2 \\ \tau_1 \\ \tau_2 \\ 2C^a \end{pmatrix}\,, \quad \widetilde{\Pi} = \frac{1}{2}\begin{pmatrix} 1 \\ \Phi^2 - S\tau_3 \\ S \\ \tau_3 \\ 2\Phi^a \end{pmatrix}\,, \tag{5.3.12}$$

where $a = 1, \ldots, 16$ and C^2 is the square of the vector C^a and similarly for Φ^2. When comparing with the type IIB setting we may identify the moduli τ_i with the complex structure modulus of the i-th torus, S with the axio-dilaton, Φ^a with the relative position of the D7-branes with respect to the O7-planes and the moduli

[8] Note that for the case of $\mathbf{K3} \times \widetilde{\mathbf{K3}}$ the second Chern class satisfies $\frac{1}{2}c_2(\mathbf{K3} \times \widetilde{\mathbf{K3}}) \in H^4(\mathbf{K3} \times \widetilde{\mathbf{K3}}, \mathbb{Z})$ and therefore the fluxes should be simply integrally quantized [179].

C^a are the additional complex structure moduli of the first **K3** surface. Using the period vectors it is straightforward to write down the Kähler potential (5.3.6) as

$$K = -\log\left[2\overline{\Pi}.M.\Pi\right] - \log\left[2\overline{\tilde{\Pi}}.M.\tilde{\Pi}\right], \qquad (5.3.13)$$

where M is the intersection matrix

$$M = \begin{pmatrix} 0 & 2 & & & \\ 2 & 0 & & & \\ & & 0 & 2 & \\ & & 2 & 0 & \\ & & & & \mathbf{1}_{16} \end{pmatrix}. \qquad (5.3.14)$$

For simplicity we may take the limit where the first **K3** becomes the orbifold $\mathbf{T}^4/\mathbb{Z}_2$, turning off the moduli C^a, and also turn off all Φ^a except one, considering a single moving D7-brane whose position is given by Φ. Then we obtain that the Kähler potential is

$$K = -\log\left[-(\tau_1-\overline{\tau}_1)((\tau_2-\overline{\tau}_2)\right] - \log\left[-(S-\overline{S})(\tau_3-\overline{\tau}_3)+(\Phi-\overline{\Phi})^2\right]. \qquad (5.3.15)$$

This Kähler potential can also be written in the form (5.3.13) using the simplified period vectors and intersection matrix

$$\Pi = \begin{pmatrix} 1 \\ -\tau_1\tau_2 \\ \tau_1 \\ \tau_2 \\ 0 \end{pmatrix}, \quad \tilde{\Pi} = \begin{pmatrix} 1 \\ \Phi^2 - S\tau_3 \\ S \\ \tau_3 \\ 2\Phi \end{pmatrix}, \quad M = \begin{pmatrix} 0 & 2 & & & \\ 2 & 0 & & & \\ & & 0 & 2 & \\ & & 2 & 0 & \\ & & & & 1 \end{pmatrix}. \qquad (5.3.16)$$

Finally, in this reduced moduli space, the most general superpotential (5.3.7) can be written as

$$l_s W = \Pi.G.\tilde{\Pi}, \qquad (5.3.17)$$

where Π, $\tilde{\Pi}$ are as in (5.3.16) and G is a matrix of integer entries containing the relevant flux quanta

$$G = \begin{pmatrix} \hat{n}_0 & m_0 & -n_0 & \hat{m}_0 & f_0 \\ \hat{n}_3 & m_3 & -n_3 & \hat{m}_3 & f_3 \\ \hat{n}_1 & m_1 & -n_1 & \hat{m}_1 & f_1 \\ \hat{n}_2 & m_2 & -n_2 & \hat{m}_2 & f_2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (5.3.18)$$

In the type IIB limit $\hat{m}_i, \hat{n}_i \in \mathbb{Z}$ can be identified with quanta of F_3, then $m_i, n_i \in \mathbb{Z}$ with quanta of H_3, and $f_i \in \mathbb{Z}$ with D7-brane worldvolume flux quanta [162]. By explicit computation one finds that the superpotential reads

$$l_s W = \hat{\mathfrak{n}} + \hat{\mathfrak{m}}\,\tau_3 - \mathfrak{n}\,S + \mathfrak{m}\left(\Phi^2 - S\tau_3\right) + 2\mathfrak{f}\,\Phi\,. \qquad (5.3.19)$$

where the calligraphic letters are functions of the moduli of the first **K3**, namely

$$\hat{\mathfrak{n}} = \hat{n}_0 + \hat{n}_1\tau_1 + \hat{n}_2\tau_2 - \hat{n}_3\tau_1\tau_2\,, \tag{5.3.20}$$

$$\hat{\mathfrak{m}} = \hat{m}_0 + \hat{m}_1\tau_1 + \hat{m}_2\tau_2 - \hat{m}_3\tau_1\tau_2\,, \tag{5.3.21}$$

$$\mathfrak{n} = n_0 + n_1\tau_1 + n_2\tau_2 - n_3\tau_1\tau_2\,, \tag{5.3.22}$$

$$\mathfrak{m} = m_0 + m_1\tau_1 + m_2\tau_2 - m_3\tau_1\tau_2\,, \tag{5.3.23}$$

$$\mathfrak{f} = f_0 + f_1\tau_1 + f_2\tau_2 - f_3\tau_1\tau_2\,. \tag{5.3.24}$$

As stressed above, using these Kähler and superpotential to compute the scalar potential for closed and open string moduli is only a good approximation in the regime of small field values for S, τ_i and Φ. Nevertheless, these supergravity quantities are quite useful to detect discrete and continuous symmetries of our system, as we will discuss in the following. Finally, the above Kähler potential will be subject to one-loop corrections, see [170–172] for details. For simplicity, in the following we will assume that such one-loop effects are negligible.

5.3.3 Monodromies and shift symmetries

Discrete symmetries and multi-branched structure

Besides providing simple expressions for the effective Kähler and superpotential, the example of $\mathbf{K3}\times\widetilde{\mathbf{K3}}$ is useful in the sense that the discrete shift symmetries characteristic of axion-monodromy systems can be easily detected. Indeed, recall form the discussion of section 5.3.1 that in any type IIB flux compactification with periodic D7-branes a multi-branched potential is expected to appear, in which closing a loop in the D7-brane moduli space is compensated by shifting some worldvolume flux quanta, and that this operation corresponds to a change in the branch of the 4d potential. Such symmetry is manifest in the DBI computation of section 5.2, since the potential and kinetic terms only depend on $\mathcal{F}$. When embedded in the toroidal model $\mathbf{T}^4/\mathbb{Z}_2\times\mathbf{T}^2$, this discrete symmetry corresponds to shifting Φ by the lattice $\Lambda=\{p+q\tau_3\}$ that describes the non-trivial loops of the $\mathbf{T^2}$ transverse to the D7-brane. Clearly, one would expect that such a discrete symmetry is also manifest in the 4d effective theory that arises from the $\mathbf{K3}\times\widetilde{\mathbf{K3}}$ F-theory lift of this compactification.

In particular, one would expect that the Kähler potential (5.3.15) is invariant per se, as in the absence of fluxes the theory is fully symmetric under lattice shifts of Φ. Indeed one sees that this Kähler potential is invariant under the transformations

$$\text{(a)} \qquad \Phi \;\to\; \Phi+1\,, \tag{5.3.25}$$

$$\text{(b)}\left\{\begin{array}{lcl} \Phi & \to & \Phi+\tau_3 \\ S & \to & S+2\Phi+\tau_3 \end{array}\right. \tag{5.3.26}$$

that generate the lattice Λ describing $\mathbf{T^2}=\mathbb{R}^2/\Lambda$, and in general under the transformation

$$\left\{\begin{array}{l} \Phi\to\Phi+p+q\tau_3 \\ S\to S+2q\Phi+q\,(p+q\tau_3) \end{array}\right. \qquad \text{with} \quad p,q\in\mathbb{Z}\,. \tag{5.3.27}$$

This discrete symmetry is easier to detect in the matrix formulation of the Kähler potential (5.3.13), as these transformations can be expressed as shifts of the period vector $\widetilde{\Pi}$

$$\widetilde{\Pi} \to \mathcal{S}.\widetilde{\Pi}\,, \tag{5.3.28}$$

where for $\widetilde{\Pi}$ as in (5.3.16) and in the case of the lattice generators we have that

$$\mathcal{S}_a = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 \end{pmatrix}\,, \qquad \mathcal{S}_b = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix}\,, \tag{5.3.29}$$

for (5.3.25) and (5.3.26), respectively. Then because

$$\mathcal{S}^T.M.\mathcal{S} = M\,, \tag{5.3.30}$$

we have that each of these shifts as well as any sequence of them leaves the Kähler potential invariant.

With respect to the superpotential, we expect that the discrete symmetry is preserved if combined with discrete shifts of the flux quanta. More precisely the shift (5.3.28) will be compensated by the opposite shift in the flux matrix

$$G \to G.\mathcal{S}^{-1}\,, \tag{5.3.31}$$

which in the case of the lattice generator (5.3.25) translates into

$$f_i \to f_i - m_i\,, \qquad \hat{n}_i \to \hat{n}_i + m_i - 2f_i\,, \tag{5.3.32}$$

and in the case of the generator (5.3.26) it becomes

$$f_i \to f_i + n_i\,, \qquad \hat{m}_i \to \hat{m}_i - n_i - 2f_i\,. \tag{5.3.33}$$

While these discrete symmetries are derived in the context of F-theory, they have an intuitive interpretation in terms of their type IIB limit. On the one hand, the shift in f_i corresponds to the shift in D7-brane worldvolume flux quanta that compensates the shift of B-field, as discussed in section 5.3.1. On the other hand, the shifts in $\hat{n}_i$, $\hat{m}_i$ correspond to shifts in the background flux F_3 due to the rearrangement of D5-brane charge.

This example allows us to readily generalize the picture of discrete shift symmetries to a generic Calabi-Yau four-fold. Here the fundamental quantity is the period vector $\Pi(z)$ of the Calabi-Yau four-fold whose entries are functions of the four-fold complex structure moduli. Notice that since in F-theory brane position moduli get unified with closed string moduli we can treat them on equal footing. In this scenario the tree-level Kähler potential is written as

$$K = -\log\left[\overline{\Pi}.M.\Pi\right]\,, \tag{5.3.34}$$

where M is the intersection matrix of integral 4-cycles in the Calabi-Yau four-fold. A discrete shift symmetry is present whenever upon performing a suitable translation in complex structure moduli space $z \to z + f(z)$ it is possible to find a matrix $\mathcal{S}$ with integer coefficients such that $\Pi(z + f(z)) = \mathcal{S}.\Pi(z)$ and $\mathcal{S}^T.M.\mathcal{S} = M$. While this clearly constitutes a symmetry of the Kähler potential it is necessary to take into account how the superpotential transforms as well if fluxes are added. The superpotential may be easily expressed in terms of the period vector as

$$l_s W = \mathsf{G}.M.\Pi(z)\,, \tag{5.3.35}$$

where G is a vector with integer coefficients. Upon performing the aforementioned discrete transformation we find that the transformed superpotential is

$$l_s W' = \mathsf{G}'.M.\Pi(z)\,, \tag{5.3.36}$$

where $\mathsf{G}' = \mathsf{G}.(\mathcal{S}^T)^{-1}$. This shows how the effect of performing a discrete shift symmetry is translated in a suitable redefinition of the integer flux quanta, a mechanism which is the avatar of axion monodromy. It is important to state that the presence of these discrete shift symmetries effectively cuts the moduli space to some *fundamental domain* which may contain some compact directions inside it: addition of fluxes effectively unfolds this compact moduli space, a signature of axion monodromy. Identification of the correct fundamental domain is in general case is a difficult exercise although in some specific cases the answer is known [180–182].

The question that remains open is when and under which conditions a discrete shift symmetry does appear. Luckily it is possible to find an answer to these questions: discrete shift symmetries are intimately tied with the presence of singular points in the complex structure moduli space.[9] In the case we have previously analysed the singularity is located at the point of large complex structure of the Calabi-Yau 4-fold, and indeed in the proximity of this point a shift symmetry appears for the complex structure moduli [162]. For simplicity we will phrase our discussion in the case of complex structure moduli space of a Calabi-Yau 3-fold $\mathbf{X}_6$, where most examples are known, although the discussion can be easily generalized to a Calabi-Yau n-fold mutatis mutandis. First we need to highlight one of the characteristics of the period vector $\Pi(z)$: namely that it behaves as a section of an appropriate vector bundle $\mathcal{H}$ over the complex structure moduli space $\mathcal{M}$. Specifically at $z \in \mathcal{M}$ the fibre of $\mathcal{H}$ is simply $H^3(X_z, \mathbb{Z})$ where X_z is the Calabi-Yau manifold X with complex structure specified by z. This vector bundle comes equipped with a flat connection ∇ called Gauß-Manin connection which allows to perform parallel transport of sections of $\mathcal{H}$ around paths on $\mathcal{M}$. While it is true that the connection is flat (and therefore parallel transport around closed cycles would give no transformations on sections of $\mathcal{H}$), it may develop some singularities at specific points in the complex structure moduli space $\hat{z}_i$ where the Calabi-Yau manifold develops a singularity. The presence of singularities in the Gauß-Manin

[9] In some cases though the presence of a singular point in the complex structure moduli space does not give discrete shift symmetries, see [41] for examples.

connection implies that upon circling these singular points a section of $\mathcal{H}$ gets acted upon by a matrix transformation which realizes the transformation of the period vector $\Pi(z)$ advocated above. This provides a mechanism to realize discrete shift symmetries in general Calabi-Yau compactifications, although the precise details of the vector period transformations are somewhat technical and here we will refrain from delving into them. The interested reader may consult for instance [183–186] and references therein for explicit examples.

Continuous shift symmetries

One well-known fact is that in the tree-level Kähler potential (5.3.15) the discrete shift symmetry (5.3.25) is promoted to the continuous shift-symmetry

$$\Phi \;\to\; \Phi + \lambda\,, \tag{5.3.37}$$

with $\lambda \in \mathbb{R}$. This continuous symmetry highlights the field direction Re Φ, and makes it a natural inflaton candidate, as considered in [161].

While (5.3.37) is an obvious shift symmetry of this Kähler potential it is strange that it is the only one. After all, it is nothing but a translation along one of the one-cycles of the $\mathbf{T}^2$ transverse to the D7-brane. Geometrically all of these one-cycles are on the same footing, and microscopically they are all similar for the D7-brane. Hence there is a priori no reason why the field direction (5.3.37) should be special. In particular we would expect to find a continuous shift symmetry like (5.3.37) for each of the points of the lattice that defines $\mathbf{T}^2$.

One can indeed see that this is the case whenever we allow for field space excursions involving S and Φ simultaneously. Indeed, let us consider our $\mathbf{K3} \times \widetilde{\mathbf{K3}}$ model with an initial point in moduli space given by (Φ_0, S_0) and with all τ_i fixed to some value. Then if we consider the one-dimensional trajectory

$$\left\{ \begin{array}{lcl} \Phi & = & \Phi_0 + \lambda\,(s + r\tau_3) \\ S & = & S_0 + r\frac{\Phi^2 - \Phi_0^2}{s + r\tau_3} \end{array} \right. \qquad \text{with varying } \lambda \in \mathbb{R}\,, \tag{5.3.38}$$

and fixed $r, s \in \mathbb{R}$, one can see that the Kähler potential (5.3.15) is left invariant. Notice that we do not have one shift symmetry but an infinite number of them, parametrized by $(r, s) \in \mathbb{R}^2$. If we take $(r, s) = (p, q) \in \mathbb{Z}^2$ then each of these trajectories connects with different lattice points of $\mathbf{T}^2$, where they reduce to (5.3.27). In particular, taking $(r, s) = (0, 1)$ and $\lambda \in \mathbb{N}$ we generate the discrete shifts that correspond to (5.3.25) and taking $(r, s) = (1, 0)$ we generate those in (5.3.26).

We then see that, when combining field excursions involving Φ and S, many shift symmetries arise, and that they are related to the periodic directions in the D7-brane moduli space. Absent some criterium that selects one among the rest, they are all equally valid as inflationary trajectory candidates and should be considered on equal footing.

The criterium to select one trajectory among all of them will in general come from the effective superpotential. Indeed, as discussed above W will transform non-trivially under discrete shifts that leave K invariant, and generically the same will happen for their continuous counterparts. Interestingly, for the case under discussion one can easily characterize whenever W selects one of the above trajectories among the others. Indeed, it is easy to check that for a superpotential of the form (5.3.19) a trajectory with fixed τ_i and

$$\begin{cases} \Phi &= \Phi_0 + \kappa \\ S &= S_0 + \mathfrak{n}\frac{\Phi^2-\Phi_0^2}{\mathfrak{m}+\mathfrak{n}\tau_3} + 2\mathfrak{f}\frac{\Phi-\Phi_0}{\mathfrak{m}+\mathfrak{n}\tau_3} \end{cases} \qquad \text{with varying } \kappa \in \mathbb{C}\,, \tag{5.3.39}$$

leaves W invariant. As a result, whenever $\mathfrak{f} = 0$ and $\mathfrak{n}\,\overline{\mathfrak{m}} \in \mathbb{R}$ there will be a field space trajectory of the form (5.3.38) that leaves both the Kähler and superpotential invariant, which signals a flat direction of the scalar potential. As discussed in Appendix 8.4 this can be made manifest by using the $SL(2,\mathbb{R})$ invariance of K.

As we will see in the following, this result will still hold when we complete K and W with the remaining ingredients to describe a compactification with full moduli stabilization. Therefore, in such a setup we will have a simple mechanism to generate flat directions in field space, which then will be useful to generate mass hierarchies among fields in the scalar potential.

5.3.4 Moduli stabilization

Following [161], one may try to embed a system with a mobile D7-brane into a type IIB compactification with the necessary ingredients for full moduli stabilization. In the case where $h^{1,1}(\mathcal{S}) = 1$ and the background flux is transverse to $\mathcal{S}$, one may capture the non-trivial kinetic term of the D7-brane position field in terms of a higher derivative correction to the Kähler potential, as done in [161, 187], and so study the stability of the inflationary trajectory by means of 4d supergravity techniques. In the case where the effect of flux flattening is important, namely when $h^{1,1}(\mathcal{S}) > 1$, such a description for the D7-brane scalar potential and kinetic terms for large values of Φ is not known. Nevertheless, one may still use 4d supergravity to analyze the stability of the inflationary trajectory at small field values, in order to estimate how important are the effects of moduli stabilization and heavy field backreaction on the naive potential computed in section 5.2.

Recovering the DBI potential at small field

In order to connect with the setup of section 5.2 let us assume a D7-brane whose moduli space of positions contains a $\mathbf{T}^2$ parametrized by the complex field Φ. Then, by analogy with the $\mathbf{K3} \times \widetilde{\mathbf{K3}}$ example, we may consider that the D7-brane and closed string dynamics is governed by an effective superpotential of the form

$$l_s W \;=\; \hat{f} - Sf + \left(\Phi^2 - SU\right) g + U\hat{g}\,, \tag{5.3.40}$$

where U is the complex structure modulus of such a $\mathbf{T}^2$ and f, g, $\hat{f}$, $\hat{g}$ are holomorphic functions of the flux quanta and the complex structure moduli of the compactification. Similarly, one would expect a Kähler potential of the form

$$K \;=\; -\log\left[(\Phi-\overline{\Phi})^2 - (S-\overline{S})(U-\overline{U})\right] + K_2\,, \tag{5.3.41}$$

where K_2 contains the dependence on the Kähler and remaining complex structure moduli.

In the absence of any superpotential for the Kähler moduli we will recover a positive definite scalar potential which, at $\Phi = 0$, reduces to the no-scale scalar potential in [95] for the axio-dilaton S and complex structure moduli. In principle, one may assume that the mass for these fields at the vacuum is much larger than that of Φ and so, following the philosophy in [188], replace such heavy fields by their vevs in (5.3.40) and (5.3.41). This strategy, followed in [83, 161], is however only a fair approximation for a restricted range of superpotential parameters in (5.3.40). Indeed, from the discussion above we have that whenever $g/f \in \mathbb{R}$ there is a flat direction of the scalar potential along $\Phi \propto f + gU$ in which the dilaton varies as[10]

$$S \;=\; S_0 + \frac{g}{f}\frac{\Phi^2}{1+\frac{g}{f}U}\,, \tag{5.3.42}$$

with S_0 the vev of S at $\Phi = 0$. Therefore, for generic g/f it is not a good approximation to assume that S will remain close to its vev S_0. This means that, in general, we cannot apply the philosophy of [188] to S.

Instead we can integrate out S by canceling its F-term, solving for it in terms of the other moduli and plugging the result back into the scalar potential. For simplicity, let us consider the Kähler and superpotential above with all the complex structure moduli including U fixed to their vev. Then the F-term for S is given by

$$D_S W \;=\; -\frac{(\Phi-\overline{\Phi})^2(f+gU)-(U-\bar{U})\left(\hat{f}-\bar{S}f+\left(\Phi^2-\bar{S}U\right)g+U\hat{g}\right)}{(\Phi-\overline{\Phi})^2-(S-\bar{S})(U-\bar{U})}\,, \tag{5.3.43}$$

where we have assumed that $\partial_S K_2 = 0$. Hence we obtain $D_S W = 0$ by demanding that

$$S \;=\; S_0 + \frac{\bar{g}}{\bar{f}}\frac{\overline{\Phi}^2}{1+\frac{\bar{g}}{\bar{f}}\overline{U}} + \frac{(\Phi-\overline{\Phi})^2}{U-\overline{U}}\,. \tag{5.3.44}$$

Plugging this expression into the scalar potential we obtain that

$$V \;=\; \frac{e^K}{\kappa_4^2}K^{\Phi\bar{\Phi}}|D_\Phi W|^2 \;=\; \frac{1}{4\pi\kappa_4^4}\frac{2\left|\left(\bar{f}+\bar{g}\bar{U}\right)\Phi-\left(\bar{f}+\bar{g}U\right)\bar{\Phi}\right|^2}{8\mathrm{Vol}^2_{\mathbf{X}_6}|U-\bar{U}||\int_{\mathbf{X}_6}\Omega\wedge\overline{\Omega}|}\,. \tag{5.3.45}$$

where in our conventions $\kappa_4^2 = l_s^2/4\pi$ and all volumes are measured in units of l_s.

In order to compare this result with the scalar potential of section 5.2 we need to canonically normalize the position field at $\Phi = 0$. Taking into account that there its kinetic term is given by $K_{\Phi\overline{\Phi}}|_{\Phi=0} = g_s/|U-\overline{U}|$, with $g_s^{-1} = \mathrm{Im}\ S_0$ we obtain that the scalar potential is

$$V_{\mathrm{SUGRA}} = \frac{g_s^{-1}}{2\pi\kappa_4^2}\frac{\left|\left(\bar{f}+\bar{g}\bar{U}\right)\Phi-\left(\bar{f}+\bar{g}U\right)\bar{\Phi}\right|^2}{8\mathrm{Vol}^2_{\mathbf{X}_6}|\int_{\mathbf{X}_6}\Omega\wedge\overline{\Omega}|}\,, \tag{5.3.46}$$

[10] This assumes that in (5.3.41) K_2 does not depend on S and Φ or, if it does, it depends through the combination $(\Phi-\overline{\Phi})^2-(S-\overline{S})(U-\overline{U})$.

where now Φ is canonically normalized at the origin. We may now compare with the DBI result (5.2.31) in the small field limit and in the 4d Einstein frame

$$V_{\text{DBI+CS}} \simeq \frac{g_s}{\kappa_4^4} \frac{|\overline{G}_{\bar{1}\bar{2}\bar{3}}\Phi - S_{\bar{3}\bar{3}}\overline{\Phi}|^2}{16\text{Vol}_{\mathbf{X}_6}} \,, \tag{5.3.47}$$

where for simplicity we have set a trivial warp factor $Z = 1$. We then obtain that

$$G_{\bar{1}\bar{2}\bar{3}} = \frac{\kappa_4}{\sqrt{\pi}} \frac{f + gU}{g_s \text{Vol}_{\mathbf{X}_6}^{1/2} |\int_{\mathbf{X}_6} \Omega \wedge \overline{\Omega}|^{1/2}} \quad , \quad S_{\bar{3}\bar{3}} = \frac{\kappa_4}{\sqrt{\pi}} \frac{\bar{f} + \bar{g}U}{g_s \text{Vol}_{\mathbf{X}_6}^{1/2} |\int_{\mathbf{X}_6} \Omega \wedge \overline{\Omega}|^{1/2}} \,. \tag{5.3.48}$$

Finally, as in [83] we may diagonalize this scalar potential as

$$\kappa_4^4 V_{\text{DBI+CS}} \simeq \frac{g_s}{16\text{Vol}_{\mathbf{X}_6}} \left[\left(|G_{\bar{1}\bar{2}\bar{3}}| - |S_{\bar{3}\bar{3}}|\right)^2 (\text{Re } \Phi')^2 + \left(|G_{\bar{1}\bar{2}\bar{3}}| + |S_{\bar{3}\bar{3}}|\right)^2 (\text{Im } \Phi')^2 \right] \,, \tag{5.3.49}$$

where

$$\Phi' = e^{-i\gamma/2}\Phi \quad , \quad \gamma = \text{Arg } (G_{\bar{1}\bar{2}\bar{3}} S_{\bar{3}\bar{3}}) \,. \tag{5.3.50}$$

Notice that using the dictionary (5.3.48) we have that $g\bar{f} \in \mathbb{R}$ is equivalent to $|G_{\bar{1}\bar{2}\bar{3}}| = |S_{\bar{3}\bar{3}}|$, which precisely is where we obtain a flat direction in the scalar potential, in agreement with our previous discussion. Away from the flat direction condition we have that the masses of the two mass eigenstates go like

$$\begin{aligned} m_{\sqrt{2}\text{Im}\,\Phi'} &= \frac{g_s^{1/2}}{2\kappa_4^2 \text{Vol}_{\mathbf{X}_6}^{1/2}} \left(|G_{\bar{1}\bar{2}\bar{3}}| + |S_{\bar{3}\bar{3}}|\right) = 2\, e^{K/2} |W_0| (1 + \varepsilon) \,, & (5.3.51) \\ m_{\sqrt{2}\text{Re}\,\Phi'} &= 2\, e^{K/2} |W_0|\, |\varepsilon| \,, & (5.3.52) \end{aligned}$$

where

$$\begin{aligned} |W_0| &= \kappa_4^{-2} |G_{\bar{1}\bar{2}\bar{3}}| \, \text{Vol}_{\mathbf{X}_6}^{1/2} \left| \int_{\mathbf{X}_6} \Omega \wedge \overline{\Omega} \right|^{1/2} = \frac{\kappa_4^{-1}}{\sqrt{\pi}} g_s^{-1} |f + gU| \,, & (5.3.53) \\ \varepsilon &= \frac{|S_{\bar{3}\bar{3}}| - |G_{\bar{1}\bar{2}\bar{3}}|}{2|G_{\bar{1}\bar{2}\bar{3}}|} \simeq \frac{\text{Im } U}{|f + gU|^2} \text{Im } (g\bar{f}) \,. & (5.3.54) \end{aligned}$$

Here ε measures the departure form the flat direction case, and whenever $|\varepsilon| \ll 1$ we have that Re Φ' is a very light compared to Im Φ'. In that case, the heaviest mode Im Φ' is in turn much lighter than the complex structure and axio-dilaton moduli whenever $\kappa_4 |W_0| \ll N$, with N the typical value for the flux quanta.[11] In particular, its mass will not be far from that of the Kähler moduli sector in standard moduli stabilization schemes. Therefore, one should be able to describe an $\mathcal{N} = 1$ effective field theory for Φ and the Kähler moduli below the flux scale, as we discuss in the following.

[11]For instance, for the choices $\kappa_4 W_0 \sim 0.1$, $|\varepsilon| \sim 0.01$, $e^K \sim 10^{-5}$ one recovers an inflaton mass of the order $m^2_{\sqrt{2}\text{Re}\,\Phi'} \sim 4 \times 10^{-11} M_P^2$ and $m^2_{\sqrt{2}\text{Im}\,\Phi'} \sim 4 \times 10^{-7} M_P^2$, while $m^2_{\text{flux}} = N^2 \times 10^{-5} M_P^2$.

Integrating out the dilaton

As mentioned above, in general it will not be a good approximation to fix the 4d axio-dilaton S at its vev S_0 in K and W, since S varies significantly as we change the value of Φ. However, when a flat direction is developed because $g\bar{f} \in \mathbb{R}$, we have that the holomorphic field redefinition

$$\hat{S} \; = \; S - \frac{g}{f}\frac{\Phi^2}{1+\frac{g}{f}U}\,, \tag{5.3.55}$$

is such that $\hat{S}$ remains constant and equal to S_0 along the flat direction. Therefore, for describing the scalar potential in a field space region around the flat direction trajectory, one may apply the strategy of [188] to this new holomorphic variable $\hat{S}$, and replace it by its vev S_0 both in K and W, as done with the complex structure moduli.[12]

Whenever the flat direction is not present because Im $(g\bar{f}) \neq 0$ then $\hat{S}$ will no longer be constant along the trajectory of minimum energy. On the one hand it will still be true that, if S is given by (5.3.44), then Re $\hat{S}$ = Re S_0 for any value of Φ. On the other hand it will happen that Im $\hat{S}$ will depart from Im S_0 as we move away from $\Phi = 0$ along the said trajectory. Nevertheless, one expects that this displacement is small as long as the mass of Im Φ', Re Φ' is much smaller than the typical mass scale induced by fluxes. In particular whenever $|\varepsilon|, \kappa_4|W_0| \ll 1$, the approximation of taking $\hat{S} = S_0$ in K and W should be accurate enough to describe the inflationary potential up to subleading backreaction effects [188].

Doing this procedure in the no-scale case we find an effective Kähler and superpotential for Φ given by

$$\begin{aligned} K \;&=\; -\log\left[-(S_0-\bar{S}_0)(U-\bar{U}) - \left(\frac{g}{f}\frac{\Phi^2}{1+\frac{g}{f}U} - \frac{\bar{g}}{\bar{f}}\frac{\overline{\Phi}^2}{1+\frac{\bar{g}}{\bar{f}}\bar{U}}\right)(U-\bar{U}) + (\Phi-\overline{\Phi})^2\right] + K_2\,, \\ W \;&=\; W_0\,, \end{aligned} \tag{5.3.56}$$

where again K_2 contains all the dependence on the Kähler moduli. In terms of the components of the field Φ' defined in (5.3.50) the first part of the Kähler potential K' reads

$$\begin{aligned} K' \;&=\; -\log\Big[-(S_0-\bar{S}_0)(U-\bar{U}) \\ &\quad +\frac{1}{2}\left(1+\frac{|f+g\bar{U}|}{|f+gU|}\right)(\Phi'-\overline{\Phi}')^2 - \frac{1}{2}\left(1-\frac{|f+g\bar{U}|}{|f+gU|}\right)(\Phi'+\overline{\Phi}')^2\Big] \\ &=\; -\log\left[-(S_0-\bar{S}_0)(U-\bar{U}) + \frac{1}{2}\left(1+\frac{|S_{\bar{3}\bar{3}}|}{|G_{\bar{1}\bar{2}\bar{3}}|}\right)(\Phi'-\overline{\Phi}')^2 - \frac{1}{2}\left(1-\frac{|S_{\bar{3}\bar{3}}|}{|G_{\bar{1}\bar{2}\bar{3}}|}\right)(\Phi'+\overline{\Phi}')^2\right]. \end{aligned} \tag{5.3.57}$$

[12] Notice that $\hat{S}$ is not only holomorphic on Φ and U, but also on all the remaining complex structure moduli through g and f. Therefore (5.3.55) can be seen as a field redefinition even at the flux scale, and one may apply the strategy of [188] to all complex structure moduli and $\hat{S}$ simultaneously. We discuss alternative definitions to the definition (5.3.55) in Appendix 8.4.

Therefore we recover an effective theory with a constant superpotential and a Kähler potential with no apparent shift symmetry for any component of Φ. Notice however that whenever $g\bar{f} \in \mathbb{R}$ or equivalently $|G_{\bar{1}\bar{2}\bar{3}}| = |S_{\bar{3}\bar{3}}|$ we recover a shift symmetry along Re Φ', which then becomes a flat direction. Finally, we can rewrite the Kähler potential in the simpler form

$$K = -3\log(T+\bar{T}) - \log\left[4su + (1+\varepsilon)(\Phi' - \overline{\Phi}')^2 + \varepsilon(\Phi' + \overline{\Phi}')^2\right] + K_2\,, \quad (5.3.58)$$

with $u = \mathrm{Im}\ U$, $s = \mathrm{Im}\ S_0$ and ε is defined as in (5.3.54). Again, notice that in the regime of interest $|G_{\bar{1}\bar{2}\bar{3}}| \simeq |S_{\bar{3}\bar{3}}|$ and so $|\varepsilon| \ll 1$.

Adding Kähler moduli stabilization

Let us now add the necessary ingredients to achieve full moduli stabilization in a semi-realistic setup. Since our setup requires $|W_0| \ll 1$ in order to decouple the D7-brane position modulus from the complex structure moduli, it is more natural to consider a KKLT-like scheme with a single Kähler modulus T, as done in [161]. We then have a Kähler potential of the form

$$K \;=\; -3\log(T+\bar{T}) - \log\left[(\Phi-\overline{\Phi})^2 - (S-\overline{S})(U-\overline{U})\right] + K'\,, \quad (5.3.59)$$

where K' contains the dependence in the complex structure moduli besides U. In addition we have a superpotential of the form

$$l_s W \;=\; l_s W_{\text{flux}} + l_s W_{\text{np}} \;=\; \left(\hat{f} - Sf + \left(\Phi^2 - SU\right) g + U\hat{g}\right) + l_s A\, e^{-aT}\,, \quad (5.3.60)$$

where $f, g, \hat{f}, \hat{g}$ depend on the flux quanta and complex structure moduli, and so may the non-perturbative prefactor A. From these two quantities we compute the supergravity scalar potential

$$V_{\text{SUGRA}} \;=\; \frac{e^K}{\kappa_4^2}\left(K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \overline{W} - 3|W|^2\right)\,, \quad (5.3.61)$$

which together with an uplifting term[13]

$$V_{\text{up}} \;=\; \frac{e^K}{\kappa_4^4}\Delta^2\,, \quad (5.3.62)$$

give us the final scalar potential

$$V \;=\; V_{\text{SUGRA}} + V_{\text{up}}\,. \quad (5.3.63)$$

Notice that if W_{np} does not depend on S and Φ the full superpotential will still be invariant under the complex shift (5.3.39). Hence, if we also assume that

[13]Here we are treating Δ^2 as a constant, as it would arise by considering, e.g., F-term uplift. As in [161] we will not delve on the actual microscopic origin of this uplifting mechanism, as it will not affect the subsequent discussions.

K' does not depend on S and Φ and follow our previous discussion, we have that whenever $g\bar{f} \in \mathbb{R}$ there will be a real shift of the form

$$\begin{cases} \Phi &= \Phi_0 + \lambda \left(1 + \frac{g}{f}U\right) \\ S &= S_0 + \frac{g}{f}\frac{\Phi^2}{1+\frac{g}{f}U} \end{cases} \qquad \text{with } \lambda \in \mathbb{R} \tag{5.3.64}$$

that leaves W and K invariant. Therefore both V_{SUGRA} and V_{up} will be invariant and this direction in field space will be a flat direction of the full scalar potential.

We may now consider relaxing the above assumptions on W_{np} and K'. For instance, let us consider a non-trivial dependence of the prefactor A on Φ, as done in [106]. In general, such a dependence may or may not be periodic in the lattice of Φ. If on the one hand it is not periodic, then it should be such that A is invariant under the discrete shift symmetry of section 5.3.3 that shifts fields and flux quanta simultaneously. Therefore, it will most likely depend on Φ^2 through a function of W_{flux}, and so it will be invariant under the real shift symmetry (5.3.64). If on the other hand the dependence is periodic it must be bounded, so we expect it to be subdominant with respect the dependence in W_{flux} for large values of Φ. The same observations apply to the potential dependence of K' on Φ, for instance through one-loop corrections, which as stated above we assume negligible. Therefore, up to this degree of approximation the full scalar potential should develop a flat direction whenever $g\bar{f} \in \mathbb{R}$, and a very light direction in field space whenever we slightly violate this condition. In the following we will consider the consequences of this feature in the simplest case, namely when A and K' do not depend on Φ.

As in our previous discussion of the no-scale case, the variable $\hat{S}$ defined in (5.3.55) remains constant and equal to its vev along such a flat direction of V, and very close to it when $|\delta| \propto \text{Im}\,(g/f)$ is very small. We may then apply the strategy of [188] to $\hat{S}$ and all the complex structure moduli, replacing them by their vevs in W and K. We thus obtain an effective potential for T and Φ of the form (5.3.63), where now V_{SUGRA} and V_{up} only depend on T and Φ, through the quantities

$$\begin{aligned} W &= W_0 + A\, e^{-aT}\,, \\ K &= -3\log(T+\bar{T}) - \log\left[4su + (1+\varepsilon)(\Phi' - \overline{\Phi}')^2 + \varepsilon(\Phi' + \overline{\Phi}')^2\right]\,, \end{aligned} \tag{5.3.65}$$

where u, s and ε are as in (5.3.58). All these quantities as well as $a \in \mathbb{R}$ and A, $W_0 \in \mathbb{C}$ are treated as constants. Notice that even if the inflaton candidate Re Φ' appears in the Kähler potential there is a priori no η-problem, as $|\varepsilon| \ll 1$ and so the kinetic term for Φ is dominated by the coefficient of Im Φ' in K.

Given this effective theory, we are able to stabilize the Kähler modulus as in the KKLT proposal [102]. Cancelling the F-term of T in the vacuum we arrive to the relation

$$D_T W = 0 \to W_0 = -\frac{1}{3} A e^{-aT_0}(2a\text{Re}\ T_0 + 3)\,, \tag{5.3.66}$$

where T_0 is the value of T at the KKLT AdS vacuum. For simplicity, in the following we will assume that W_0, $A \in \mathbb{R}$, so that Im $T_0 = 0$. The introduction of the uplifting term (5.3.62) will shift the Kähler modulus vev. For instance, in order to obtain a

Minkowski vacuum state one should minimize the scalar potential for every field in the vacuum and impose $V|_{\rm tot}^{\rm vac}=0$ from which we obtain the following relations

$$A=-\frac{3W_0e^{at}(at-1)}{2a^2t^2+4at-3}\quad,\quad \Delta^2=\frac{12a^2t^2\left(a^2t^2+at-2\right)}{\left(2a^2t^2+4at-3\right)^2}W_0^2\kappa_4^2\,, \tag{5.3.67}$$

describing implicitly the new value for $t=\langle\text{Re }T\rangle$, while $\langle\text{Im }T\rangle$ still vanishes.

We can see that the ingredients for Kähler moduli stabilization do not change significantly the mass hierarchies obtained in the no-scale case. Indeed, if we denote by φ and ξ the canonically normalized components Re Φ' and Im Φ', respectively, we find that in the vacuum

$$m_\varphi^2=\frac{\varepsilon^2W_0^2}{8ust^3}+\mathcal{O}\left(t^{-4}\right)\,,\, m_\xi^2=\frac{W_0^2(1+\varepsilon)^2}{8ust^3}+\mathcal{O}\left(t^{-4}\right)\,, \tag{5.3.68}$$

$$m_{\text{Re}T}^2=\frac{a^2W_0^2}{8ust}-\frac{5\left(aW_0^2\right)}{8ust^2}+\mathcal{O}\left(t^{-3}\right)\,,\, m_{\text{Im}T}^2=\frac{a^2W_0^2}{8ust}-\frac{3\left(aW_0^2\right)}{8ust^2}+\mathcal{O}\left(t^{-3}\right)\,. \tag{5.3.69}$$

which reproduces (5.3.51), (5.3.52) and the usual mass for the Kähler modulus in KKLT-like schemes. Again, the mass of the inflaton candidate is strongly suppressed with respect the other moduli by the parameter ε, and the mass of its partner ξ is of the same order of magnitude as the Kähler moduli sector. Multifield effects during inflation will then be negligible as long as

$$|\varepsilon|<10^{-2}\,. \tag{5.3.70}$$

Given these expressions, one is able to accommodate a realistic setup by for instance taking the following set of parameter values

$$\kappa_4A=-1.6\,,\quad a=\frac{2\pi}{15}\,,\quad \kappa_4W_0=0.09\,,\quad su=10\,,\quad \varepsilon=2.3\times10^{-2}\,, \tag{5.3.71}$$

so that the Minkowski vacuum is found for

$$t=10.8\ ,\ \Delta^2=0.0148\,, \tag{5.3.72}$$

and the above masses are given by

$$m_\varphi=6.4{\times}10^{-6}M_{\rm P}\ ,\ m_\xi=2.8{\times}10^{-4}M_{\rm P}\ ,\ m_{\text{Re}T}=8.1{\times}10^{-4}M_{\rm P}\ ,\ m_{\text{Im}T}=9.9{\times}10^{-4}M_{\rm P}\,. \tag{5.3.73}$$

Inflaton potential and backreaction

Let us now analyze the effect of moduli stabilization and backreaction during inflation. First notice that, even in this supergravity description, the kinetic term for the inflaton candidate $\phi=\text{Re }\Phi'$ depends on itself due to the breaking of the shift symmetry. The definition of the canonically normalized variable

$$\varphi=\int\sqrt{2K_{\Phi\bar\Phi}}\,d\phi\,, \tag{5.3.74}$$

is non-trivial. In particular, for the case at hand we see that

$$\sqrt{2K_{\Phi\bar{\Phi}}} = \frac{\sqrt{su + \varepsilon(1+2\varepsilon)\phi^2}}{su + \varepsilon\phi^2} \,. \tag{5.3.75}$$

which admits an analytic integral but it does not admit an analytic inverse. However, since $|\varepsilon| \ll 1$ we may approximate this expression by

$$\sqrt{2K_{\Phi\bar{\Phi}}} \simeq (su + \varepsilon\phi^2)^{-1/2}\left(1 + \frac{\varepsilon^2\phi^2}{su + \varepsilon\phi^2}\right) = \frac{1}{\sqrt{su}}\left(1 - \frac{\varepsilon\phi^2}{2su}\right) + \mathcal{O}(\varepsilon^2)\,, \tag{5.3.76}$$

where in the second equality we have expanded around $\varepsilon = 0$. Integrating the last expression we arrive to

$$\varphi = \frac{\phi}{\sqrt{su}}\left(1 - \frac{\varepsilon\phi^2}{6su}\right)\,, \tag{5.3.77}$$

whose inverse involves roots of a polynomial of degree 3. Since this effective 4d supergravity description is supposed to be valid in the small field limit we may assume that

$$|\varepsilon|\phi^2 \ll 6su \to \phi \sim \sqrt{su}\,\varphi\,, \tag{5.3.78}$$

and use this relation in the following.

Let us now address the backreaction effects of the Kähler modulus and the inflaton partner ξ. For this we will employ perturbation theory, where we define

$$\text{Re}\ T = t + \delta\text{Re}T(\varphi)\ ,\ \text{Im}\ T = 0 + \delta\text{Im}T(\varphi)\ ,\ \xi = \langle\xi\rangle + \delta\xi(\varphi)\,, \tag{5.3.79}$$

with t, and $\langle\xi\rangle = 0$ are vevs of the backreacting fields in the Minkowski vacuum. Assuming that the fluctuations are small and minimizing the scalar potential for them we find that

$$\delta\text{Re}T(\varphi) = \frac{3\varepsilon^2\varphi^2}{2a^3t^2} + \mathcal{O}\left(\frac{H^2}{m_T^2}\right)\ ,\ \delta\text{Im}T(\varphi) = 0\ ,\ \delta\xi(\varphi) = 0\,. \tag{5.3.80}$$

Notice that the backreaction of Re T is suppressed by a factor of t^2 as compared to similar setups, like e.g. in [161]. The main reason is that in our setup the Kähler modulus is not coupled to the inflaton neither via the superpotential nor the kinetic terms. It is only coupled via the overall factor of e^K in the scalar potential. One way to check the consistency of this result is to plot the scalar potential in the plane $(\text{Re}\,T, \varphi)$ for the benchmark set of parameter values (5.3.71), as done in figure 5.5. Indeed, there we see that the trajectory of minimum energy (represented by the darkest blue colour) is at this level of approximation a straight line in the $(\text{Re}\,T, \varphi)$ plane. This means that the Kähler modulus backreaction effects are essentially negligible. Numerically we have that

$$\delta\text{Re}T(\varphi) \sim 10^{-4}\varphi^2\,, \tag{5.3.81}$$

and the leading order contribution in the scalar potential will be $V_{back} \sim -1.55 \times 10^{-16}\varphi^4$.

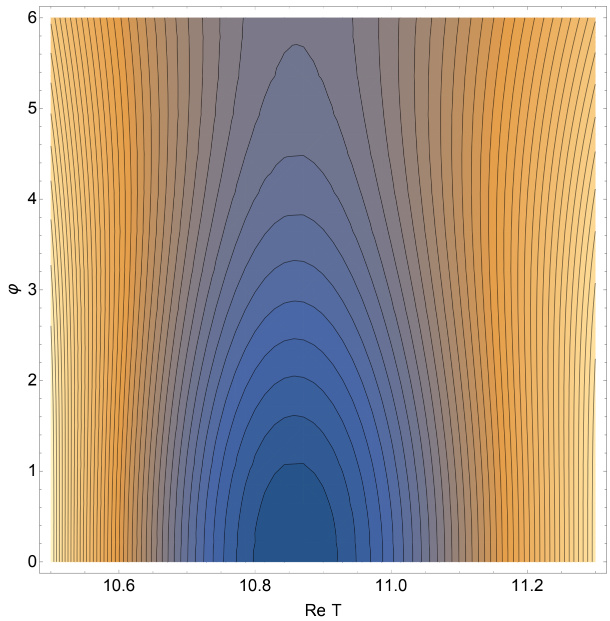


Figure 5.5: Scalar potential evaluated in the $(\mathrm{Re}\, T, \varphi)$ plane for the set of parameters (5.3.71) where colder colours mean smaller values of V.

The scalar potential taking into account both backreaction effects and the flattening induced by the kinetic term is then

$$V = \frac{\varepsilon^2 W_0^2}{16ust^3}\left[\varphi^2 - 2\varepsilon\varphi^4\right] + \mathcal{O}\left(\varepsilon^4, \frac{1}{t^4}\right) , \tag{5.3.82}$$

where the φ^4 term in the former expression arises only due to the non-trivial kinetic term, and not to the backreaction of heavy moduli. Unfortunately, when we plug the set of parameters (5.3.71) into this potential we find a supergravity model where the slow-roll conditions cannot occur for more than $\Delta\varphi \sim 6M_P$ and so the necessary number of e-folds cannot be attained. Of course, this supergravity description is only valid for the small-field limit. At large-field values we should not trust the supergravity scalar potential, which should be replaced by the DBI potential of section 5.2. By the analysis of subsection 5.2.5 we obtain that the corresponding flux-flattened potential would indeed attain the 60 e-fold of inflation with cosmological observables within current experimental bounds. The above analysis should then be understood as a means to estimate the magnitude of the backreaction effects. Indeed, if this magnitude is already negligible for (5.3.82) we expect it to be even less important for the DBI scalar potential, since the effect of flux-flattening will lower the potential energy. We have found this to be a general feature of the effective supergravity models of the kind (5.3.65), irrespective of the set of effective parameters chosen. In fact, for a different choice of parameters one may easily construct models where 60 e-folds of inflation are attained and with realistic cosmological observables, already at the supergravity level.[14]

[14]Indeed, had we chosen the set of parameters

$$\kappa_4 A = -1.05\,, \quad a = \frac{2\pi}{26}\,, \quad \kappa_4 W_0 = 0.48\,, \quad su = 1.05\,, \quad \varepsilon = 6.3\times 10^{-4}\,, \quad t = 9.27\,, \quad \Delta^2 = 0.28\,,$$

we would have also found mass scales similar to (5.3.73) and a supergravity potential of the form (5.3.82). However this potential would now be such that 60 e-folds are attained starting from $\varphi_* = 14.16 M_{\mathrm{P}}$, and with CMB observables with values $r = 0.069$ and $n_s = 0.960$. Again, the backreaction effects will be negligible, more precisely of the order $V_{back} \sim -3.13\times 10^{-18}\varphi^4$. Hence, this example constitutes a 4d supergravity model of large-field inflation of interest on its own.

Part IV

Moduli Stabilization and backreaction

6 Moduli stabilization and large-field inflation

Describing inflation with low-energy effective string actions can often be split into two problems. On the one hand, obtaining a comparably light scalar field with a suitable scalar potential. The latter must be able to generate at least 50 to 60 e-folds of inflation at a characteristic scale H in accordance with CMB measurements. On the other hand, stabilizing all remaining moduli in a Minkowski or de Sitter vacuum at a mass scale greater than H. In this chapter we will focus on the latter problem and its implications for the former.

As we have seen, the models of inflation that we have discussed in Chapters 4 and 5 are F-term axion monodromy realizations where the inflaton candidate is an open-string modulus. For large values of the inflaton candidate they are described by the DBI action, which takes into account all α' corrections. The common denominator of both models is that they realize models of quadratic chaotic inflation, i.e. $V_{\mathrm{sugra}} \approx \frac{1}{2}m^2\varphi^2$ in the $\mathcal{N}=1$ low-energy regime.

We have seen two different descriptions in supergravity of which give that inflationary potential. In Chapter 4 chaotic inflation was realized by means of the so-called 'stabilizer' field coupled in a bilinear superpotential to the inflaton candidate. In Chapter 5 chaotic inflation was realized by means of a quadratic superpotential for the inflaton candidate.

Consistent models of inflation imply that all the closed-string moduli arising due to the compactification should be stabilized during inflation. Thus, it will be crucial to have control over the non-trivial interplay between moduli stabilization and inflation. We will see that both realizations of chaotic inflation will have a different interplay with moduli stabilization and supersymmetry breaking.

In the following section we will focus on moduli stabilization schemes with spontaneous supersymmetry breaking applied to both realizations of chaotic inflation which will be useful for Chapter 8. Afterwards we will review supersymmetric moduli stabilization for chaotic inflation which will be applied in Chapter 7.

6.1 Combining moduli stabilization, chaotic inflation and supersymmetry breaking

Moduli stabilization and inflation In many string-effective inflation models the inflaton and the moduli interact even if the moduli are much heavier than the dynamical scale of inflation. Through supergravity couplings this even happens in models where the superpotential splits into

$$W = W_{\text{inf}}(\Phi_i) + W_{\text{mod}}(N_i)\,, \tag{6.1.1}$$

where Φ_i collectively denotes the superfields involved in the inflationary part of the theory and N_i closed string moduli. Many models of this type have been constructed in the recent literature, from various different corners of string theory. The effect of stabilizing and integrating out the fields N_i has been systematically studied in [189]. In cases where all N_i appear logarithmically in the Kähler potential, the effective potential for the fields N_i at leading order reduces to the scalar potential of the inflation sector alone, as if the moduli had not been present as dynamical degrees of freedom. This is true as long as all moduli masses, determined by the second derivatives of $W_{\text{mod}}(N_i)$, lie above the Hubble scale H, determined by W_{inf} and its first derivatives.

In general one could classify moduli stabilization schemes in two broad groups depending whether they break supersymmetry spontaneously or not. In this chapter we will focus on moduli stabilization schemes which spontaneously break supersymmetry [1], we refer the reader to Section 3.4.2 and [76] for more details. More concretely, we will apply these moduli stabilization schemes to chaotic inflation setups in supergravity. Typically, in these cases the scale of supersymmetry breaking is above the Hubble scale and the effects on the inflationary dynamics will not be negligible in any case. For models of chaotic inflation which fit into this group, typically, the inflaton will receive soft masses controlled by the gravitino mass.

In the following we will focus backreaction analysin on the two different realizations of chaotic inflation in $\mathcal{N} = 1$ supergravity that we have already seen: via quadratic term of the inflaton superfield, Φ, on the superpotential and using the so-called 'stabilizer' fields which we will call S.

Chaotic inflation with $W_{\text{inf}} \supset \Phi^2$ In these type of models the $\mathcal{N} = 1$ description is typically given by

$$\begin{aligned} K &= K_1\left(N_i, \bar{N}_i\right) + K_2\left(N_i, \bar{N}_i, \left(\Phi + \bar{\Phi}\right)\right)\,, && (6.1.2)\\ W &= W_{\text{mod}}\left(N_i\right) + m\Phi^2\,, && (6.1.3)\end{aligned}$$

where N_i denotes the closed-string sector coming due to the compactification except the inflaton candidate which we denote as $\Phi = \chi + i\phi$. In these cases the F-term

[1] In models which don't induce supersymmetry breaking the mass scale of the stabilized moduli at a high scale and the moduli could decoupled at first order from the inflationary dynamics [18 Clem]. We will review this fact in section 6.2.

scalar potential is generically unbounded from below due to the term $-3e^K|W|^2$. The F-term scalar potential obtained from (6.1.3) could be written as

$$V_{\text{F-term}} + V_{\text{up}} = V_0\left(N_i, \bar{N}_i, \chi\right) + V_1\left(N_i, \bar{N}_i, \chi\right) m\varphi^2 + V_2\left(N_i, \bar{N}_i, \chi\right) m^2\varphi^4 \,, \quad (6.1.4)$$

where φ denotes the canonically normalized inflaton candidate. Note that we have added an uplifting term denoted by V_{up} (which is independent of the inflaton candidate φ) which allows to obtain a Minkowski or dS vacuum state after inflation,i.e. $\Phi = 0$. In general, the straightforward way to handle this problem is just to minimize the scalar potential in terms of all the scalar fields in the theory except the inflaton candidate φ and plug it back into the scalar potential. The main issue to this approach is that in stringy compactifications the number of scalar fields obtained after compactification is too huge in order to handle those systems of equations and only numeric approximations could be performed.

One approach is to stabilize all moduli at its supersymmetric point solving the F-term condition $D_{N_i}W = 0$ while switching off the inflaton field $\Phi = 0$. Solving these system of equations will give us a set of vevs N_i^0. Backreaction effects will come by perturbation theory. We consider that every moduli during inflation will be displaced from its minimum by

$$N_i = N_i^0 + \delta N_i\left(\varphi\right) \ , \ \chi = \chi^0 + \delta\chi\left(\varphi\right) \,, \quad (6.1.5)$$

and we expand (6.1.4) at leading order in perturbations $\delta N_i\left(\varphi\right)$, $\delta\chi\left(\varphi\right)$ which we call V_{eff}. Note that this procedure will be valid as long as $\delta N_i \ll N_i^0$, $\delta\chi \ll \chi^0$. After doing that we have to minimize V_{eff} in terms of the perturbations which will have an explicit dependence on the inflaton field φ. After that, we plug it back into the scalar potential and the backreacted scalar potential will be of the form

$$V_{\text{back}} = \frac{1}{2}\hat{m}^2\varphi^2\left(1 + \lambda\frac{m_{3/2}}{\hat{m}} - \frac{3}{8}\varphi^2\right) + \mathcal{O}\left(\frac{H^2}{m_N^2}\right) \,, \quad (6.1.6)$$

where $\hat{m}$ denotes the mass of the canonically normalized inflaton, $m_{3/2}$ is the gravitino mass, m_N denote the mass scale of the moduli which we are integrating out and λ is, at this level of approximation, a constant which depends on the moduli stabilization procedure and the supergravity setup. We see that this behavior is different from the one naively expected when one neglects moduli stabilization during inflation. Thus, we see straightforwardly that moduli stabilization with supersymmetry breaking is an important task to address in order to allow 60 e-folds of inflation with the former scalar potential. Thus, we conclude that having control under backreaction effects will be crucial in any model of large-field inflation. We will follow this procedure in order to obtain corrections to the scalar potential from backreaction effects in Chapter 8.

Models with stabilizer fields Models of chaotic inflation based on stabilizer fields, which we call S, typically have the following description in supergravity

$$\begin{aligned} K &= K_1\left(N_i, \bar{N}_j\right) + K_2\left(N_i, \bar{N}_i, \left(\Phi + \bar{\Phi}\right)\right) + K_{S\bar{S}}\left|S\right|^2 + K_{S\bar{S}S\bar{S}}\left|S\right|^4 \,, &(6.1.7)\\ W &= W_{\text{mod}}\left(N_i\right) + mS\Phi \,. &(6.1.8)\end{aligned}$$

In these type of models the inflaton potential is generated by the F-term of the stabilizer field which decouples from the inflationary dynamics. In order to achieve that, the moduli stabilization procedure should guarantee to set it to a high scale. Typically this large mass-term needed is provided trough a large quartic derivative term of the Kahler potential.

As we did before, the inflaton candidate will be the axionic component of Φ. In this case the stabilizer field will be set at zero vev during inflation. With this at hand, the F-term scalar potential obtained from (6.1.8) could be written schematically in powers of the canonically inflaton φ

$$V_{\text{F-term}} + V_{\text{up}} = V_0\left(N_i, \bar{N}_i, \chi\right) + V_1\left(N_i, \bar{N}_i, \chi\right) m\xi\varphi + V_2\left(N_i, \bar{N}_i, \chi\right) m^2\varphi^2 \,. \quad (6.1.9)$$

where ξ is the saxionic component of the stabilizer field before stabilizing it to zero vev. Note that, thanks to the stabilizer field, the F-term scalar potential in these kind of setups is not unbounded from below as happened before. Taking perturbations around the minimum of every moduli

$$N_i = N_i^0 + \delta N_i\left(\varphi\right)\ ,\ \chi = \chi^0 + \delta\chi\left(\varphi\right)\,. \quad (6.1.10)$$

and expanding (6.1.9) at leading order in perturbations and minimizing it with respect perturbations we find that the most of the moduli will be shifted from its minimum and, in particular, the stabilizer will play an important role on the backreacted scalar potential. In this way, the resulting scalar potential at leading order in φ could be written as

$$V_{\text{back}} \sim \frac{1}{2}\hat{m}\varphi^2\left(1 - \alpha\frac{m_{3/2}^2}{m_{3/2}^2 + \hat{m}^2}\right) + \dots\,. \quad (6.1.11)$$

where α is an order-one coefficient which depends on the specific details of the setup and the dots represent higher powers in φ. Note that, for moduli stabilization schemes where supersymmetry breaking occurs at a high scale, and thus $m_{3/2} \gg \hat{m}$, chaotic inflation with stabilizer fields is highly constrained and compromised. Thus, one can see that these kind of setups are viable from the point of view of moduli stabilization with low-scale supersymmetry breaking like [20,32-34 CLEM 1407]. For this reason we will employ a supersymmetric stabilization scheme in the analysis performed in Chapter 7.

6.2 A shortcut to integrate out heavy moduli supersymmetrically

In this section we will discuss how supersymmetrically integrating out heavy moduli is, to leading order, equivalent to replacing them by their vacuum expectation values in the Kähler potential and superpotential. This result was claimed in [188].

In the following we will consider setups in which all moduli are stabilized in a supersymmetric Minkowski vacuum.[2] For our purposes it suffices to leave the precise mechanism unspecified, and instead assume the existence of a superpotential piece $W_{\rm mod}(\rho_i) \subset W$ which satisfies $\langle D_{\rho_i} W_{\rm mod}\rangle = 0$ for all relevant moduli fields ρ_i. Examples are known in the literature, they include the famous racetrack setup of [101], and a mechanism using an additional stabilizer field [190].[3]. As we mentioned before, in cases where the moduli appear logarithmically in the Kähler potential and are provided by a sufficient mass hierarchy, the effective potential for the fields Φ_i will be reduced to the inflaton sector alone. Since $\langle D_{\rho_i} W_{\rm mod}\rangle = 0$ this confirms a naive expectation fueled by old QFT arguments: if they are heavy enough and do not break supersymmetry, the moduli completely decouple. This statement is true up to sub-leading corrections which arise in powers of H/m_{ρ_i}, cf. [189] for details. These corrections are under control whenever the moduli can be safely integrated out. Still they may be sizeable and lead to slightly changed predictions of a given model, such as the CMB observables. In particular, the higher-order terms arising in powers of H/m_{ρ_i} lead to a flattening of the potential [80, 189].

Despite the interesting effects that these corrections may have, in this section we aim to analyze the stability of the inflationary trajectories after moduli backreaction, for which it suffices to focus on the leading-order result for the effective action. In [189] and subsequent publications this has been obtained by computing the supergravity potential and solving the inflaton-dependent equations of motion for the moduli fields. Depending on the details of the setup, this can be a tedious exercise. Therefore we wish to point out here that the leading-order effective potential, taking moduli backreaction into account, can be obtained via a simple shortcut. The key is the confirmation that integrating out the heavy ρ_i is equivalent to fixing all ρ_i in W and K at their expectation values in the vacuum, and subsequently computing the scalar potential for the remaining fields ϕ_i. The result corresponds to the full effective potential in the limit $m_{\rho_i} \to \infty$. Clearly, however, corrections due to the finiteness of m_{ρ_i}—such as the flattening corrections mentioned above—cannot be obtained in this way.

6.2.1 A no-scale toy model

Let us demonstrate this claim in a few simple examples. Consider a simple no-scale model with a single Kähler modulus T and an inflaton multiplet Φ,

$$K = -3 \log \left[T + \bar{T} - \frac{1}{2}(\Phi + \bar{\Phi})^2 \right] , \qquad W = m\Phi^2 + W_{\rm mod}(T) \,. \tag{6.2.1}$$

For a similar illustration this toy model has already been considered in [193]. It corresponds to a boiled-down variant of some of the F-term axion monodromy models

[2] For specific realizations we refer the reader to Section 3.4.2.

[3] Note that supersymmetry is necessarily broken in the original setup of [102] once the vacuum is uplifted to a Minkowski or de Sitter background. The same applies to the extensions of [113] and [191, 192], in which the breaking scale is typically very high.

from the recent literature.[4] The corresponding scalar potential reads

$$V(\varphi,t) = \frac{1}{6t}\left[\left(\frac{1}{6}m^2 + \frac{1}{2}mW'_{\rm mod}(t)\right)\varphi^2 - \frac{3W_{\rm mod}(t)W'_{\rm mod}(t)}{t} + W'_{\rm mod}(t)^2\right] , \quad (6.2.2)$$

where φ is the canonically normalized inflaton field and $t = \mathrm{Re}\,T$. The other two real scalars do not play a role in this case and have been set to zero. They do not have linear terms in V and do not displace the inflaton. Moreover, their masses are positive and large compared to H.

At first sight, this theory has a supersymmetric Minkowski vacuum at $t = t_0$ with $W'_{\rm mod}(t_0) = 0$ and $W_{\rm mod}(t_0) = 0$. On the inflationary trajectory, then, (6.2.2) reduces to a simple quadratic potential for φ. However, this is not really true because (6.2.2) contains non-trivial interaction terms between t and φ. In particular, minimizing the full potential with respect to t leads to

$$t_{\rm min} \simeq t_0 - \frac{m\varphi^2}{4W''_{\rm mod}(t_0)} + \mathcal{O}\left(\frac{m^2\varphi^2}{W''_{\rm mod}(t_0)^2}\right) , \quad (6.2.3)$$

at leading order in powers of H/m_t, where $m_t \sim W''_{\rm mod}(t_0)$. Plugging this back into (6.2.2) leads to the proper effective potential for the inflaton,

$$V(\varphi) = \frac{1}{18t_0}\left(\frac{1}{2}m^2\varphi^2 - \frac{3}{16}m^2\varphi^4\right) + \mathcal{O}\left(\frac{m\varphi}{W''_{\rm mod}(t_0)}\right) . \quad (6.2.4)$$

Evidently, the interaction during inflation interferes with the cancellation of the negative definite term in the supergravity potential. Taking the backreaction of t into account re-introduces the term proportional to $-3|W|^2$, which makes the model fail.

Most importantly, we could have seen this much faster. Instead of setting $t = t_0$ in the scalar potential, which leads to the wrong result, we must replace $T = t_0$ in K and W defined in (6.2.1). Treating only Φ as dynamical, we observe that

$$K = -3\log\left[2t_0 - \frac{1}{2}(\Phi+\bar{\Phi})^2\right] , \qquad W = m\Phi^2 + W_{\rm mod}(t_0) , \quad (6.2.5)$$

leads to the correct leading-order potential

$$V(\varphi) = \frac{1}{18t_0}\left(\frac{1}{2}m^2\varphi^2 - \frac{3}{16}m^2\varphi^4\right) . \quad (6.2.6)$$

As stressed before, this simplified treatment corresponds to taking $m_t \sim W''_{\rm mod}(t_0) \to \infty$, and thus it is insufficient for computing corrections.

[4]For the purposes of this discussion the precise form of K does not matter. In particular, our results remain unchanged whether Φ and T mix kinetically or not.

6.2.2 A no-scale toy model with stabilizer field

In the following, we will analyze a second example which contains a stabilizer field S. While this eliminates the dangerous term proportional to $-3|W|^2$, effects of the moduli backreaction are important and can be observed using our shortcut. Consider

$$K = -3\log\left[T + \bar{T} - \frac{1}{2}(\Phi + \bar{\Phi})^2\right] + \frac{1}{2}(S + \bar{S})^2\,, \qquad W = mS\Phi + W_{\text{mod}}(T)\,, \tag{6.2.7}$$

which is a simplified version of some of the effective theories that arise in D-brane inflation which we will analyze in Chapter 7. Neglecting the explicitly modulus-dependent terms proportional to W_{mod} and its first derivative for now, we find the following scalar potential.

$$V(S,\varphi,t) = \frac{1}{12t^2}\left[\frac{1}{2}m^2\varphi^2 + \frac{1}{2}(m^2 + 3m^2\varphi^2)s_1^2 + \frac{1}{2}m^2 s_2^2 + \mathcal{O}(W_{\text{mod}}(t), W'_{\text{mod}}(t))\right]\,, \tag{6.2.8}$$

where we have expanded in the relevant fields up to quadratic order. Notice that we have written $S = (s_1 + is_2)/\sqrt{2}$. At this level the picture seems to be the following: φ, s_1, s_2 have equal supersymmetric masses. In addition, s_1 receives a supersymmetry-breaking mass term through its Kähler potential coupling to the inflationary vacuum energy. While s_2 is not heavy enough to satisfy a single-field treatment of inflation for arbitrary initial conditions, the model appears consistent. This would remain true if we naively set $t = t_0$, which entails $W_{\text{mod}}(t_0) = W'_{\text{mod}}(t_0) = 0$.

The consistency no longer holds when we take the backreaction of t into account by setting $T = t_0$ in eqs. (6.2.7). What we find for the leading-order effective potential of the S - Φ system, using

$$K = -3\log\left[2t_0 - \frac{1}{2}(\Phi + \bar{\Phi})^2\right] + \frac{1}{2}(S + \bar{S})^2\,, \qquad W = mS\Phi + W_{\text{mod}}(t_0)\,, \tag{6.2.9}$$

is instead

$$V(S,\varphi) = \frac{1}{12t^2}\left[\frac{1}{2}m^2\varphi^2 + \frac{1}{2}\left(m^2 + \frac{3}{4}m^2\varphi^2\right)s_1^2 + \frac{1}{2}\left(m^2 - \frac{3}{4}m^2\varphi^2\right)s_2^2\right]\,. \tag{6.2.10}$$

One can check that the same result is found after consistently minimizing $T = T_{\text{min}}(S,\varphi)$ during inflation. Notice that s_2 is actually a tachyonic direction during inflation. While s_1 is saved from the same fate by its soft mass term proportional to H^2, the model never yields successful slow-roll inflation due to the tachyonic direction along s_2. This is ultimately due to the shift symmetry of the stabilizer field, and was only concealed by a would-be no-scale cancellation in the modulus sector. As we will explore in the next chapter, this is exactly what causes the D6-brane inflation model analyzed in Chapter 4 to fail.

7

D6-brane inflation and backreaction of closed-string moduli

In the following we would like to apply the general remarks seen in Section 6.2 to examine string theory models of large-field inflation. In particular, in this chapter we will focus on the proposal pointed out in Chapter 4 were the models of chaotic inflation were argued to embed stabilizer fields in type IIA compactifications with D6-branes. As we will see, taking into account the shift symmetries of the model and applying the shortcut to integrate out heavy fields seen in Chapter 6.2 leads to tachyonic directions within the inflationary system which, as in the toy model above, spoil slow-roll inflation. This analysis will allow us to discuss, in Chapter 7.2, whether it is possible or not the embedding of stabilizer fields in type II compactifications. We will illustrate whether the needed conditions are satisfied away from the large-complex structure limit.

7.1 D6-brane inflation

In Chaptar 4 we argued the possibility to embed models of large-field inflation in string theory based on the property of certain D-branes to generate bilinear superpotentials for open- and closed-string axions 4.2.6. In essence the setup features a D6-brane that creates an inflationary potential for a B-field axion and the Wilson line of the brane. Near the supersymmetric vacuum the low-energy supergravity is that of chaotic inflation with a stabilizer field, as first proposed in [131] and generalized in [132]. As discussed in [143] the D6-brane couples to the background in such a way that the following superpotential is developed

$$W_{\text{inf}} = n_a T^a \Phi = T\Phi\,, \tag{7.1.1}$$

where $n_a \in \mathbb{Z}$, Φ is the superfield containing the D6-brane Wilson line, and $T = n_a T^a$ is a linear combination of Kähler moduli such that $b = \operatorname{Im} T$ is the B-field axion that couples to the D6-brane. Following [131] it is clear that such a superpotential can yield an effective description of chaotic inflation if at least one of the two chiral fields is light enough (usually through the appearance of a shift symmetry) and the other one is significantly heavier.

The shift symmetries of this system can be analyzed through the effective Kähler potential for the closed- and open-string moduli in type IIA orientifold compactifications, first discussed in [121, 122] and more recently in [120]. There it was argued that $K = K_{\rm K} + K_{\rm Q}$, where on the one hand

$$K_{\rm K} = -\log\left[\frac{1}{6}\kappa_{abc}(T^a+\bar{T}^a)(T^b+\bar{T}^b)(T^c+\bar{T}^c)\right], \tag{7.1.2}$$

with T^a the Kähler moduli of the compactification and κ_{abc} the corresponding triple intersection numbers.[1] On the other hand for a choice of Calabi-Yau three-form symplectic basis we can write $K_{\rm Q}$ as [122]

$$K_{\rm Q} = -2\log\left(\frac{1}{16}\mathcal{F}_{KL}\left(U'^K+\bar{U}'^K\right)\left(U'^L+\bar{U}'^L\right)\right), \tag{7.1.3}$$

where $\mathrm{Re}\,U'^K$ are defined in terms of the periods of the three-form $\mathrm{Re}\,\Omega$, and $\mathcal{F}_{KL}$ are real functions that only depend on their quotients, such that they are invariant under the overall rescaling $U'^K \to \lambda U'^K$. The most involved part in describing $K_{\rm Q}$ is determining how the geometric quantities U'^K depend on the holomorphic variables of the four-dimensional effective theory. By the analysis of [120] one obtains that

$$U'^K = U^K + \frac{1}{2}T^a\mathbf{H}_a^K, \tag{7.1.4}$$

where U^K is the new holomorphic variable and $\mathbf{H}_a^K$ a homogeneous function of zero degree in $\mathrm{Re}\,T^a$, $\mathrm{Re}\,\Phi$ and $\mathrm{Re}\,U^K$. The leading-order term is of the form

$$\mathbf{H}_a^K = -\frac{1}{2}Q^K\eta_a\frac{(\Phi+\bar{\Phi})^2}{[\eta_a(T^a+\bar{T}^a)]^2}+\dots, \tag{7.1.5}$$

where Q^K and η_a can be taken to be constants that depend on the D6-brane embedding. Putting all this together we obtain the following approximate expression,

$$K_{\rm Q} = -2\log\left\{\frac{1}{16}\mathcal{F}_{KL}\left[U^K+\bar{U}^K-\frac{1}{8}\tilde{Q}^K(\Phi+\bar{\Phi})^2\right]\left[U^L+\bar{U}^L-\frac{1}{8}\tilde{Q}^L(\Phi+\bar{\Phi})^2\right]+\dots\right\}, \tag{7.1.6}$$

where we have defined $\tilde{Q}^K = Q^K/(\eta_a\mathrm{Re}\,T^a)$. This expression for $K_{\rm Q}$ resembles the one used in [31, 130] except for the fact that here $\tilde{Q}^K$ is moduli-dependent. This fact will not be important when applying the philosophy of Section 6.2, since upon integrating out all the closed-string moduli except T we will obtain an effective Kähler potential where $\tilde{Q}^K$ become constant.[2]

[1] In order to connect with the standard notation in the 4d supergravity literature used in Section 6.2, our conventions differ from those in [120–122] and are such that $T^a = t^a + ib^a$, with b^a the B-field axions of the compactification. The same applies to the complex structure moduli, with Im U'^K containing the axionic piece of the field.

[2] When $\tilde{Q}^K$ also depends on the stabilizer field T the discussion is a bit more involved. The coupling of Φ and T in K introduces additional interactions in the scalar potential. However, one can check that these interaction terms arise first at $\mathcal{O}(T^3)$ in the action, which makes them irrelevant to the following discussion. We can thus safely treat $\tilde{Q}^K$ as constants in this case as well.

What it will be relevant in the following is the fact that the Kähler potential only depends on $\mathrm{Re}\, T^a$, $\mathrm{Re}\, U^K$ and $\mathrm{Re}\, \Phi$ and therefore it displays several shift symmetries. This fact is true in general, even without the simplifying assumptions that took us to the expression (7.1.6), and it only relies on considering type IIA at large compactification volumes compared to the string scale [120]. These shift symmetries imply that in principle either $\mathrm{Im}\, T$ or $\mathrm{Im}\, \Phi$ could play the role of the inflaton field; both scenarios have been considered in [31]. Unfortunately this also means that the other field cannot play the role of the stabilizer field, a fact missed in the analysis of [31] where backreaction effects of the heavy closed-string moduli were not taken into account. To see this point in detail we analyze the scalar potential for the inflaton system first from the viewpoint of [31]. Then, in Section 7.1.2, we revisit the scalar potential by applying the philosophy of Section 6.2 to see how backreaction effects destabilize the inflationary trajectory.

7.1.1 The scalar potential without backreaction

Let us consider the scenario in which the D6-brane Wilson line $\phi = \mathrm{Im}\, \Phi$ is the inflaton candidate, and so $\mathrm{Re}\, \Phi = T = 0$ defines the would-be inflationary trajectory. On this trajectory the superpotential (7.1.1) generates a quadratic potential for ϕ. The pressing issue at hand, however, is the stabilization of the closed-string moduli U^K and T^α, where the index α runs over all the Kähler moduli except T. In order to implement such a stabilization W_{inf} must be accompanied by an additional piece $W_{\mathrm{mod}}(U^K, T^\alpha)$, which lifts the corresponding flat and run-away directions. As in [31, 130] we consider the case where none of these moduli break supersymmetry in the vacuum,[3] that is when

$$D_{U^K} W_{\mathrm{mod}}\Big|_{\Phi=0} = D_{T^\alpha}\ , \ W_{\mathrm{mod}}\Big|_{\Phi=0} = 0\,, \tag{7.1.7}$$

and then expand the full F-term scalar potential around the inflationary trajectory to find an effective potential for T and Φ. In [193] it was shown that (7.1.7) is actually a necessary assumption in these kinds of setups. Allowing the moduli to break supersymmetry in the vacuum leads to additional terms, essentially soft terms, proportional to $\langle W_{\mathrm{mod}} \rangle$ and $\langle W'_{\mathrm{mod}} \rangle$. If one of them, or equivalently the scale of supersymmetry breaking, becomes too large the model fails due to a backreaction of the stabilizer field T.

[3]Explicit moduli stabilization schemes with this property do exist in the literature. Cf. [101] for the racetrack proposal, and [190] for a less fine-tuned mechanism involving another stabilizer field.

At quadratic order in the fields the resulting scalar potential of [31] reads[4]

$$V = e^K \left[K^{\Phi\bar{\Phi}} |\partial_\Phi W_{\rm inf}|^2 + K^{T\bar{T}} |\partial_T W_{\rm inf} + T\partial_T^2 W_{\rm mod}| + 4(\operatorname{Im} T)^2 (\operatorname{Im}\Phi)^2 \right] . \quad (7.1.8)$$

where we have assumed that $W_{\rm mod}$ is very small or vanishing at the vacuum. Taking the potential (7.1.8) at face value one can show that $\operatorname{Re}\Phi$ and both components of T have masses parametrically larger than the Hubble scale H, which means they can be safely integrated out during inflation, leading to the desired quadratic potential for ϕ. Note that for $b = \operatorname{Im} T$ this is due to the last piece in (7.1.8), which appears as a remnant of the no-scale symmetry in the closed-string sector. In terms of canonically normalized fields (7.1.8) reads

$$V = \frac{1}{2} m^2 \varphi^2 + \left(\frac{1}{2} m^2 + m^2 \varphi^2 \right) \sigma^2 + \left(\frac{1}{2} m^2 + 2m^2\varphi^2 \right) t_1^2 + \left(\frac{1}{2} m^2 + \frac{8}{3} m^2 \varphi^2 \right) t_2^2 \,, \quad (7.1.9)$$

where t_1 and t_2 are the components of the stabilizer fields, φ denotes the canonically normalized inflaton field, and σ its saxionic partner. In this form the scalar potential mostly depends on the mass parameter m, which in turn depends on the constants in K and the volume of the compact manifold. In this form the desired mass hierarchy $m_\varphi \ll m_\sigma, m_{t_1}, m_{t_2}$ during inflation is evident.

Finally, we may also consider the scenario where we take $b = \operatorname{Im} T$ to be the inflaton candidate. Applying the approach of [31] and expanding the F-term potential along the new inflationary trajectory $\operatorname{Re} T = \Phi = 0$, we obtain a similar scalar potential but with the roles of Φ and T exchanged. More precisely, we obtain (7.1.8) but with the interchange $\Phi \leftrightarrow T$. Needless to say, this leads to the same potential (7.1.9) for canonically normalized fields and therefore to the same naive mass hierarchies as in the previous scenario.

7.1.2 Backreaction of closed-string moduli

As explained above the scalar potential (7.1.8) is obtained via a two-step approach [31]. First one assumes that all closed-string moduli except T are stabilized to a certain value by a suitable superpotential $W_{\rm mod}$ via the condition (7.1.7). Second, the full F-term scalar potential is expanded around the inflationary trajectory to derive the leading-order potential in Φ and T. While this procedure gives the correct result for the potential along the inflationary trajectory where the stabilizer is fixed at the origin, it misses important mass terms for the stabilizer field which arise during inflation. In the following we implement the approach of Section 6.2 to

[4]Here we exhibit the result obtained in [31], which assumed a Kähler potential of the form (7.1.6) and, following [122], that $\tilde{Q}^K$ are moduli-independent. Had we taken into account the correct moduli dependence of these quantities and applied the same procedure a scalar potential different from (7.1.8) would have been obtained, although the subsequent discussion based on it would have been similar. The fact that the calculation of [31] yields effective scalar potentials after changing the dependence of heavy fields in the initial Kähler potential indicates that those heavy fields are not being integrated out consistently.

integrate out the closed-string moduli at tree level to obtain the correct effective potential. As in the toy examples studied earlier, the interaction between moduli and inflaton during inflation leads to tachyonic modes for the stabilizer field which eventually cause the model to fail. This unpleasant effect is ultimately due to the shift symmetries present in the Kähler potentials (7.1.2) and (7.1.6), as already suggested by the toy models of Section 6.2.

Backreaction in the Wilson line scenario

In order to show the importance of backreaction effects in the above models of D6-brane inflation let us focus on the scenario in which the Wilson line $\phi = \mathrm{Im}\,\Phi$ is the inflaton candidate. To illustrate the computation of the effective potential it suffices to consider the case of a single complex structure/dilaton modulus U and two Kähler moduli T_v and T, that respectively parameterize the complexified overall volume and the orthogonal combination of Kähler moduli. Taking into account the general expressions (7.1.2) and (7.1.6) we are lead to the following toy model

$$K_\mathrm{K} = -\log\left[\frac{1}{6}(T_\mathrm{v}+\bar{T}_\mathrm{v})^3 - \frac{1}{2}(T_\mathrm{v}+\bar{T}_\mathrm{v})(T+\bar{T})^2\right], \tag{7.1.10a}$$

$$K_\mathrm{Q} = -4\log\left\{\frac{1}{4}\left[U+\bar{U}-\frac{\tilde{Q}}{8}(\Phi+\bar{\Phi})^2\right]\right\}, \tag{7.1.10b}$$

$$W = T\Phi + W_\mathrm{mod}(U, T_\mathrm{v}), \tag{7.1.10c}$$

in which we have taken simple choices for the triple intersection numbers and defined $\tilde{Q} = 2Q/(T_\mathrm{v}+\bar{T}_\mathrm{v})$. In this parameterization the vacuum of the theory is

$$\langle\Phi\rangle = \langle T\rangle = 0\,, \quad \langle T_\mathrm{v}\rangle = \mathcal{V}^{1/3}\,, \quad \langle U\rangle = \mathcal{V}^{1/2}\,, \tag{7.1.11}$$

where $\mathcal{V}$ denotes the volume of the compact manifold. The full scalar potential defined by (7.1.10) is a complicated expression which is not particularly illuminating. The important parts are however the inflaton couplings at linear and higher order in U and T_v, respectively. Such couplings displace the fields U and T_v from the vacuum (7.1.11) and cause a backreaction into the inflationary system. To see its effect we can expand the scalar potential in terms of this displacement by writing $U = \mathcal{V}^{1/2} + \delta U(\Phi, T)$ and $T_\mathrm{v} = \mathcal{V}^{1/3} + \delta T_\mathrm{v}(\Phi, T)$, where $\mathcal{V}$ is treated as a constant fixed by the details of W_mod. Expanding the action and minimizing the result with respect to the fluctuations δU and δT_v leads to the following effective potential

$$\begin{aligned} V = {} & \frac{1}{2}m^2\varphi^2 + \left(\frac{1}{2}m^2 + m^2\varphi^2\right)\sigma^2 + \left(\frac{1}{2}m^2 - \frac{3}{4}m^2\varphi^2\right)t_1^2 + \left(\frac{1}{2}m^2 - \frac{3}{4}m^2\varphi^2\right)t_2^2 \\ & + \mathcal{O}\left(\frac{m\varphi}{\partial_U^2 W_\mathrm{mod}}, \frac{m\varphi}{\partial_U\partial_{T_\mathrm{v}} W_\mathrm{mod}}, \frac{m\varphi}{\partial_{T_\mathrm{v}}^2 W_\mathrm{mod}}\right), \end{aligned} \tag{7.1.12}$$

at quadratic order in the canonically normalized variables. In this derivation we have again assumed that W_mod and its first derivatives are small or vanishing in the vacuum, so that the second derivatives define the mass matrix of the closed-string sector. In this case the mass parameter behaves as $m \sim Q^{-1/2}\mathcal{V}^{-3/4}$.

Notice the important difference with respect to the naive result (7.1.9): here both components of the stabilizer field are tachyonic during inflation, destabilizing the would-be inflationary trajectory. This is because the “remnant" mass terms for the stabilizer found in the two-step procedure of [31], are actually not present. In particular we find that the last term on the right-hand side of (7.1.8) is absent, something which is only visible after considering the backreaction of U and $T_{\rm v}$ as discussed above.

These backreaction effects can be directly seen by applying the shortcut discussed in Section 6.2. In particular, the leading-order potential (7.1.12) is easily obtained by treating U and $T_{\rm v}$ as constants from the beginning. The scalar potential obtained from

$$K = -\log\left[\frac{4}{3}\mathcal{V} - \mathcal{V}^{1/3}(T+\bar{T})^2\right] - 4\log\left\{\frac{1}{2}\left[\mathcal{V}^{1/2} - \frac{Q}{16\mathcal{V}^{1/3}}(\Phi+\bar{\Phi})^2\right]\right\}\,, \quad (7.1.13)$$

$$W = T\Phi\,. \quad (7.1.14)$$

is identical to the first line of (7.1.12). This way, if one is not interested in the corrections suppressed by powers of m_U and $m_{T_{\rm v}}$ one can save a lot of effort to compute the back-reacted effective potential. Notice that from this viewpoint it is obvious that the moduli dependence of $\tilde{Q}$ does not play an important role for computing the effective potential. Finally, in this form it is obvious that the cancellation which removes the dangerous negative terms does not take place as expected. What we are left with after backreaction effects are taken into account is a variation of the original inflationary theory of [131], but with a shift-symmetric Kähler potential for the stabilizer field. Following Appendix D we can see that in all theories with $K = K(\Phi+\bar{\Phi}, T+\bar{T})$ and the given superpotential the desired mass hierarchy between the inflaton and the stabilizer field cannot be obtained. This applies in particular to the D6-brane inflation scenario in which the inflaton candidate is the B-field, and which fails for the same reason as the case just studied.

A few comments are in order with respect to these findings. First, following standard supergravity computations done in Section D.3 one can easily verify that including different powers of $(T+\bar{T})$ in (7.1.10a) does not solve the problem of the tachyonic directions. Second, the corrections to the leading-order potential in the second line of (7.1.12) can never lift the problematic directions. For the theory to be consistent it must be that $m_U, m_{T_{\rm v}} \gg H \sim m\varphi$, so that these corrections are always subleading. Third, the previous statement is true even in the case when the conditions (7.1.7) are violated, i.e., if the closed-string moduli are permitted to break supersymmetry. This is more tedious to prove because, in this case, there is no complete decoupling of the heavy fields and the computation of the back-reacted potential is more involved. This analysis has been done in [76] for a variation of the model at hand, and in [194] more generally. There are indeed “remnant" terms after integrating out U and $T_{\rm v}$ in this case, which are proportional to $W_{\rm mod}$ and its first derivatives. However, none of them break the shift symmetry of T, so the tachyonic directions cannot be lifted.

We conclude that both Wilson line and Kähler moduli are unsuitable candidates for stabilizer fields in large field inflationary models, due to the shift symmetry that they display in the Kähler potential. Notice however that such shift symmetries are not fundamental, but an artefact of considering type IIA compactifications with large volumes compared to the string scale. Had we considered compactifications of stringy size, the shift symmetries for the Kähler moduli would be generically broken by worldsheet instanton effects and they could in principle serve as stabilizer fields. Nevertheless, the difficulty in that scheme would be to formulate a mechanism that fixes all the heavy moduli. Indeed, in the large volume limit the agent lifting closed-string moduli is a combination of NS and RR fluxes, and implementing the presence of the latter at small volumes remains a challenge. These difficulties will be however absent if we consider the mirror setup of type IIB compactifications at large volume and small complex structure, as we do in the next section.

7.2 Could geometrical moduli act as a 'stabilizer' fields?

In the previous section we have learned that a shift symmetry of the stabilizer field is detrimental to realizing inflation. Whenever the stabilizer field is a Kähler modulus in type IIA theories this shift symmetry is inherent to the large volume regime—the desired regime to use ten-dimensional supergravity to treat compactifications with RR fluxes. The mirror dual statement holds for complex structure moduli in type IIB compactifications with O3/O7-planes: shift symmetries are present whenever we consider the large complex structure limit. However, in such a theory one can explore arbitrary regions of the complex structure moduli space—where the shift symmetries are absent—without sacrificing the ten-dimensional supergravity picture.

Using the former argument one could try to see if there is any way to describe stabilizer fields using geometrical moduli. The aim of this section is to shed some light on this issue. Our starting point will be the mirror dual model to the one described by the D6-brane Wilson line.

One may then conceive a model of large-field inflation in which the role of the stabilizer field is played by a complex structure modulus with no shift symmetries, such that the stability problems discussed in the previous section no longer arise. As we discuss below, these fields can have superpotential bilinear couplings to D7-brane Wilson lines, which would then contain the inflaton candidate.

However, even when this obstacle can be overcome in type IIB setups, a bigger one remains: since the warping close to the locus of the brane does not enter the kinetic term of the D7-brane Wilson line in the way that it does for the D6-brane, the necessary mass hierarchies to justify a four-dimensional effective description of single-field inflation cannot be obtained. As explained in more detail below, the mass of the Wilson line axion is generically close to the Kaluza-Klein scale. This seems to render any attempt of realizing chaotic inflation with stabilizer fields in this way futile. For this reason in the following section we will focus only in the requirements that a type II compactification should satisfy in order to be possible to describe stabilizer fields.

7.2.1 Setting the basics

Useful definitions on type IIB complex structure sector

Here we are going to give a brief recap about the complex structure sector in type IIB orientifolds. For more details we refer the reader to see Chapter 3.

As we have seen the complex structure sector define a special Kahler submanifold in the ambient Calabi-Yau which Kähler potential is given in the Kähler coordinates $z^{\hat{K}}$ by

$$K_{\mathrm{cs}} + \log|X^0|^2 = -\log\left[i\left[(z^a - \bar{z}^a)\left(\mathcal{F}_a + \bar{\mathcal{F}}_a\right) - 2(\mathcal{F} - \bar{\mathcal{F}})\right]\right] = -\log\left(i\Pi^T\Sigma\Pi\right)\,, \tag{7.2.1}$$

where the piece $\log|Z^0|^2$ can be removed by a Kähler transformation, and

$$\Pi = \begin{pmatrix} \mathcal{F}_0 \\ \vdots \\ \mathcal{F}_{h^{2,1}} \\ z^0 \\ \vdots \\ z^{h^{2,1}} \end{pmatrix}\,, \tag{7.2.2}$$

and Σ is the symplectic matrix

$$\Sigma = \begin{pmatrix} 0 & \mathbf{1_3} \\ -\mathbf{1_3} & 0 \end{pmatrix}\,. \tag{7.2.3}$$

with $z^0 = 1$ and $\mathcal{F}_0 = 2\mathcal{F}(z) - z^a\mathcal{F}_a(z)$. In this form, the invariance of the Kähler potential (7.2.1) under $\mathrm{Sp}(2\,(h^{2,1}+1)\,,\mathbb{Z})$ transformations of the periods is manifest.

In practice, one way to compute the periods (3.1.22) in terms of the complex structure moduli is to solve a system of coupled partial differential equations called Picard-Fuchs equations. These arise from the relations among the derivatives of Ω with respect to the complex structure moduli, due to the fact that the dimension of the third cohomology group of $\mathcal{M}$ is finite.

Nevertheless, a relatively simple expression is obtained in the large complex structure limit $z^a \gg 1$ where, as expected from mirror symmetry, one obtains a prepotential of the form

$$\mathcal{F}(z) = \frac{1}{3!}\kappa_{abc}z^a z^b z^c + \frac{1}{2}S_{ab}z^a z^b + P_a z^a + Q + \mathcal{F}_{\mathrm{exp}}\,. \tag{7.2.4}$$

Here κ_{abc}, S_{ab}, P_a and Q are constants and $\mathcal{F}_{\mathrm{exp}}$ contains exponentially suppressed contributions which in the mirror manifold are identified with world-sheet instantons in the large volume limit. Together with (7.2.1) this leads to the well-known expression for the Kähler potential for the complex structure moduli,

$$K_{\mathrm{cs}} = -\log\left[\frac{1}{6}\kappa_{abc}\left(U^a + \bar{U}^a\right)\left(U^b + \bar{U}^b\right)\left(U^c + \bar{U}^c\right) + f_{\mathrm{exp}}\right]\,. \tag{7.2.5}$$

where we have defined $U^a = iz^a$ in order to connect with the conventions of Section 6.2. In this form the shift symmetry of the real part of the U^a is obvious, broken only by exponentially suppressed contributions. It is this shift symmetry – or its type IIA dual – that caused the problems in the D-brane model seen in Chapter (4).

D7-brane Wilson lines

As we have seen in Section 3.5.2, in order to cancel the RR tadpole induced by O7-planes these compactifications typically contain D7-branes wrapping holomorphic four-cycles $\mathcal{S}_A$. If these four-cycles have non-trivial three-cycles $(\tilde{A}^a, \tilde{B}_b)$ within them, the D7-brane has continuous Wilson line moduli that redefine the holomorphic variables T_α (3.5.21). To express such Wilson lines as chiral coordinates one first considers a basis of harmonic one-forms on the four-cycle $\mathcal{S}_A$ wrapped by the D7-brane. Let us, for simplicity, assume the minimal setup in which $b_1(\mathcal{S}_A) = 2$, which corresponds to a single Wilson line. We denote these two harmonic forms as $(\tilde{\alpha}, \tilde{\beta})$ and take them to be in the Poincaré dual class of the three-cycles $(\tilde{A}, \tilde{B})$ of $\mathcal{S}_A$. Then the unique harmonic $(1,0)$-form of $\mathcal{S}_A$ can be expressed as

$$\gamma = (\mathrm{Re}\, f)^{-1} \left[\tilde{\alpha} - i f(U)\, \tilde{\beta}\right] , \tag{7.2.6}$$

with f a holomorphic function on the bulk complex structure moduli U^a. Finally, we can express the D7-brane Wilson line as

$$A = \frac{\pi}{l_s} \mathrm{Re}[\Phi\, \overline{\gamma}] = \frac{\pi}{l_s} \left[-\theta_\beta\, \tilde{\beta} + \theta_\alpha \tilde{\alpha}\right] , \tag{7.2.7}$$

where $i\Phi = \theta_\beta + i f \theta_\alpha$ is the complexification of the real Wilson lines θ_α, θ_β. Following [120], one then sees that such complexified Wilson lines modify the definition of the Kähler variables (3.5.21) as

$$T_\alpha = T'_\alpha + \frac{1}{4} \sum_A \frac{\mathcal{C}^A_\alpha}{\mathrm{Re}\, f^A}\, \Phi^A\, \mathrm{Re}\, \Phi^A\, , \tag{7.2.8}$$

where A runs over the different four-cycles $\mathcal{S}_A$ wrapped by the D7-branes with Wilson lines, and $\mathcal{C}^A_\alpha = l_s^{-4} \int_{\mathcal{S}_A} \omega_\alpha \wedge \tilde{\alpha} \wedge \tilde{\beta}$ is a coupling independent of the moduli. The Wilson lines then enter in the corresponding Kähler potential of the Kähler sector by performing the following replacement

$$T'_\alpha + \overline{T}'_\alpha \;\to\; T_\alpha + \overline{T}_\alpha - \frac{1}{8} \sum_A \frac{\mathcal{C}^A_\alpha}{\mathrm{Re}\, f^A} \left(\Phi^A + \overline{\Phi}^A\right)^2 . \tag{7.2.9}$$

where we have set $G^a = 0$ and we have neglected D7-brane moduli deformations $\zeta^A = 0$. In the simple case where there is only one field T and one Wilson line Φ the Kähler potential becomes

$$K_{\mathrm{K}} = -3 \log \left[T + \bar{T} - \frac{\mathcal{C}}{8\, \mathrm{Re}\, f} (\Phi + \bar{\Phi})^2\right] + \log 8\, . \tag{7.2.10}$$

In general, $\mathcal{V}^2 = \nu(t'_\alpha)$ is a homogeneous function of degree three in $t'_\alpha = \mathrm{Re}\,T'_\alpha$. After the substitution (7.2.9) and expanding up to second order in $\mathrm{Re}\,\Phi^A$ we find that

$$\begin{aligned} K_{\mathrm{K}} &= -\log\left[\nu(t_\alpha) - \frac{\partial_{t_\alpha}\nu}{16}\sum_A \frac{\mathcal{C}^A_\alpha}{\mathrm{Re}\,f^A}\left(\Phi^A + \overline{\Phi}^A\right)^2 + \ldots\right] \\ &\simeq -\log\left[\nu(t_\alpha) - \frac{\mathcal{V}}{16}\sum_A \frac{\mathcal{C}^A}{\mathrm{Re}\,f^A}\left(\Phi^A + \overline{\Phi}^A\right)^2 + \ldots\right], \end{aligned} \tag{7.2.11}$$

where $t_\alpha = \mathrm{Re}\,T_\alpha$ and $\mathcal{C}^A = v^\alpha \mathcal{C}^A_\alpha = l_{\mathrm{s}}^{-4}\int_{\mathcal{S}_A} J \wedge \tilde{\alpha} \wedge \tilde{\beta}$. Since $\mathcal{C}^A$ scales like $[\mathrm{Vol}(\mathcal{S}_A)]^{1/2}$, the larger the four-cycle wrapped by the D7-brane, the larger the coefficient of the kinetic term for its Wilson line, and the smaller the associated canonical masses.

In general it is a difficult problem to determine the form of $f(z^a)$ but, as discussed in [120], whenever a Wilson line appears in the open string superpotential it also needs to satisfy a certain condition with respect to the bulk periods. More precisely the superpotential is given by

$$l_{\mathrm{s}} W_{\mathrm{D7}} = -\sum_a \frac{1}{\pi l_{\mathrm{s}}^2}\int_{\mathcal{S}_A} \Omega \wedge A = -i\sum_a \theta_\beta\left[c_{A\,a}U^a - h^a_A \mathcal{F}_a\right] + \theta_\alpha\left[d^a_A\,\mathcal{F}_a - p_{A\,a}\,U^a\right]\,, \tag{7.2.12}$$

where now $\mathcal{F} \equiv \mathcal{F}(U)$ and $(c_{A\,a}, h^b_A, d^a_A, p_{a\,A})$ are the integrals of (α_a, β^b) over the three-cycle $(\tilde{A}, \tilde{B})$ in $\mathcal{S}_A$. Specifically,

$$\begin{aligned} c_{A\,a} &= l_{\mathrm{s}}^{-4}\int_{\mathcal{S}_A} \alpha_a \wedge \tilde{\beta}\,, & d^a_A &= l_{\mathrm{s}}^{-4}\int_{\mathcal{S}_A} \beta^a \wedge \tilde{\alpha}\,, \\ h^a_A &= l_{\mathrm{s}}^{-4}\int_{\mathcal{S}_A} \beta^a \wedge \tilde{\beta}\,, & p_{a\,A} &= l_{\mathrm{s}}^{-4}\int_{\mathcal{S}_A} \alpha_a \wedge \tilde{\alpha}\,. \end{aligned} \tag{7.2.13}$$

Demanding that W_{D7} is holomorphic in Φ amounts to imposing

$$i f^A(U) = \frac{d^b_A \mathcal{F}_b\, - p_{A\,a}U^a}{c_{A\,a}U^a - h^a_A \mathcal{F}_a}\,, \tag{7.2.14}$$

which determines f^A. Moreover, if we want to impose that (7.2.12) is linear in the U^a some of these intersection numbers need to vanish, in particular we need to impose $h^a_A = c_{A\,0} = 0$. Finally, in the limit of very large complex structure one expects that f^A tends to a linear function of the U^a so that K_{K} respects the shift symmetry of the U^a in the large complex structure limit and one recovers the results in type IIA vacua with D6-branes [120]. Away from that limit, however, we expect higher powers of U^a to appear in f. This will be an important ingredient for describing stabilizer fields, as we discuss next.

7.2.2 Engineering stabilizer fields in type IIB

From D6-brane inflation to D7-brane inflation

We are now in position to consider a mirror-dual version of the D6-brane model defined by (7.1.10). The stabilizer field S is now one of the complex structure moduli

U^a, whereas the D7-brane Wilson line modulus Φ that contains the inflaton appears in K_{K} as described above. For simplicity, let us assume $h^{1,1} = 1$ and $h^{2,1} = 2$, the generalization to more complicated backgrounds being quite straightforward. In the presence of a suitable superpotential W_{mod} the Kähler modulus T and the second complex structure modulus are stabilized supersymmetrically at a high scale. Therefore, we treat them as in Section 7.1.2, meaning we integrate them out at the level of K and W. This leaves us with an effective theory defined by

$$K = -\log\Big[1-(S+\bar{S})^2\Big] - \log\left[\mathcal{V} - \frac{\mathcal{C}(\Phi+\bar{\Phi})^2}{f(S)+\overline{f(S)}}\right] - \log 8\mathcal{V}\,, \tag{7.2.15a}$$

$$W = S\Phi\,. \tag{7.2.15b}$$

where we have set the two heavy closed-string fields to their vacuum expectation values in terms of appropriate powers of the volume. We have also, without loss of generality, set the constant coefficients to simplify the final expression.[5]

In the large complex structure limit $f(S)$ is linear and the theory in (7.2.15) has the same problems as its dual version discussed in Section 7.1.2. However, away from this regime we may assume the general expansion $f(S) = a_0 + a_1 S + a_2 S^2 + \dots$, which leads to a breaking of the shift symmetry for S proportional to the parameter a_2. However, there are still two obstacles to overcome. First, since the breaking term is proportional to Φ^2 in K, the breaking is introduced at too high orders in Φ and S to sufficiently stabilize the tachyonic directions discovered in Section 7.1.2. Second, when leaving the large complex structure regime K_{cs} becomes more complicated than depicted in the first piece of (7.2.15a). While a general form of the prepotential $\mathcal{G}$ is not known, an expansion of (7.2.1) around the origin yields the following terms for the stabilizer field,

$$\begin{aligned} K_{\mathrm{cs}} = -\log\Big[&\alpha_0 + \alpha_1(S+\bar{S}) + \alpha_2|S|^2 + \alpha_3(S^2+\bar{S}^2) \\ &+ \alpha_4(S\bar{S}^2 + S^2\bar{S}) + \alpha_5(S^3+\bar{S}^3) + \dots\Big]\,. \end{aligned} \tag{7.2.16}$$

The coefficients α_i depend on the details of $\mathcal{G}$ and its derivatives, as well as the vacuum expectation value of the second complex structure modulus. While some terms explicitly break the shift symmetry (as expected at small complex structure) others – like the ones proportional to odd powers of S – have a destabilizing effect on the scalar potential. They introduce linear couplings of S to powers of the inflaton field, causing a backreaction of S on the inflationary trajectory. This is analogous to the backreaction of heavy moduli fields that we have encountered before.

Nevertheless, both obstacles can be overcome by including the contribution of a second Wilson line scalar in K_{K}, stemming from a second D7-brane wrapping a different four-cycle $\mathcal{S}_B$, and which also develops a bilinear superpotential coupling.

[5] In general, the Gukov-Vafa-Witten potential contained in W_{mod} may also depend on the stabilizer field S. However, as discussed in a similar example in [70], it is possible to eliminate this dependence by some discrete choices.

Indeed, let us consider a system of the form

$$K = K_{\text{cs}}(S,U,\tilde{U}) - \log\left[\nu(t_\alpha) - \frac{\mathcal{V}}{16}\frac{\mathcal{C}^A(\Phi_A+\bar{\Phi}_A)^2}{\operatorname{Re} f^A(U,\tilde{U},S)} - \frac{\mathcal{V}}{16}\frac{\mathcal{C}^B(\Phi_B+\bar{\Phi}_B)^2}{\operatorname{Re} f^B(U,\tilde{U},S)}\right], \tag{7.2.17a}$$

$$W = S\Phi_A - \tilde{U}\Phi_B + W_{\text{mod}}(U,\tilde{U}), \tag{7.2.17b}$$

with Φ_B a Wilson line arising form the D7-brane wrapping $\mathcal{S}_B$. If W_{mod} contains a term of the form $M\tilde{U}$ then Φ_B will be stabilized at $\langle\Phi_B\rangle \sim M$, and if $\mathcal{C}^B \lll \mathcal{C}^A$ the stabilization will occur at a high scale. We may then replace T_α, U, $\tilde{U}$ and Φ_B by their vevs and obtain an effective theory described by

$$\begin{aligned} K &= -\log\left[1-(S+\bar{S})^2\right] - \log 8\mathcal{V} \\ &\quad - \log\left[\mathcal{V} - \mathcal{V}^{1/3}(\Phi+\bar{\Phi})^2 + \frac{\mathcal{V}^{1/3}\mathcal{M}^2}{2a_0 + a_2(S^2+\bar{S}^2)}\right], \end{aligned} \tag{7.2.18a}$$

$$W = S\Phi, \tag{7.2.18b}$$

where $\mathcal{M} = 2\operatorname{Re}\langle\Phi_b\rangle\sqrt{\mathcal{C}^B/\mathcal{C}^A}$ and $\Phi = \Phi_A$. To simplify the subsequent supergravity analysis we have dropped the S-dependence of the term dividing $(\Phi+\bar{\Phi})^2$. Up to the aforementioned higher-order terms it merely amounts to a field redefinition of Φ. Moreover, in $\operatorname{Re} f^B$ we have removed the linear dependence on S, since neither does it contribute to the breaking of the shift symmetry nor alter the results. As we will argue, this effective system could be possible to describe models of chaotic inflation with stabilizer fields.

Supergravity analysis

Let us take a more detailed look at how the term proportional to c breaks the shift symmetry and lifts the tachyonic directions of the stabilizer field. To treat this term as a correction to the overall volume in K_{K}, we can expand the scalar potential in powers of $\epsilon \equiv \mathcal{M}^2/2a_0\mathcal{V}^{2/3}$. At leading order in ϵ and $\mathcal{V}^{-1}$ we find for the relevant real scalar fields

$$\begin{aligned} V &= \frac{1}{2}m^2\varphi^2 + \frac{1}{2}m^2 s_1^2\left(1-\frac{3}{2}\varphi^2\right) + \frac{1}{2}m^2 s_2^2\left(1-\frac{3}{2}\varphi^2\right) \\ &\quad + \epsilon\left(\frac{3a_2^2}{8a_0^2}m^2 s_1^2\varphi^2 + \frac{3a_2^2}{8a_0^2}m^2 s_2^2\varphi^2\right) + \mathcal{O}(\epsilon^2,\varphi^3,s_1^3,s_2^3), \end{aligned} \tag{7.2.19}$$

in the same notation as in Section 7.1.2. Notice how nicely the first line agrees with the first line of the dual model in (7.1.12). The second line, for nonzero c and a_2 introduces positive definite mass terms for both components of S. One can easily verify that it is possible to choose $\mathcal{V}$, c, and a_0 reasonably such that $\epsilon < 1$ and the expansion is consistent. At the same time, with a mild hierarchy $a_2 > a_0$ the positive mass terms are much larger than the negative ones in the first line, without leaving the convergence radius of the ϵ-expansion. Then both components of S can be integrated out safely at the origin and

$$V = \frac{1}{2}m^2\varphi^2 \,, \tag{7.2.20}$$

is the correct effective theory for the inflaton field.

What remains to be checked is that the destabilizing terms discussed in (7.2.16) do not spoil this nice theory. As mentioned before, using the full Kähler potential in (7.2.16) instead of the reduced version in (7.2.18a) leads to additional terms in the scalar potential of the schematic form

$$\Delta V = \xi s_1 m^2\varphi^2 + \mathcal{O}(s_1^3\varphi^2) \,. \tag{7.2.21}$$

These lead, during the inflationary phase, to a displacement of s_1 away from the origin. ξ is proportional to α_1, α_3, and α_4. This induces a backreaction on the inflaton potential in the same way the heavy moduli potentially do. This kind of backreaction of a stabilizer field has been analysed in similar models in [193, 194]. In this case, due to the mass terms for s_1 and s_2 we have found above, it is always possible to achieve a hierarchy between the mass of s_1 and m.[6] Therefore, one finds that in large regions of parameter space the backreaction induced by 7.2.21 is under control and the model successfully yields 60 or more e-folds of single-field slow-roll inflation in the potential (7.2.20).[7]

7.2.3 A different approach: Stabilizer fields in the Picard-Fuchs basis

There is a different way to obtain low-energy theories at small complex structure with strongly broken shift symmetry for at least one of the complex structure moduli. Following techniques developed in [180, 195, 196] one can expand the complex structure moduli space around a critical point, the so-called Landau-Ginzburg point, and, for a few example manifolds, compute the Kähler potential explicitly in a particular field basis, the Picard-Fuchs basis. In this last section of the paper we wish to briefly review this technique and outline possible toy models that arise and that have promising features in terms of their application to inflation. This description is complementary to the approach taken in Section 7.2.2. The integral symplectic basis of the complex structure used there is related to the Picard-Fuchs basis employed here by a nontrivial field redefinition. Of course, since the same string theory ingredients are taken into account both theories should be equivalent. Note that here we only present the results which are most important for the supergravity analysis, referring to Appendix E and the original literature for details.

[6] For illustrative reasons we are assuming in this section that m is a tunable coefficient.

[7] We have verified this statement via a numerical analysis of the full scalar potential with general coefficients α_i.

The Landau-Ginzburg point and the Picard-Fuchs basis

Finding an explicit expression for K_{cs} away from the large complex structure limit is challenging. In the end, everything boils down to computing the periods of the compact manifold, cf. (7.2.1). These are known in the literature for a few examples in the vicinity of critical points in moduli space. For simplicity we restrict ourselves to two-parameter compactifications, i.e. manifolds with $h^{2,1} = 2$, and the vicinity of the Landau-Ginzburg point. The study of the corresponding moduli space and periods has been carried out for certain Fermat hypersurfaces in [195, 196], following the classic treatment of the quintic and its mirror in [180]. Regarding flux compactifications on such manifolds we recommend [197] for details.

A central observation is that on certain hypersurfaces constructed as mirror duals of weighted projective spaces, the periods can be computed by direct integration of the three-form Ω along a carefully chosen contour, making use of the residue formula. The first step towards the computation of Π is the fundamental period ϖ_0. For two complex structure moduli S and U one can, in the vicinity of the Landau-Ginzburg point at $S = 0$, calculate the fundamental period

$$\varpi_0\left(S,U\right) = -\frac{2}{d}\sum_{n=1}^{\infty}\frac{\Gamma\left(\frac{2n}{d}\right)\left(-dS\right)^n u_{-\frac{2n}{d}}\left(U\right)}{\Gamma\left(n\right)\Gamma\left(1-\frac{n}{d}\left(k_1-1\right)\right)\Gamma\left(1-\frac{k_2 n}{d}\right)\Gamma\left(1-\frac{k_3 n}{d}\right)\Gamma\left(1-\frac{k_4 n}{d}\right)}\,, \tag{7.2.22}$$

where, for $|U| < 1$,

$$u_\nu\left(U\right) = \frac{e^{i\pi\nu/2}\Gamma\left(1+\frac{\nu(k_1-1)}{2}\right)}{2\Gamma(-\nu)}\sum_{m=0}^{\infty}\frac{e^{i\pi m/2}\Gamma\left(\frac{m-\nu}{2}\right)\left(2U\right)^m}{m!\Gamma\left(1-\frac{m-\nu k_1}{2}\right)}\,. \tag{7.2.23}$$

Γ denotes the Euler gamma function and d is the dimension of the defining polynomial of the weighted projective space. Note that u_ν is in general a combination of hypergeometric functions. The infinite sum converges in the vicinity of $S = 0$, for $|U| < 1$, and far from the conifold point located at the locus where $P = 0 = \mathrm{d}P$. From the fundamental period one can construct the remaining periods as follows,

$$\varpi_j\left(S,U\right) = \varpi_0\left(\lambda^j S, \lambda^{jd/2} U\right)\,, \qquad j = 0,\ldots,d-1\,, \tag{7.2.24}$$

where λ is the generator of $\mathbb{Z}_d$. The periods (7.2.24) are solutions of the Picard-Fuchs equations. To distinguish the field basis spanned by S and U from the fields z^a encountered before, we call it the Picard-Fuchs basis. Note that the number of independent periods is again $2\left(h^{2,1}+1\right)$, as expected. A useful period vector in this basis turns out to be

$$\varpi^T = -\frac{\left(2\pi i\right)^3}{S}\left(\varpi_0,\ldots,\varpi_{2(h^{2,1}+1)}\right)\,, \tag{7.2.25}$$

where we have rescaled the 3-form Ω by means of a gauge transformation

$$\Omega \to \frac{1}{S}\Omega\;,\; K \to K + \log\left(S\bar{S}\right)\,. \tag{7.2.26}$$

This period vector, in particular its functional dependence on the complex structure moduli, seems far more complicated than the one found in the symplectic basis in (3.1.23). While this means that the computation of the four-dimensional effective theory is more involved, we will see that the Kähler potential in these variables has a form which displays the breaking of the shift symmetry very explicitly.[8]

In order to actually compute the Kähler potential we have to perform a basis transformation from the Picard-Fuchs to the symplectic basis of the manifold, and then use (7.2.1) to compute the Kähler potential. This basis change can be written in terms of a matrix m_{PF}, which depends on the monodromy matrices of the manifold in the vicinity of the GL point [195,196],

$$\Pi = m_{\mathrm{PF}} \cdot \varpi \,. \tag{7.2.27}$$

Note that $m_{\mathrm{PF}} = [m_{\mathrm{PF}}] / \mathrm{Sp}\left(2(h^{2,1}+1), \mathbb{Z}\right)$. Finding m_{PF} for a given manifold is a difficult task, and we refrain from repeating the details of the examples worked out in the literature. (7.2.27) implies that the coordinates z^a introduced earlier are complicated non-linear functions of S and U. In the end, after expanding the periods in terms of the moduli as follows,

$$(\varpi)^j = 2 \cdot (2\pi i)^3 \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} f_{n,m} \lambda^{nj} \left(-1\right)^{jm} S^{n-1} U^m \,, \tag{7.2.28}$$

where $f_{n,m}$ is given in (E.1.8). One can show that the Kähler potential has the following form,

$$K_{\mathrm{cs}} = -\log\left(\alpha\left(U, \bar{U}\right) + \beta\left(U, \bar{U}\right)|S|^2 + \gamma\left(U, \bar{U}\right)|S|^4 + \cdots\right) \,. \tag{7.2.29}$$

This makes the breaking of the shift symmetry of the modulus S manifest.[9] Moreover, none of the destabilizing terms of Section 7.2.2 arise in the Kähler potential. The functions α, β, γ, depend on the parameters of the projective space and the entries of m_{PF} and are shown in (E.2.6). In the following supergravity computations we treat them as constants, and consider the modulus U to be stabilized at a high scale by fluxes.

While the Picard-Fuchs basis seems to present us with a quite simple and useful expression for K, the expression for W in this basis becomes more complicated. That is because a D7-brane superpotential that is bilinear in the symplectic basis may not be bilinear in the Picard-Fuchs basis. Indeed, in the latter S and U are complicated non-linear functions of z^1 and z^2, and so the superpotential piece involving the D7-brane Wilson line will rather have the form

$$W_{\mathrm{D7}} = i\Phi(a_0 + a_1 S + a_2 S^2 + a_3 S^3 + \dots) \,, \tag{7.2.30}$$

[8] Besides, note that the prepotential $\mathcal{G}(z^a)$, which is not known in general, can be a very complicated function as well.

[9] Note that S is somewhat special compared to U in the parameterization we chose. This is because of the way it rescales the holomorphic three-form.

where the a_i implicitly depend on U. We refer to Appendix E.3 for more details. What is more, the Gukov-Vafa-Witten superpotential [177]

$$W_{\mathrm{GVW}} = \int (F_3 - \tau H_3) \wedge \Omega \,, \tag{7.2.31}$$

now also necessarily depends on S, and the method outlined in [70] to avoid this depencence no longer works. In total, for an appropriate flux choice we expect that the relevant piece of the superpotential reads

$$W = (i\Phi + N)(a_0 + a_1 S + a_2 S^2 + a_3 S^3 + \dots) \,, \tag{7.2.32}$$

where $N \in \mathbb{N}$ and we assume U to be stabilized supersymmetrically by the remaining terms in W_{GVW}. As we analyze in the next section, obtaining a working model of inflation from this low-energy effective supergravity is highly non-trivial.

Supergravity analysis

In order to study the impact of the additional superpotential couplings arising in the Picard-Fuchs basis, let us study a simple toy model once more. Assuming again that all Kähler moduli, as well as U, are decoupled we can describe the effective action by

$$K = \frac{1}{2}(\Phi + \bar{\Phi})^2 + \alpha |S|^2 - \beta |S|^4 + \dots \,, \tag{7.2.33a}$$
$$W = (i\Phi + N)(a_0 + a_1 S + a_2 S^2 + a_3 S^3 + \dots) \,, \tag{7.2.33b}$$

after, for simplicty, expanding (7.2.10) and (7.2.29) around the origin and neglecting factors of $\mathcal{V}$. From the point of supergravity (7.2.33a) looks appealing: There is no shift symmetry for the stabilizer field, and a coefficient $\beta \gtrsim 1$ will introduce a large mass for both real scalar components of S during inflation. As we explore now, however, the problem here is the more general form of the superpotential. In particular, the constant term $a_0 N$ and the non-linear couplings of S to Φ spoil inflation in general. To see this, let us expand the scalar potential to second order in the relevant fields,

$$\begin{aligned} V = {}& a_0^2(1 - 3N^2) + a_1^2 N^2 + \frac{1}{2}(a_1^2 - 3a_0^2)\varphi^2 + \frac{1}{2} m_{s_1}^2 s_1^2 + \frac{1}{2} m_{s_2}^2 s_2^2 \\ & + a_1 s_1 \left[a_0 \left(\sqrt{2} - 2\sqrt{2} N^2 \right) + 2\sqrt{2} a_2 N^2 + \sqrt{2} \varphi^2 (a_2 - a_0) \right] \,, \end{aligned} \tag{7.2.34}$$

where, again, φ is the real inflaton field and $s_{1,2}$ denote the real and imaginary part of S, respectively. The mass parameters $m_{s_{1,2}}$ depend on the a_i, N, β, and φ. They can be made large (and positive) by choosing β to be large, as expected. Notice that none of the a_i with $i > 3$ enter the action at this level. The second line of (7.2.34) once again displaces the real part of S from the origin and introduces a φ-dependent backreaction. But what is more troubling about this potential is the mass term of the inflaton, in particular the negative contribution. Imposing the cancellation of the cosmological constant at $\varphi = 0$ by eliminating (and tuning), for example, the parameter a_1, we find after integrating out both s_1 and s_2

$$V_{\text{eff}}(\varphi) = -\frac{a_0^2}{2N^2}\varphi^2 + \frac{a_0\varphi^2\left[a_0\left(-8N^4 - 4N^2\left(\varphi^2-1\right)+\varphi^2\right)+4a_2N^2\left(2N^2+\varphi^2\right)\right]}{8\beta N^4\left(2N^2+\varphi^2\right)} + \mathcal{O}(\beta^{-2})\,. \tag{7.2.35}$$

We have expanded in inverse powers of β for the sake of illustration. Notice that the leading-order term is negative. This can only be avoided if $a_0 = 0$ and at the same time $N = 0$, which corresponds to a very special choice and tuning. While the next-to-leading order term proportional to $1/\beta$ may include a positive mass term for φ if $a_2 \gg a_0$, the model can never work: If the correction is larger than the leading-order term, the higher-order terms we have omitted here are even more important. A numerical analysis of the full potential reveals that there is no parameter regime where the negative mass term can be overcome. As mentioned above, in this case it is the superpotential which spoils the model. The Kähler potential itself has the correct symmetry structure to achieve a mass hierarchy between the inflaton and both components of S. The inner workings of the setup are, however, extremely sensitive to non-linear superpotential couplings of the stabilizer field. One may be fooled into thinking that a field redefinition $\Phi \to \Phi - N$ makes the problem less hard. But this only shifts the minimum value of φ in (7.2.34) and does not eliminate the dangerous interaction terms between the inflaton and the stabilizer field.

We should stress, however, that if a manifold exists which admits $N = a_0 = a_2 = a_3 = 0$ without severe tuning, or some mechanism which realizes this in the known examples, we would immediately be left with a stable string theory version of the original version of chaotic inflation with a stabilizer field conceived in [131]. In general, however, more ingredients seem to be necessary to obtain 60 e-folds of inflation from a theory written in the Picard-Fuchs basis. For example, this may be additional open-string fields as in the previous successful model. After all, there should exist an analog description of the setup in Section 7.2.2 in terms of the Picard-Fuchs basis. Since the two possible bases of the complex structure are related by a field redefinition, the observable physics should be equivalent in both. In practice, however, finding the precise field redefinition is very challenging. As explained above, it is thus far only known in very few examples.

7.2.4 Mass hierarchies and challenges for large-field inflation

While the problems involving tachyonic directions in the type IIA scenario seem to be avoidable in the type IIB picture, a new problem arises in this setup. Whenever one describes models of single-field inflation as effective theories of string compactifications, there should be a mass hierarchy of the form

$$M_{\text{string}} > M_{\text{KK}} > M_{\text{cs}},\; M_{\text{Kahler}} > H_{\text{inf}}^{\star}\,, \tag{7.2.36}$$

to guarantee control of the various effective field theories. $H_{\rm inf}^\star$ denotes the value of the inflationary Hubble parameter at the point of horizon crossing, i.e., evaluated at the field value $\varphi_\star$ at which the CMB observables are generated. In the large volume regime of a compact manifold with volume $\mathcal{V}$ it is, therefore, instructive to consider the volume scaling of the different mass scales. For sufficiently isotropic internal manifolds with appropriate fluxes one has, in natural units, $M_{\rm string} \propto \mathcal{V}^{-1/2}$, $M_{\rm KK} \propto \mathcal{V}^{-2/3}$, and $M_{\rm cs} \propto N\mathcal{V}^{-1}$, where N is an $\mathcal{O}(1)$ coefficient related to the relevant flux quanta [95]. Moreover, in Kähler moduli stabilization schemes where the T^α break supersymmetry, like KKLT [102] or the Large Volume scenario [113], one typically has a mass scale $\propto W_0\mathcal{V}^{-1}$ for many moduli, while the mass scale of others may be suppressed compared to that, meaning $M_{\rm Kahler} \propto W_0\mathcal{V}^{-3/2}$. Here W_0 is usually the vacuum expectation value of the Gukov-Vafa-Witten superpotential. By a tuning of fluxes one can achieve $W_0 \ll 1$, so that a hierarchy $M_{\rm cs} > M_{\rm Kahler}$ is possible as well. In the schemes that we consider, i.e., the ones where the Kähler moduli do not break supersymmetry, $M_{\rm Kahler}$ is typically unrelated to W_0, but related to other quantities in $W_{\rm mod}$ which may be of $\mathcal{O}(1)$ or smaller, so that the same structure is preserved [101, 190].

This very successful scheme ensures the first two inequality signs in (7.2.36). So how does the inflationary Hubble parameter scale in the discussed models of D6- or D7-brane inflation? In large-field inflation with a quadratic potential one has, up to $\mathcal{O}(1)$ factors, $H_{\rm inf}^\star = m\varphi_\star$. Here m is the mass of the canonically normalized inflaton field φ, and it is this parameter that must be suppressed compared to the remaining M_i above. For the case of D6-brane inflation it was shown in [31] that for a D6-brane wrapping a maximally large three-cycle of size $\mathcal{V}^{1/2}$,

$$m \propto \frac{1}{Q\mathcal{V}^{3/4}} \,. \tag{7.2.37}$$

Moreover, it was argued in [31] that in strongly warped regions of the compactification the warp factor enters the coefficient Q linearly. This means that strong warping can suppress m and make up for the lack of volume suppression compared to $M_{\rm cs}$ and $M_{\rm Kahler}$. Therefore, the hierarchy (7.2.36) can be achieved and the effective field theories of the model are under control.

In the case of D7-brane inflation in a type IIB dual theory as outlined in Section 7.2 the picture is different. Warping does not affect the Kähler potential of the D7-brane Wilson line [154]. Expanding the open-string Kähler potential as in (**??**) and computing the canonically normalized mass then leads to

$$m \propto \frac{1}{\mathcal{V}^{1/2}\,\mathrm{Vol}_{\mathcal{S}_A}^{1/4}} \sim \frac{1}{\mathcal{V}^{2/3}} \,, \tag{7.2.38}$$

where for simplicity we have assumed that $\mathrm{Vol}_{\mathcal{S}_A} \sim \mathcal{V}^{2/3}$, which is obviously the case for compactifications with a single Kähler modulus. In the type IIB case there is no additional suppression of this term because all coefficients that enter are intersection numbers of $\mathcal{O}(1)$. This means that, at least naively, the inflationary Hubble scale in the type IIB dual description is generically of the same order as the Kaluza-Klein scale and larger than the moduli scales.[10] This makes a controlled four-dimensional description of single-field inflation impossible.

[10] Similar control issues have been encountered in setups involving only closed-string fields, cf. [71, 152, 198].

8

D7-brane inflation, moduli stabilization and backreaction

As we have seen in Chapter 6 it is crucial to analyze the interplay between moduli stabilization, supersymmetry breaking and large-field inflation in order to build consistent models. In this chapter we will show, illustratively, how this interplay works in an explicit model. The supergravity description of the setup to analyze will be the one shown in Chapter 5. Contrary to the approach taken in Section 5.3, in this chapter we will not consider large displacements of the dilaton, S. The following analysis will be similar to ones performed in [72,74,161]. In this chapter we will consider, in first place, backreaction of the Kähler moduli. Afterwards we will consider backreaction effects coming from the complex structure plus axio-dilaton sector and discuss the validity of the two-step process. In this chapter the moduli stabilization procedure will be a KKLT-like scheme but a similar analysis could be performed using the LARGE volume scenario.

8.1 $\mathcal{N}=1$ supergravity description

In this section, we will show the supergravity lagrangian which we will analyze. It will be based on the supergravity description displayed in Chapter 5. This means that we will consider a toy model based on D7-brane chaotic inflation [32] on toroidal orientifolds. In presence of a periodic D7-brane the Kahler potential considered will be given by

$$\begin{aligned} K &= -\log\left(T^3+\bar{T}^3\right)-\log\left(\left(S-\bar{S}\right)\left(U^3-\bar{U}^3\right)-\frac{1}{2}\left(\Phi-\bar{\Phi}\right)^2\right) && (8.1.1)\\ &- \sum_{j=1}^{2}\log\left(U^j-\bar{U}^j\right)-\sum_{j=1}^{2}\log\left(T^j+\bar{T}^j\right) . && (8.1.2)\end{aligned}$$

The superpotential will be sourced by ISD background fluxes which will generate, through the Gukov-Vafa-Witten superpotential, W_{flux}. Since this is an illustrative example we will consider that the brane position modulus of the D7-brane, Φ will be source by a μ-term which, for simplicity, will be a tunable coefficient. Finally all the Kahler moduli in this setup will be stabilized by non-perturbative effects, more concretely by gaugino condensate of D7-branes. Thus, we will consider the following

$$W = W_{\text{flux}}\left(U^i, S\right) + W_{\text{np}}\left(T^i\right) + W_{\text{inf}}\left(\Phi^2\right) . \tag{8.1.3}$$

In the following we will consider a KKLT-like moduli stabilization procedure, where after finding a supersymmetric AdS vacuum we will perform a suitable uplifting by means of a nilpotent goldstino.

8.2 Looking for a minimum

First of all, we will follow the standard procedure of KKLT and we will consider all complex structure moduli and the axio-dilaton stabilized through fluxes to a supersymmetric point, i.e. $D_{U^i}W_{\text{mod}} = 0 = D_S W_{\text{mod}}$, and we will consider dynamically only the Kahler and inflaton sectors. This assumption, implicitly implies a cutoff scale in our theory defined by the mass scale of the complex structure. For simplicity we will consider the three Kähler modulus on 'equal footing' which implies $T^i = T$ for $i = 1, \ldots, 3$.

8.2.1 Stabilizing Kähler sector in a KKLT-like scheme

Our first step will be to find a stable AdS vacuum state. It is straightforward to see that the F-term of the inflaton superfield is cancelled at $\Phi = 0$. Now, using this fact we will treat the Kahler modulus. Using the former assumptions the low-energy effective field theory will be described by

$$\begin{aligned} K &= K^Q|_0 - 3\log\left(T+\bar{T}\right) , & (8.2.1)\\ W &= W_0 + Ae^{-aT} , & (8.2.2)\end{aligned}$$

where $K^Q|_0$ denotes the Kahler potential for the complex structure sector and the inflaton candidate evaluated at its vev in the vacuum. Analogously, W_0 will be the vev of the superpotential W_{mod} evaluated at the supersymmetric point. As we saw in Section 3.4.2 we have introduced a non perturbative term coming from gaugino condensation in order to stabilize the Kahler modulus. For simplicity we will consider that ImT is stabilized at 0 and that W_0 is real definite. Then, minimizing the F-term for the Kähler modulus and considering Im$T = 0$ one finds

$$D_T W = 0 \rightarrow W_0 = -Ae^{-aT_0}\left(1 + \frac{2}{3}aT_0\right) , \tag{8.2.3}$$

where we have defined $T_0 = \text{Re}T|_0$. Plugging this result into the scalar potential one finds the usual AdS vacuum found in KKLT scenarios

$$V_{\text{AdS}} = -e^{K^Q|_0}\frac{a^2A^2e^{-aT_0}}{6T_0} . \tag{8.2.4}$$

Now, we need to introduce an uplifting term to reach at least Minkowski or deSitter vacuum . As discussed before in Section 3.4.2, there are several options to perform the uplifting. For simplicity, we will consider a nilpontent goldstino superfield, X, as considered in [111]. The supergravity description of this toy model in presence of the nilpotent goldstino is

$$\begin{aligned} K &= K^Q|_0 - 3\log\left(T+\bar{T}\right) + X\bar{X}\,, && (8.2.5)\\ W &= W_0 + Ae^{-aT} + \Delta X\,. && (8.2.6)\end{aligned}$$

Using this description the uplifting potential will be

$$V_{\rm up} = e^K \Delta^2\,, \tag{8.2.7}$$

which coincides with the expression given in the original KKLT scenario [101] as we have been already discussed. The corrections to the original theory given by the uplfiting term will be subleading, in volume powers, and thus choosing one F-term uplifting instead of other will not modify the underlying physics. For instance, in order to obtain a Minkowski vacuum state one should minimize the scalar potential for every field in the vacuum and impose $V|_{\rm tot}^{\rm vac} = 0$ from which we obtain the following relations

$$A = -\frac{3W_0 e^{aT_0}(aT_0-1)}{2a^2T_0^2+4aT_0-3} \quad , \quad \Delta^2 = \frac{12a^2T_0^2\left(a^2T_0^2+aT_0-2\right)}{\left(2a^2T_0^2+4aT_0-3\right)^2}W_0^2\,, \tag{8.2.8}$$

describing implicitly the new value for $T_0 = \langle \mathrm{Re}\ T\rangle$, while $\langle \mathrm{Im}\ T\rangle$ is stabilized at the origin. The effect of adding an uplifting to the scalar potential will shift the minima that we found in (8.2.3) and thus, Kähler moduli will break supersymmetry. This effect δT_{up} is at leading order

$$\delta T_{up} \sim \frac{1}{a^2 T_0}\,. \tag{8.2.9}$$

With these results at hand, our next step will be to evaluate the scale of supersymmetry breaking given by the gravitino mass and the Kahler moduli scale at leading order in volume which will be given, respectively, by

$$m_{3/2} = e^{K/2}\,|W| = e^{K^Q|_0/2}\frac{aA}{3\left(2T_0\right)^{1/2}}e^{-aT_0} \approx e^{K^Q|_0/2}\frac{W_0}{\left(2T_0\right)^{3/2}}\,, \tag{8.2.10}$$

and

$$m_T = e^{K^Q|_0}\frac{\sqrt{2T}}{9}W_{np}|_{T_0} = 2aT_0 m_{3/2}\,. \tag{8.2.11}$$

In order to assure the consistency of the stabilization with a model of single-field inflation one should assure that the mass scale of the Kähler modulus is at least $m_T > H$. Also, as we will see, assuring a sufficiently large hierarchy of scales will keep under control backreaction effects. The mass of the Kähler modulus could be rewritten as

$$m_T \approx e^{K^Q|_0/2}\frac{aW_0}{\left(2T_0\right)^{1/2}}\,, \tag{8.2.12}$$

where $a = \frac{2\pi}{N}$. One could deduce that, in order to assure a consistent model fo inflation, it is necessary high-scale SUSY breaking. This is opposed to the case analyzed in Chapter 7, where models of chaotic inflation with stabilizer fields were only compatible with low-scale SUSY breaking moduli stabilization schemes in order to be compatible. In this case, stability of the minimum during inflation, will imply that the gravitino mass should, at least, be or order the Hubble scale in order to not decompactify.

With these considerations at hand, one may consider the following choice of parameters

$$W_0 \sim 0.0547 \; , \; N = 24 \; , \; A = -1.7 \; , \; \Delta = 5.664 \times 10^{-3} \; , \; e^{K^Q|_0} = 1.35 \, . \qquad (8.2.13)$$

With the former choice of parameters one can plot the scalar potential in terms of the volume modulus for both, considering or not the upliting.

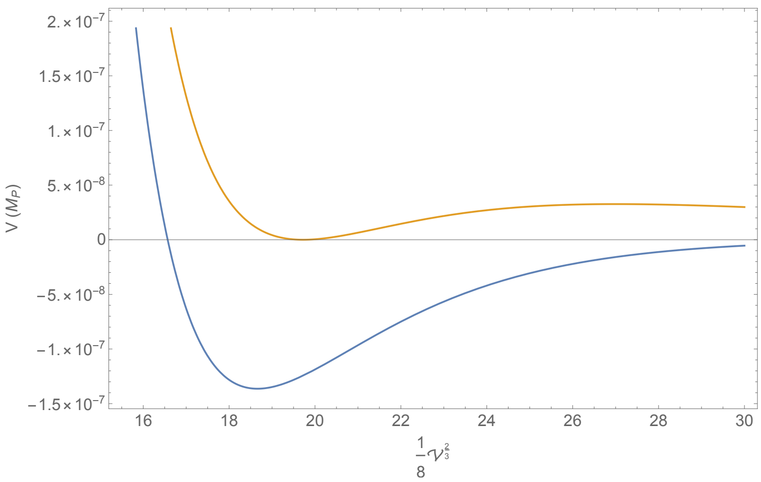


Figure 8.1: In yellow we see the scalar potential for the Kahler modulus with the uplifting shown before. In blue we see the original scalar potential with an AdS vacua. It has been plotted in terms of Re$T = \frac{1}{8}\mathcal{V}^{\frac{2}{3}}$

One can extract from the former figure the shift in the Kähler modulus minimum through the addition of the uplifting mechanism. More concretely, the vev of the Kahler modulus in the AdS is $T_0^{\rm AdS} = 18.66$ and in the Minkowski vacuum $T_0^{\rm Mink} = 19.72$.

8.2.2 Considering complex strucuture sector

As we mentioned before, we considered all complex structure stabilized at a high scale through fluxes. The choice of coefficients shown in (8.2.13) implicitly implied the stabilization of the complex structure sector at some specific vevs. In this section we will give a concrete setup were the complex structure moduli will be stabilized at some vev which coincides with the example given in last section.

Apart from giving an specific example, we will be able to see if we are able to build a consistent mass hierarchy with inflation as discussed in Section 2.3.2.

As mentioned above, in this section we will concentrate only into finding a consistent vacuum after inflation. Since in this illustratively example we are assuming a toroidal orientifold, and the GVW superpotential will have the following form $W_{\text{flux}} = A\,(U^i) + B\,(U^i)\,S$. In this case it will be given by

$$\begin{aligned} W_{\text{flux}} &= (n_0 - \hat{n}_0 S) + U^1\,(n_1 - \hat{n}_1 S) + U^2\,(n_2 - \hat{n}_2 S) + U^3\,(n_3 - \hat{n}_3 S) \\ &+ U^1 U^2 U^3\,(m_0 - \hat{m}_0 S) + U^2 U^3\,(m_1 - \hat{m}_1 S) + U^1 U^3\,(m_2 - \hat{m}_2 S) \\ &+ U^1 U^2\,(m_3 - \hat{m}_3 S)\;. \end{aligned} \tag{8.2.14}$$

By means of landscape arguments one may assume that this superpotential fixes all the complex structure modulus perturbatively at some vev. In the following we will give an explicit setup. Considering the following set of fluxes

$$n_0 = 0\;,\; n_1 = 0\;,\; n_2 = 0\;,\; n_3 = 1\;,\; m_0 = 6\;,\; m_1 = -2\;,\; m_2 = -2 \tag{8.2.15}$$
$$m_3 = 5\;,\; \hat{n}_0 = 1\;,\; \hat{n}_1 = 2\;,\; \hat{n}_2 = 2\;,\; \hat{n}_3 = -3\;,\; \hat{m}_0 = 0\;,\; \hat{m}_1 = 0 \tag{8.2.16}$$
$$\hat{m}_2 = 0\;,\; \hat{m}_3 = 6\,, \tag{8.2.17}$$

we find the following vevs for the complex structure moduli plus the axio-dilaton

$$S = 0 + i\frac{\sqrt{5}}{3\sqrt{2}}\;,\; U^3 = 0 + i\frac{\sqrt{5}}{3\sqrt{2}}\;,\; U^1 = 0 + i\frac{1}{\sqrt{6}}\;,\; U^2 = 0 + i\frac{1}{\sqrt{6}}\,. \tag{8.2.18}$$

With the former setup we see that its contribution to the RR tadpole cancellation condition will be $N_{\text{flux}} = 35$. Note that this choice of fluxes does not imply the introduction of $\overline{D3}$-branes.

We see that the dilaton, S, is stabilized at a vev smaller than 1 which will imply that we should take into account g_s corrections to our toy model. In order to avoid that we will consider the S-dual theory which will be given by

$$F_3 \to -H_3\;,\; S \to \frac{1}{S}\,. \tag{8.2.19}$$

One can check that this transformation will not change the superpotential and the minimization given above. In the S-dual description the dilaton will be stabilized at

$$S = 0 + i\frac{3\sqrt{2}}{\sqrt{5}} \approx 0 + i1.83\,. \tag{8.2.20}$$

On the other hand, we see that the flux superpotential for the complex structure moduli evaluated at its minimum is the same given in the former section, i.e.

$$W_0 = 0.0547\,. \tag{8.2.21}$$

8.2.3 Mass hierarchies in the vacuum

Once we have been able to stabilize explicitly all the moduli in our toy model one may be able to compute the masses of all the fields in the uplifted vacuum state.

Thus, the masses for the canonically normalized fields in the vacuum show the following hierarchy (once we take into account the proper eigenvectors)

$$m_{\text{cx. str.}} \sim 10^{-2} M_{\text{P}} \ , \ m_T \sim 10^{-3} M_{\text{P}} \, . \tag{8.2.22}$$

More concretely, the eigenvalues of the mass matrix are

$$m_1 = 2.26 \times 10^{-2} M_{\text{P}} \ , \ m_2 = 2.26 \times 10^{-2} M_{\text{P}} \ , \ m_3 = 2.25 \times 10^{-2} M_{\text{P}} \tag{8.2.23}$$
$$m_4 = 2.25 \times 10^{-2} M_{\text{P}} \ , \ m_5 = 2.23 \times 10^{-2} M_{\text{P}} \ , \ m_6 = 2.23 \times 10^{-2} M_{\text{P}} \tag{8.2.24}$$
$$m_7 = 2.27 \times 10^{-2} M_{\text{P}} \ , \ m_8 = 2.27 \times 10^{-2} M_{\text{P}} \ , \ m_9 = 2.97 \times 10^{-3} M_{\text{P}} \tag{8.2.25}$$
$$m_{10} = 2.41 \times 10^{-3} M_{\text{P}} \, . \tag{8.2.26}$$

Looking at this mass eigenvalues one can take two considerations. First of all one could argue the validity of the two-step process employed since the complex structure sector is stabilized at a higher scale compared to the Kähler scale. In this concrete example the mass scale will be ten times bigger. On the other hand, assuming that exists a way to set the mass of the inflaton in order to fit COBE normalization (1.2.1), which is around the GUT scale, the mass hierarchy between those scales will be sufficiently large to neglect backreaction effects, at leading order, of the complex structure moduli during inflation.

Also, in order to ensure the consistency of the toy model, one should consider the string scale and the KK scale which, will scale as

$$M_s \sim \frac{1}{\mathcal{V}^{1/2}} \sim 7 \times 10^{-2} M_{\text{P}} \ , \ M_{KK} \sim \frac{1}{\mathcal{V}^{2/3}} \sim 3 \times 10^{-2} M_{\text{P}} \, . \tag{8.2.27}$$

One can see that, with this example at hand, there exists a mild hierarchy between KK and complex structure scale which is on the edge of the validity regime of our theory.

8.3 Moduli stabilization during inflation and backreaction

In the former section we have been able to obtain a Minkowski vacuum state compatible with the model of large-field inflation at hand. In this section we will analyze backreaction of the closed string sector during inflation. First of all, as we have done in last section, we will analyze backreaction of the Kähler moduli sector while assuming all complex structure moduli stabilized at a high scale. Afterwards we will analyze the validity of this approach by analyzing the backreaction of all the closed-string moduli. The former analysis will be performed only numerically. We refer the reader to Appendix F for an analytic approach in a simplified version of the model considered in this section.

8.3.1 Backreaction of the Kahler modulus

Once we have computed explicitly moduli stabilization in the vacuum of all the closed string moduli, in the spirit of what we have seen in Chapter 6 in this chapter we will consider backreaction during inflation of the Kahler modulus. We will first take this approach since it corresponds to the lightest scale of stabilized moduli and, in principle, it would contain the most important effects of backreaction if a hierachy between complex structure and Kähler sectors is assured.

Next, we will consider the presence of the open-string sector sourced in the superpotential μ-term[1] which modifies the setup shown in Section 8.2.1 in the following way

$$\begin{aligned} K &= -3\log\left(T+\bar{T}\right) - \log\left(4u_2^1u_2^2\right) - \log\left(4s_2u_2^3 + \frac{1}{2}\left(\Phi-\bar{\Phi}\right)^2\right) + X\bar{X} \quad (8.3.1)\\ W &= W_0 + Ae^{-aT} + \mu\Phi^2 + \Delta X\,, \quad (8.3.2) \end{aligned}$$

where s_2, u_2^i denote the vevs of the saxionic components of the axio-dilaton and the respective complex structure labeled by i. Note that this compactification contains 15 more D7-branes which, for simplicity, we have stabilized at the top of the respective orientifold planes.

As we have discussed before, we will consider backreaction effects as perturbations around the minimum where the corresponding closed string modulus is stabilized, i.e. $T \approx T_0 + \delta T$. We will assume that the perturbations satisfy $\delta T \ll T_0$. Expanding the scalar potential into perturbations and minimizing the scalar potential with respect them, i.e. $\partial_{\delta T} V = 0$ we find that

$$\begin{aligned} T &\approx T_0 + \delta T = T_0 - K_0^{\Phi\bar{\Phi}}\frac{\mu}{2aW_0}\varphi^2 + \mathcal{O}\left(\frac{H}{m_T}\right)^2\,, \quad (8.3.3)\\ \mathrm{Im}T &\approx \mathrm{Im}T_0 + \delta\mathrm{Im}T = 0 = \mathrm{Re}\Phi_0 + \delta\mathrm{Re}\Phi \approx \mathrm{Re}\Phi\,. \quad (8.3.4) \end{aligned}$$

In order to have under control backreaction effects we need to satisfy

$$\delta T \ll T_0 \to K_0^{\Phi\bar{\Phi}}\mu\varphi_\star^2 \ll 2aW_0T_0\,, \quad (8.3.5)$$

where $\varphi_\star \sim 15M_{\mathrm{P}}$ denotes the initial condition which gives us 60 e-folds of inflation for chaotic inflation. The mass of the inflaton after backreaction is given by

$$m_\Phi^2 \sim \frac{1}{\mathcal{V}^2}\left(2K_0^{\Phi\bar{\Phi}}\mu^2 + 3fW_0\right)\,. \quad (8.3.6)$$

Considering $\mu = 5\cdot 10^{-6}$ we obtain the mass of the inflaton satisfying the COBE normalization 1.2.1

$$m_\varphi = 6\times 10^{-6}M_{\mathrm{P}}\,,\ m_{\mathrm{Im}(\Phi)} = 1.4\times 10^{-3}M_{\mathrm{P}}\,. \quad (8.3.7)$$

[1] We will not specify here the details of the μ-term. One could consider a diluted flux as in [161] or a combination of complex structure moduli stabilized at some vev, where one should be aware of [71].

We see that the mass of the saxionic partner is even above the Hubble scale, so our considerations about a single-field inflationary model will hold. Note that, in this case the mass of the inflaton during inflation is driven by the gravitino mass as we have seen in Chapter 6. This fact only can be seen once we have computed the backreaction. Plugging back the backreacted moduli (8.3.3) into the effective scalar potential one finds a result similar to the one discussed in Chapter 6. In fact one can obtain

$$V_{\text{back}}(\varphi) = \frac{1}{\mathcal{V}^2}\left(K_0^{\Phi\bar{\Phi}}\mu^2 + \frac{3}{2}\mu W_0\right)\varphi^2 - \frac{3}{8\mathcal{V}^2}K_0^{\Phi\bar{\Phi}}\mu^2\varphi^4 + \mathcal{O}\left(\frac{H}{m_T}\right)^2 . \tag{8.3.8}$$

Note that this expression reassembles into (6.1.6). Plugging into the former expression the set of parameters given in the former section (8.2.13) one can see that the backreacted scalar potential will be given by

$$V_{back}(\varphi) \quad = \quad 3.6 \times 10^{-11}\varphi^2 - 6.2 \times 10^{-15}\varphi^4 . \tag{8.3.9}$$

One could compare this result with the one naively obtained neglecting backreaction effects. To do so, we will show in the next figure a plot of the leading-order backreacted scalar potential, the naive scalar potential and the backreacted scalar potential at all orders

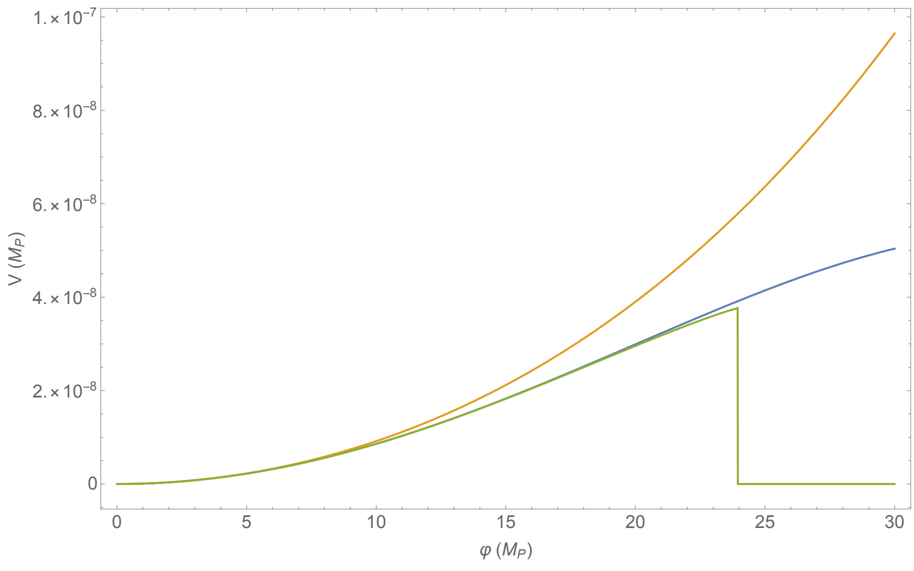


Figure 8.2: In yellow we see the scalar potential without taking into account backreaction effects. In blue we see the backreacted scalar potential which allows 60 e-folds of inflation. The green line is the numeric backreacted scalar potential taking into account all orders in $\delta(T)$

One could see that the backreacted scalar potential allows for 60 e-folds of inflation and also we see that is in completely agreement with the numeric backreacted scalar potential taking into account all orders in backreaction.

For completitude, one can see that the gravitino mass in this case is given by

$$m_{3/2} = 2.1 \times 10^{-4} M_{\text{P}} \sim 4H . \tag{8.3.10}$$

This result assures, which is related with the barrier of potential of the metastable vacuum, us the stability of the compactification along 60 e-folds of inflation.

8.3.2 Backreaction of the closed-string sector

In this section we will consider all the closed-string sector moduli as dynamical. In this case we will consider that the upliting mechanism also depends on the complex structure moduli and we will find a minimum where all moduli will break supersymmetry. Afterwards, following the same strategy of the former section, we will consider backreaction of all the closed-string sector during inflation. Due to the large number of fields in this toy model we will employ a numerical approximation. Finally we will compare the results obtained with the ones shown in last section. It will give us the opportunity to discuss about the validity of the approximations taken in sections where we considered all complex structure moduli already stabilized.

First of all we have to note that the mass of the complex structure moduli is around $10^{-1}M_{KK}$ and backreaction of the scalar potential will not be almost affected with respect the one that we have shown in the last section. This fact could be seen using the argument shown in [188] where we argued that the leading order backreaction of the heavy moduli (i.e. taking its mass to infinity) is obtained just putting those moduli at its vev in the Kahler potential and superpotential and computing the scalar potential using the standard $\mathcal{N} = 1$ supergravity formula. In that sense we can consider the results shown in the last section as the combination of the leading order backreacted scalar potential for the complex structure moduli and the next-to-leading order in backreaction for the Kahler moduli. So, in principle we can consider that this model is safe under backreaction effects due to the fact that the next-to-leading order in backreaction of the complex structure moduli will be subleading with respect to the contribution coming from the Kahler sector. We will perform in this example a numeric backreaction analysis were all moduli will be dynamical.

This computation is more challenging than the shown in the previous section, because we have to minimize the scalar potential with respect fluctuations of all real fields. Since, now, we have to consider all the moduli dynamically we will not consider here the two-step process and stabilize all moduli at the same time. Due to the mass hierarchy in the vacuum shown in the former sections our hint is that the results obtained in this section will not vary strongly with respect the ones shown in the former section.

First of all we will obtain a metastable dS solution using the same uplifting term as we used in the former section. Since all the saxionic components of all moduli will appear in the Kahler potential all moduli will break supersymmetry

$$V = V_{\text{F-term}}\left(T, U^i, S\right) + e^K \Delta^2 \,. \tag{8.3.11}$$

Note that in the following example we will use the flux choice shown in (8.2.17). Minimizing numerically the former scalar potential we see that the vevs in the vacuum of the Kahler and complex structure moduli and Δ are barely shifted compared to the ones obtained before.

$$U^3 = 0 + i0.54 \ , \ S = 0 + i1.88 \ , \ U^2 = 0.632 \ , \ U^1 = 0.632 \ , \ T = 0 + i19.72 \qquad (8.3.12)$$

The next step is to compute the backreaction effects coming from the heavy moduli. In this case we have to expand all the real fields in perturbations dependent on φ. We compute the backreacted scalar potential at leading order and we see that the backreacted scalar potential at next-to-leading order in backreaction, now taking into account all moduli dynamically coincides approximately with the one obtained just taking into acccount just the Kahler moduli. This is because the contribution of the complex structure moduli because of backreaction is highly supressed by itself because the system enjoys a mass hierarchy of order $\sim 10^3 H$.

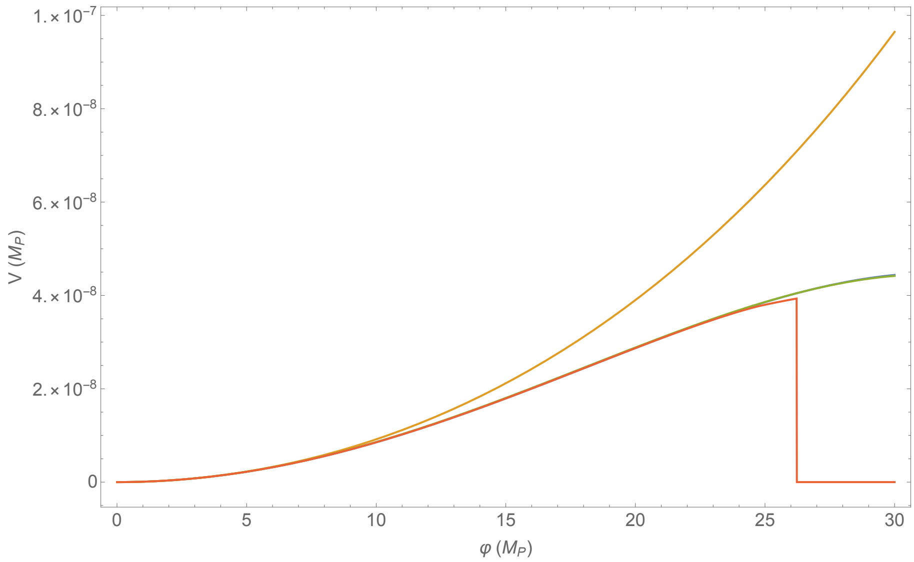


Figure 8.3: In yellow we see the scalar potential without taking into account backreaction effects. In green we see the backreacted scalar potential computed considering dynamically all moduli which allows 60 e-folds of inflation, it overlaps the blue line wich only takes into account the backreaction of the Kahler moduli. The red line is the numeric backreacted scalar potential taking into account all orders in backreaction of all moduli

Note that the red line takes into account backreaction effects to all orders in perturbation theory. We see clearly that backreaction effects coming from complex structure moduli are negligible and that with the former setup we are able to hold 60 e-folds of inflation.

In conclusion, we have checked that the two-step procedure used before naively is consistent due to the mass hierarchies achieved. Also we see that the highest contribution to the backreaction of the scalar potential is the one coming from the lightest field, which is the Kahler moduli.

Analysis of the transplanckian field range As a final remark we will analyze the consistency of the transplanckian field range following the arguments given in

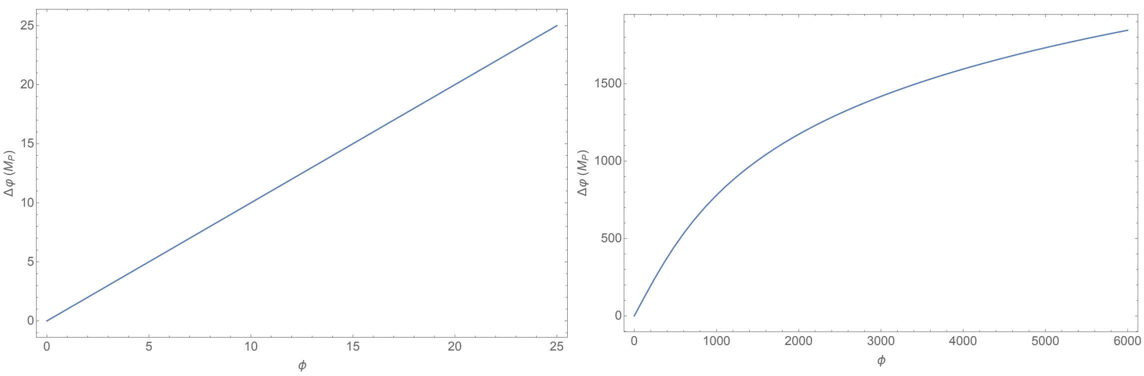
[69, 72, 74, 199]. To do so, one should focus on backreaction effects shown in the kinetic term of the inflaton candidate rather than the backreacted scalar potential. The field displacement of the canonically normalized inflaton is given by

$$\Delta\varphi = \int \frac{1}{\sqrt{K^{\Phi\bar{\Phi}}}} d\phi \,. \tag{8.3.13}$$

where $K^{\Phi\bar{\Phi}}$ is the appropriate entry in the inverse Kähler metric once one takes backreaction effects into account. Clearly, from the Kähler potential employed in this example one can see that this entry in the vacuum is given by

$$K^{\Phi\bar{\Phi}}|_{\varphi=0} = 2\left(-\mathrm{Im}\,(\Phi)^2 + u_2 s_2\right) \,. \tag{8.3.14}$$

Once one takes into account the backreaction effects that we discussed in former sections this term shows an explicit dependence on the inflaton candidate. Taking into account this dependence and plugging it into (8.3.13) one observes a logarithmic dependence on the field range with respect the inflaton candidate as pointed in [69,72,74,199]. In the following plots we will show the numerical field range obtained in the example considered



a) $\phi \in [0, 25 M_{\mathrm{P}}]$ **b)** $\phi \in \left[0, 6\times 10^3 M_{\mathrm{P}}\right]$

Figure 8.4: General field range for the axion for different domains

One can extract from the former figures that the logarithmic dependence, seen for large displacements, in the field range could be delayed by a sufficient mass hierarchy between the inflaton candidate and closed-string sector. In Figure 8.4 one could see that the field range does not sense the logarithmic dependence during the 60 e-folds of inflation. This result was pointed in [72, 74]. But as it was stressed in [72] one should be able to give a proper description for the μ-term in order to be consistent with this fact. From four-fold considerations the μ-term should be a quantized flux sourcing the brane position modulus in the superpotential and in that case the necessary mass hierarchy will be impossible to achieve and thus, the logarithmic dependence will shown in the process of inflation such that it will invalidate the theory. A proper description of consistent μ-terms which allow the necessary behavior are beyond the scope of this text and we leave it for future work.

8.4 $SL(2,\mathbb{R})$ transformations of the Kähler and superpotential and alternative effective theories

One naive possibility to achieve the necessary small μ-term could be obtained by employing $SL(2,\mathbb{R})$ transformations of the theory as we will explain below. What we are going to discuss in this section is, at this stage, far from being a final answer since it is necessary to analyze carefully backreaction effects and amount of tuning necesary as discussed in [71]. But it seems worthy to discuss it.

First of all, let us consider a Kähler potential of the form

$$K \;=\; -\log\Big[(\Phi-\overline{\Phi})^2-(S-\overline{S})(U-\overline{U})\Big]+K_2\,, \tag{8.4.1}$$

where K_2 does not contain any dependence on U, S, Φ. Then following [200–203] we see that K is invariant under a $SL(2,\mathbb{R})_U$ symmetry up to a Kähler transformation. More precisely we have that by under the following field redefinitions

$$U \;\to\; \frac{aU+b}{cU+d}\,, \tag{8.4.2}$$

$$S \;\to\; S-c\frac{\Phi^2}{cU+d}\,, \tag{8.4.3}$$

$$\Phi \;\to\; \frac{\Phi}{cU+d}\,, \tag{8.4.4}$$

with $a, b, c, d \in \mathbb{R}$ and $ad-bc=1$, the Kähler potential transforms as

$$K \to K+\log|d+cU|^2\,. \tag{8.4.5}$$

Let us now take a superpotential of the form

$$W=\hat{\mathfrak{n}}+\hat{\mathfrak{m}}\,U-\mathfrak{n}\,S+\mathfrak{m}\left(\Phi^2-SU\right)+2\mathfrak{f}\,\Phi+W_2\,, \tag{8.4.6}$$

where W_2 and the calligraphic letters are functions of other moduli but not of U, S, Φ. Applying the above set of field redefinitions and taking into account the Kähler transformation (8.4.5) we obtain

$$W\to W' \;=\; \hat{\mathfrak{n}}'+\hat{\mathfrak{m}}'\,U-\mathfrak{n}'\,S+\mathfrak{m}'\left(\Phi^2-SU\right)+2\mathfrak{f}\,\Phi+(cU+d)W_2\,, \tag{8.4.7}$$

where

$$\mathfrak{n}'=d\mathfrak{n}+b\mathfrak{m}\,,\quad \mathfrak{m}'=a\mathfrak{m}+c\mathfrak{n}\,,\quad \hat{\mathfrak{n}}'=d\hat{\mathfrak{n}}+b\hat{\mathfrak{m}}\,,\quad \hat{\mathfrak{m}}'=a\hat{\mathfrak{m}}+c\hat{\mathfrak{n}}\,. \tag{8.4.8}$$

In particular, if $\mathfrak{n}$ and $\mathfrak{m}$ have the same phase we can always choose a and c such that $\mathfrak{m}'=0$. In this case, for $\mathfrak{f}=0$ we have a flat direction along $\mathrm{Re}\,\Phi$. One can then see that, in terms of the original variables this precisely corresponds to the trajectory (5.3.38), with $r/s=-c/a$.

Interestingly, one can use this freedom to obtain an expression for W and K more suitable for the purposes of section 5.3.4, namely to obtain an effective theory for the fields Φ and T in order to analyse moduli stabilisation. For this, recall that $\mathfrak{n}$, $\mathfrak{m}$, $\hat{\mathfrak{n}}$, $\hat{\mathfrak{m}}$ are functions of the complex structure moduli of the compactification. Let us now denote their numerical value at the vacuum $\Phi = 0$ by their non-calligraphic version. That is,

$$n = \mathfrak{n}|_{\Phi=0}\,, \qquad m = \mathfrak{m}|_{\Phi=0}\,, \qquad \hat{n} = \hat{\mathfrak{n}}|_{\Phi=0}\,, \qquad \hat{m} = \hat{\mathfrak{m}}|_{\Phi=0}\,. \tag{8.4.9}$$

Now, as these quantities are numbers we can do the field redefinition (8.4.2-8.4.4) with parameters

$$a \,=\, 1 \qquad ,b \,=\, 0 \qquad ,c \,=\, -\mathrm{Re}\,\left(\frac{m}{n}\right) \qquad ,d \,=\, 1\,. \tag{8.4.10}$$

In terms of the new variables we have the same Kähler potential (8.4.1), and the new superpotential

$$W \,=\, \hat{\mathfrak{n}} + \left(\hat{\mathfrak{m}} - \mathrm{Re}\,\left(\frac{m}{n}\right)\hat{\mathfrak{n}}\right)\, U - \mathfrak{n}\, S + \left(\mathfrak{m} - \mathrm{Re}\,\left(\frac{m}{n}\right)\mathfrak{n}\right)\,\left(\Phi^2 - SU\right) + \ldots\,, \tag{8.4.11}$$

and so, if we write this superpotential in the form (5.3.40) we have that at the vacuum

$$\frac{g}{f} \,=\, i\mathrm{Im}\,\left(\frac{m}{n}\right) \,=\, i\left|\frac{m}{n}\right| \sin\,(\theta_n - \theta_m)\,, \tag{8.4.12}$$

where θ_m, θ_n are the phases of m and n, respectively. By our assumptions of the main text this difference of phases is very small and so this is a very small number. We then recover a shift-symmetric Kähler potential and a superpotential with new modulus dependent coefficients. Near the vacuum the coefficient for Φ^2 is much smaller than those for the closed string moduli, and a slight misalignment of phases plays the role of an effective μ-term. This μ-term is in particular much smaller than the coefficient of S and with a phase that differs by $e^{i\pi/2}$. Under these circumstances it seems quite reasonable to apply the strategy of [188] to the new complex structure and dilaton S, with the latter differing slightly from the variable (5.3.55). After that we obtain an effective theory for Φ given by

$$W \,=\, W_0 + \mu\Phi^2 + \ldots \qquad ,\mu = in\mathrm{Im}\,\left(\frac{m}{n}\right)\,, \tag{8.4.13}$$

and a Kähler potential of the form (8.4.1), where now S and U are replaced by their vevs. As in section 5.3.4 one may add the contribution from the Kähler moduli sector to address full moduli stabilisation below the flux scale. For instance, in a KKLT-like scenario one would obtain an effective potential of the form

$$W \,=\, W_0 + \mu\Phi^2 + Ae^{-aT}\,, \tag{8.4.14}$$

and a Kähler potential given by

$$K \,=\, -3\log\left[T + \bar{T}\right] - \log\left[4su + (\Phi - \overline{\Phi})^2\right]\,, \tag{8.4.15}$$

with $s = \langle \mathrm{Im}\, S\rangle$ and $u = \langle \mathrm{Im}\, U\rangle$. The computational details of the complex structure and Kähler moduli backreaction and the conditions needed in order to have trans-Planckian field ranges would be, then, similar to the ones discussed in in this chapter and already obtained in [161].

Part V

Conclusions & Appendices

Conclusions

In this thesis we have studied different realizations of large-field inflation in type II string compactifications within the context of axion monodromy. Such models are described by the DBI action for large vevs of the inflaton candidate and, using their supergravity description, we have been able to study moduli stabilization and backreaction of the closed-string sector.

In Part I we have presented, briefly, most of the ingredients needed in the following chapters. In Chapter 1 we have reviewed the cosmological standard problem while paying attention to several inherent problems regarding this model, like the flatness and horizon problems. Inflation, and the slow-roll approximation, is a compelling approach which solves several of these problems by means of an accelerated expansion in an early epoch of our universe. Model building of these models is constrained because inherent UV sensitivity problems, like the eta-problem. Most of these problems could be solved by the introduction of a continuous shift symmetry for the inflaton candidate. Thus, from a bottom-up perspective one could argue that axions are perfect candidates to drive large-field inflation. We have seen that a scalar potential compatible with chaotic inflation could be generated, in a gauge invariant way, through the coupling of the axion with a four-form by means of the Kaloper-Sorbo lagrangian.

Besides that, we know that String Theory is a theory of Quantum Gravity and also an outstanding candidate to unify all fundamental forces of nature. In Chapter 2 we have discussed different ways to realize inflation in string theory, where these approaches could be classified depending on the microscopic nature of the inflaton candidate. However, there are several challenges in string theory that one should address in order to describe a consistent model of inflation. Model building constraints come, essentially, from: UV sensitivity, like the eta-problem, moduli stabilization and Quantum Gravity considerations, like the WGC. Since type II string compactifications give rise to a plethora of axions in four-dimensions it seems natural to focus our analysis in models of string inflation based on axions. We have argued that, in analogy to the EFT approach, in string theory one could generate a gauge invariant mass term for the axion to drive inflation. This mechanism, which is called axion monodromy, consists on lifting the periodic direction of the axion by means of fluxes or branes. Typically the scalar potential a multi-branched structure. There exists a subclass of models known as F-term axion monodromy which admits an $\mathcal{N} = 1$ supergravity description given by an F-term scalar potential. This framework naturally embeds the Kaloper-Sorbo formalism. Finally in Chapter 3 we have introduced some necessary details regarding type II compactifications. We have described the geometrical moduli space in $\mathcal{N} = 2$ and $\mathcal{N} = 1$ compactifications and how the moduli space is modified by the introduction of D6- and D3-/D7-branes. Finally we have reviewed compactifications in the presence of background fluxes and several schemes of moduli stabilization both in type IIA and type IIB String Theory.

Afterwards, in Parts II and III we have studied several models of string inflation where the inflaton candidate belongs to the open-string sector. The scalar potential in both cases is given by the DBI action. Both models are explicit realizations of F-term axion monodromy and thus, it permits to perform well-established techniques of moduli stabilization of the closed-string sector during inflation by means of its supergravity description. Next, we will explain the details of each model.

In Chapter 4 we developed a proposal to realize models of large field inflation by including D-branes that generate an open-closed bilinear superpotential. As discussed in Section 4.2, this superpotential is generated by the presence of D6-branes wrapping special Lagrangian three-cycles containing a non-trivial two-cycle in the ambient Calabi-Yau. For concreteness, we have focused on type IIA compactifications with D6-branes, although most of our results are also valid in dual setups like type IIB/F-theory compactifications with 7-branes. Since the bilinear superpotential shown in Section 4.2 has been used extensively in the 4d supergravity literature to build models of large-field inflation using the so-called 'stabilizer' fields, we have considered compactifications whose inflaton sector resembles such supergravity models as much as possible. The 4d supergravity description is, however, only valid for small inflaton vevs. For trans-Planckian vevs, α' effects may induce important corrections to the scalar potential. We have been able to compute in Section 4.3 such corrections for the scenario where the inflaton descends from a B-field component, obtaining a flattened potential with a linear behaviour for large inflaton values. Such flattening of the potential has a non-trivial effect on the cosmological parameters of the model. In particular it lowers the value of the tensor-to-scalar ratio with respect to the quadratic potential of the related 4d supergravity models, allowing to fit the resulting ratio within current experimental bounds.

In Chapter 5 we have analyzed an interesting class of models of F-term axion monodromy inflation that arise in type IIB/F-theory flux compactifications with mobile D7-branes. The main observation, made in section 5.2, that has triggered our analysis is that the flux-induced potential on the D7-brane position field, Φ, presents large flattening effects at large field values, due to the structure of the DBI+CS action. We have found that when one considers the most generic flux background the flattening effects are much larger compared to similar scenarios. This effect, dubbed *flux flattening*, arises due to the different dependence that the inflaton potential and kinetic terms have on Φ in the presence of generic background fluxes. It occurs that the kinetic terms grow equally or faster than the potential and so, upon canonical normalization and at large field values, we find a potential that displays either a linear or smaller power-law behaviour. In Section 5.2.3 we have made a rough estimate, based on moduli stabilization considerations, for the range of values of this parameter and have shown that the related potentials nicely reproduce CMB observables within the current experimental bounds, attaining values for the tensor-to-scalar ratio as low as $r \sim 0.04$. In section 5.3 we have used the example of F-theory on $\mathbf{K3} \times \mathbf{K3}$ to develop our intuition on this system, and in particular on which kind of discrete and continuous symmetries will it exhibit. This picture has served to formulate under which conditions the 4d supergravity scalar potential of a compactification with a mobile D7-brane will contain a flat or a very light direction involving a particular component of Φ, which we then identify with the inflaton field.

In fact, we have found in Section 5.3.2 that the corresponding inflationary trajectory also involves large displacements of the dilaton field S. Finally, we have analyzed the compatibility with Kähler moduli stabilization in a very particular KKLT-like scheme.

The common denominator of both models is that, at low energies, they describe models of chaotic inflation in $\mathcal{N} = 1$ supergravity. On the one hand, the model discussed in Chapter 4 is based on 'stabilizer' fields. On the other hand, the model discussed in Chapter 5 is based on the presence of the inflaton superfield quadratically in the superpotential. In order to describe a consistent model of inflation it is necessary to stabilize and consider backreaction effects of all the moduli in our theory. We tried to shed some light to these issues in Part IV.

In Chapter 6 we have analyzed the interplay of both type of models of chaotic inflation with moduli stabilization and supersymmetry breaking. We argued that, due to backreaction effects, models with stabilizer fields are consistent with low-scale SUSY breaking while models with quadratic superpotential with high-scale SUSY breaking. Moreover, we have discussed the modification of the inflationary scalar potential once one takes into account backreaction effects of the closed-string sector. On the other hand, in Section 6.2 we have stressed that integrating out such heavy moduli supersymmetrically is, to leading order, equivalent to treating the moduli as constants in the Kähler and superpotential. This provides a simple way to take the leading-order backreaction into account.

After reviewing the basics of backreaction in models of chaotic inflation in supergravity, we have analyzed in detail in Chapter 7 the backreaction of the closed-string sector during inflation for the model proposed in Chapter 4. We have argued that the shift symmetry of the stabilizer field is detrimental to realize large-field inflation while pointing the failure of the 'two-step' process claimed in Section 4.4. Specifically, the shift symmetry of the stabilizer field in the large volume limit forbids the necessary large mass terms which stabilize the inflationary trajectory. This result served us as starting point to realize whether it is possible to realize 'stabilizer' fields arising from the closed-string sector in type II string compactifications. We argued that while this possibility seems far from being reasonable in type IIA, in its mirror type IIB dual could be achieved by complex structure moduli away from the large-complex structure limit. In this scenario, the standard Kähler potential derived in Section 3.1 is modified by terms which explicitly break the shift symmetry of the complex structure moduli. The necessary bilinear superpotential could be realized by means of the open-closed bilinear analyzed in Section 4.2. Also, we have argued in Section 7.3.2 that the Landau-Ginzburg point seems an interesting point in the complex structure moduli space to realize stabilizer fields. The complex structure moduli around this special point are expanded around the origin while the Kähler potential displays a complete absence of shift symmetries. The discussion held in section 7.3 was far away from proposing a complete model of large field inflation since the inflaton candidate was realized as the complexified D7-brane Wilson line which is difficult to realize a way to create hierarchies.

Finally, in Chapter 8 we have analyzed, illustratively, moduli stabilization and closed-string backreaction in a model with quadratic superpotential. It served us to see in detail how moduli stabilization and supersymmetry breaking are involved in a model of large-field inflation. We have argued that these kind of models need high-scale supersymmetry breaking scale in order to be consistent with moduli stabilization. Also, we have been able to analyze the validity of the usual two-step process employed. To do so we have compared the corresponding results obtained both considering the complex structure sector already integrated out or not. We have also argued, that a sufficient mass hierarchy between the inflaton candidate and the closed-string sector implies milder backreaction effects and thus allows 60 e-folds of inflation. Finally, in Section 8.2.3 we have analyzed the validity of the transplanckian field range by means of the Refined Swampland Conjecture. We have argued that the detrimental logarithmic behavior of the field range could be delayed by means of a sufficient mass hierarchy, in accordance with recent papers on the topic. It still remains a question whether it is possible to generate in string theory this kind of hierarchy with tunning and backreaction effects under control.

After these results, natural questions arise which could be considered as future research prospects.

Regarding Chapter 4, based on our results, there is a number of open problems and further developments that need to be addressed in order to construct concrete models and obtain precise predictions out of them. For instance, one important development would be to construct explicit examples of special Lagrangian three-cycles that contain two-cycles which are non-trivial in the bulk geometry. As we have seen such topological condition is necessary to generate the bilinear superpotential and, therefore, the scalar potential for the inflation system. While examples of these three-cycles can be obtained in simple toroidal orbifold geometries, it would be desirable to gain a better understanding of their properties by constructing them in smooth Calabi-Yau geometries. In particular, it would be very interesting to compute the DBI potential for such explicit examples. One could then see if the assumptions made to arrive to the square-root potential are realized in practice or if on the contrary a different sort of of flattened potential is obtained.

From what we have seen in Chapter 5, there is a series of directions which would deserve further attention in order to render our flux flattening scenario more precise. First, including Kähler moduli stabilization will induce the presence of imaginary anti-self-dual (IASD) background fluxes, which will in turn modify the DBI+CS D7-brane action. Since in our supergravity analysis the backreaction effects of Kähler moduli are negligible for our setup, we expect the same to be true for the contribution of IASD fluxes. Nevertheless, it would be interesting to generalise the D7-brane action computation of section 5.2 to include the presence of IASD fluxes in order to directly verify this expectation. Moreover, in order to perform a more accurate analysis of backreaction effects along the inflationary trajectory, it would be interesting to describe the DBI+CS D7-brane potential and kinetic terms purely in terms of 4d supergravity. Due to the complicated square root dependence that arises due to the DBI action this seems in general quite a challenging task, but it may be achievable for the simplified expressions that arise for the choice of parameters made in subsection 5.2.4.

From Chapter 7, even with the failure of the models proposed we believe that our analysis provides several points worth investigating in the future regarding a possible realization of stabilizer fields in Type II compactifications. First, can a breaking of the shift symmetry of the stabilizer field be achieved in the type IIA picture, where all mass hierarchies are under control? Without sacrificing the large volume regime, possible sources could include α' or g_s corrections. Second, is there a mechanism which could restore the desired mass hierarchies in the type IIB picture, where the tachyonic directions can be lifted? Due to the appearance of the Wilson line modulus in the Kähler potential, one may investigate if this is possible in a highly anisotropic region of complex structure moduli space.

Finally, from Chapter 8, naturally the first question that one should answer is whether there is any way to create a sufficiently large mass hierarchy between the inflaton candidate and the closed-string sector in order to guarantee the validity of the transplanckian field excursion of the inflaton. One interesting possibility is the one discussed in Section 8.3 which realizes $SL\left(2,\mathbb{R}\right)$ symmetries in order to obtain a small μ-term. But even if this option succeeds one should address the amount of tuning, if any, necessary.

Conclusiones

En esta tesis hemos estudiado diferentes realizaciones de modelos inflacionarios de 'campo grande' en compactificaciones de cuerdas de tipo II en el marco de monodromía de axiones. Estos modelos son descritos para grandes valores del inflatón por medio de la acción DBI y, haciendo uso de su descripción en supergravedad, hemos sido capaces de estudiar estabilización de *moduli* y efectos de *backreaction* del sector de cuerda cerrada.

En la Parte I hemos presentado, brevemente, todos los ingredientes necesarios en el resto de capítulos. En el Capítulo 1 hemos revisado el modelo estándar de cosmología prestando atención a varios problemas inherentes en este modelo, como pueden ser el problema del horizonte y el problema de la planitud. Infación, y la aproximación *slow-roll*, es una aproximación prometedora que resuelve varios de estos problemas dabido a la expansión acelerada del universo en una época temprana del universo. Estos modelos presentan de manera inherente problemas en cuando se trata de realizar una compleción ultravioleta. Estos problemas constriñen, como por ejemplo el problema η, las posibilidades de construir modelos inflacionarios. La construcción de este tipo de modelos se encuentra constreñida por los inherentes problemas de sensibilidad ultravioleta, como el problema η. La mayor parte de estos problemas puede ser resuelta por la introducción de una simetría contínua del candidato a inflatón. Por tanto, desde una perspectiva heurística uno puede argüir que los axiones son perfectos candidatos para llevar a cabo inflación de 'campo grande'. Hemos visto que el potencial escalar compatible con inflación caótica puede ser generado, de una manera invariante *gauge*, a través del acoplo del axion a una cuatro-forma realizando así el lagrangiano de Kaloper-Sorbo.

Por otro lado, sabemos que la Teoría de Cuerdas es una teoría de la Gravedad Cuántica y, además, una candidata a unificar todas las fuerzas fundamentales. En el Capítulo 2 hemos discutido cómo uno puede realizar inflación en teoría de cuerdas. Hemos visto distintos enfoques, dependiendo del origen microscópico del candidato a inflatón. Pero, existen diversos retos en teoría de cuerdas que han de ser resueltos para describir modelos de inflación de manera consistente. La construcción de modelos se ve constreñida esencialmente por: sensibilidad ultravioleta, como el problema η, estabilización de *moduli* y consideraciones de Gravedad Cuántica, como puede ser la WGC. Dado que las compactificaciones de cuerdas de tipo II dan lugar una plétora de axiones en cuatro dimensiones parece natural cetrar nuestro análisis en modelos de inflación en teoría de cuerdas basados en axiones. Hemos argüído que, análogamente al caso de teorías de campos efectivas, en teoría de cuerdas uno puede generar un término de masas en una forma invariante *gauge* de tal manera que pueda llevar a cabo inflación. Este mecanismo, denominado monodromía de axiones, consiste en elevar la energía de la dirección periódica del axión por medio de flujos o branas. Existe una subclase de este tipo de modelos llamado *F-term axion monodromy* el cual admite un potecial escalar de F-term en $\mathcal{N}-1$ supergravedad. Este marco reproduce de manera natural el formalismo de Kaloper-Sorbo y por tanto uno puede asumir que las correcciones debidas a operadores supradimensionales no renormalizables se encuentran bajo control. Finalmente en el Capítulo 3 hemos introducido algunos detalles necesarios sobre compactificaciones de cuerdas de tipo II. Hemos descrito el espacio de *moduli* geométrico en compactificaciones $\mathcal{N} = 2$ y $\mathcal{N} = 1$ y hemos mostrado cómo este espacio se modifica debido a la introducción de D6- y D3-/D7-branas. Por último hemos revisitado compactificaciones en presencia de flujos de fondo y varios esquemas de estabilización de moduli en los tipos de cuerdas IIA y IIB.

Posteriormente, en las partes II y III hemos discutidos varios modelos de inflación de 'campo grande' donde el candidato a inflación pertenece al sector de cuerda abierta. Por tanto, el campo escalar en ambos casos viene dado por la acción de DBI para grandes valores del campo. A bajas energías vienen descritos por un potencial escalar de *F-term*, el cual nos ha permitido utilizar técnicas de estabilización de moduli bien establecidas con tal de establecer el sector de cuerda cerrada en una escala de energías superior. A continuación explicaremos los detalles de cada modelo.

En el Capítulo 4 hemos propuesto un modelo para realizar inflación de 'campo grande' incluyendo D-branas que generan un superpotencial bilineal de cuerda abierta-cerrada. Tal como discutimos en la Sección 4.2, este superpotencial está generado por *backreaction* de D6-branas enrollando tres-ciclos *special Lagrangian* en presencia de un dos-ciclo no trivial en el Calabi-Yau ambiente. Más específicamente, nos hemos centrado en compactifiaciones de tipo II con D6-branas aunque la mayoría de los resultados mostrados son válidos en compactificaciones tipo II de cuerdas y teoría F en presencia de D7-branas. Dado que el superpotencial mostrado en la Sección 4.2 has sido empleado extensivamente en la literatura de supergravedad en cuatro dimensiones para construir modelos de inflación usando los llamados campos 'estabilizadores', hemos considerado compactificaciones donde el sector de inflación se asemeja todo lo posible a los modelos de supergravedad. Aunque, la descripción dada por supergravedad es únicamente válidad para pequeños desplazamientos del inflatón. Para desplazamientos transplanckianos, las correcciones α' induce importantes modificaciones en el potencial escalar. Hemos sido capaces de describir en la Sección 4.3 estas correciones en el escenario en el que el inflatón proviene de una componente del campo B, obteniendo un potencial escalar 'aplanado' con un comportamiento lineal para valores grandes del inflatón. Este tipo de 'aplanamiento' del potencial escalar tiene un efecto no trivial en los parámetros cosmológicos del modelo. En particular decrece el ratio entre perturbaciones escalares y tensoriales con respecto al obtenido en potenciales cuadráticos obtenidos en supergravedad, haciendo así que sea relativamente sencillo coincidir con los actuales datos experimentales.

En el Capítulo 5 hemos analizado una interesante clase de modelos de inflación basada en *F-term axion monodromy* descrita en compactificaciones de cuerdas tipo II y teoría F con la presencia de flujos y D7-branas móviles. La principal observación, hecha en la sección 5.2, que ha llevado a cabo nuestro análisis es el potencial inducido por flujos en el campo de la posición de la D7-brana, Φ, presenta un gran aplanamiento para valores grandes del campo, debido a la estructura de la acción de DBI+CS. Hemos encontrado que cuando uno considera el conjunto de flujos más genérico los afectos de aplanamiento son mucho mayores que en escenarios similares, donde no son considerados. Este efecto, denominado *Flux-Flattening*, surge debido a la distinta dependencia que manifiestan el potencial y los términos cinéticos con respecto al inflatón en presencia de estos flujos genéricos. Tal como observamos, los términos cinéticos crecen igual o más que el potencial escalar a medida que el inflatón se desplaca por lo que, bajo normalización canónica y a grandes valores del inflatón, obtenemos un potencial escalar con un ratio entre componentes escalares y tensoriales menor que el obtenido para un modelo de inflación lineal. En la Sección 5.2.3 hemos realizado una estimación, a grosso modo, basado en consideraciones provenientes de estabilización de *moduli*, en la que el espacio de parámetros resultante de nuestro modelo es compatible con los datos experimentales extraídos del fondo cósmico de microondas. Bajo estas consideraciones hemos sido capaces te obtener un ínfimo al ratio mencionado de $r \sim 0.04$. En la Sección 5.3 empleando el ejemplo en teoría F en $\mathbf{K3} \times \mathbf{K3}$ para desarrollar nuestra intuición en este sistema, y en particular para analizar las óías discreta y contínuas que aparecen. Esta imagen nos ha servido para formular las condiciones bajo las que el potencial escalar en supergravedad en cuatro dimensiones conteniendo una D7-brana móvil contiene una dirección plana o una dirección muy ligera para una componente particular de Φ, la cual identificaremos con el inflatón. De hecho, hemos encontrado en la Sección 5.3.2 que las correspondientes trayectorias inflacionaras también involucran grandes desplazamientos del dilatón S. Finalmente, hemos analizado la compatibilidad nuestros resultados con la estabilización del sector Kähler en un esquema de estabilización a la KKLT.

Los modelos presentados presentan un común denominador en su descripción en $\mathcal{N} = 1$ supergravedad. El modelo analizado en el Capítulo 4 está basado en inflación caótica con campos estabilizadores, los cuales requieren una escala de ruptura de superóía baja, En el capítulo 5 hemos analizado modelos de inflación caótica donde el supercampo de inflación aparece cuadráticamente en el superpotencial. Tal como hemos visto, debido a la compactificación una plétora de campos escalares aparecen en la teoría efectiva en cuatro dimensiones. Es neceario estabilizar todos los *moduli* y analizar sus efectos de *backreaction* con tal de describir un modelo de inflación consistente. Hemos tratado de arrojar algo de luz a estos problemas en la Parte IV.

En el Capítulo 6 hemos analizado la relación entre ambas descripciones de inflación caótica con estabilización de *moduli* y ruptura de superóía. Hemos discutido que modelos basados en campos estabilizadores son consistentes con ruptura de superóía a baja escala mientras que modelos con un campo cuadrático en el superpotencial son consistentes con ruptura de superóía a alta escala. También hemos visto cómo el potencial escalar inflacionario se ve modificado una vez se tienen en cuenta efectos de *backreaction* del sector de cuerda cerrada. Por otra parte, en la Sección 6.2 hemos analizado el hecho de que integrar supersimétricamente *moduli* pesados es, en primera aproximación, equivlente a tratar estos campos como constantes en el potencial Kähler y superpotencial. Esto provee una manera simple de obtener las correcciones dominates de backreaction.

Tras haber revisitado los aspectos básicos de *backreaction* en modelos de inflación caótica en supergravedad hemos analizado en detalle en el Capítulo 7 la *backreaction* del sector de cuerda cerrada durante inflación en el modelo propuesto en el Capítulo 4. Hemos observado que la simetría de cruce del campo estabilizador en el potencial Kähler es contraria a la realización de inflación de 'campo grande' en este tipo de modelos. Además este resultado ha mostrado que el proceso 'en dos pasos' de estabilización de *moduli* empleado en la Sección 4.4 no es válida cuando se toma en consideración efectos de backreaction. Más específicamente, esta simetría de cruce del campo estabilizador en el límite de volumen grande prohibe los necesarios términos de masa grandes que estabilizan la trayectoria inflacionaria. Este resultado nos ha servido de punto de partido para discutir si es posible realiar campos estabilizadores en comapctificaciones de cuerdas de tipo II en el sector de cuerda cerrada. Hemos discutido que mientras que esta posibilidad parece lejos de ser razonable en el tipo IIA, en su dual IIB puede ser conseguido identificándolo con el sector de estructura compleja fuera del límite de estructura compleja grande. En este escenario, el potencial Kähler estándar obtenido en la Sección 3.1 se ve modificado por términos que rompen explícitamente la simetría de curce del sector de estructura compleja. El necesario superpotencial bilineal puede ser descrito por medio del superpotencial descrito la Sección 4.2. Además, hemos observado en la Sección 7.3.2 que el punto de Landau-Ginzburg parece un punto interesante en el espacio de *moduli* para describir campos estabilizadores. Los *moduli* de estructura compleja en este punto son expandidos alrededor del origen mientras que el potencial Kähler muestra una completa ausencia de simetría de cruce. La discussión realizada en la Sección 7.3 se encuentra lejos de describir un modelo inflacionario viable con campos estabilizadores en teoría de cuerdas dado que el candidado a inflación propuesto fue la complexificación de *Wilson lines* de D7-branas para las cuales es complicado crear la necesaria jerarquía de masas compatible con inflación.

Finalmente, en el Capítulo 8 hemos analizado de manera ilustrativa la estabilización *backreaction* del sector de cuerda cerrada en un modelo de inflación caótica con ruptura de supersimetría a alta escala. Esto nos ha servido para observar en detalle cómo el procedimiento de estabilización y la ruptura de supersimetría modifican el potencial escalar. Además hemos sido capaces de analizar la validez del usual proceso 'en dos pasos' dado que hemos comparado los resultados obtenidos en los casos en los que hemos considerado el sector de estructura compleja integrado o no. Además, hemos observado que una jerarquía de masas sufciente entre el inflatón y el sector de cuerda cerrada implica efectos de backreaction más suaves y por tanto, permitiendo 60 e-folds de inflación. Finalmente, en la Seccieon 8.2.3 hems analizado la validez del rango transplanckiano del inflaton haciendo uso de la llamada *Refined Swampland Conjecture*. Hemos discutido que el perjudicial comportamiento logarítmico en el rango del campo puede ser retrasado por medio de una jerarquía de masas suficientemente grande, estos resultados se encuentran de acuerdo con recientes artículos en este tema. Todavía es una incógnita si es posible crear esta jerarquía de masas en teoría de cuerdas consistente con efectos de *backreaction* y afinación.

Depués de estos resultados aparecen diversas preguntas naturales que pueden ser consideradas líneas de investigación futuras.

En referencia al Capítulo 4, basado en nuestros resultados, existen un número de problemas abiertos que deben ser resueltos para la construcción de modelos concretos y obtener predicciones precisas de ellos. Una interesante línea de investigación sería la construcción explícita de ejemplos de tres-ciclos *special Lagrangian* que contienen dos-ciclos no triviales en el en la variedad compacta. Tal como hemos visto, esa condición topológica es condición necesaria para generar el superpotencial bilineal y por tanto el potencial escalar inflacionario. Mientras que estos tres-ciclos ha sido obtenidos en geomtrías toroidales simples, sería deseable obtener un mejor entendimiento de sus propiedades mediante la construcción de estos ciclos en geometrías de Calabi-Yau sin singularidades. En particular, sería muy interesante obtener el potencial de la acción de DBI para esos ejemplos explícitos. Esto implicaría ver si las suposiciones realizadas en el Capítulo 4 son correctas o si, por el contrario, otro tipo de dependencia del inflatón es encontrado.

De lo que hemos observado en el Capítulo 5, exiten diversas direcciones que merecen especial atención con objetivo de establecer con mayor generalidad el fenómeno de *Flux-Flattening*. Primero, incluyendo estabilización del sector Kähler induce la presencia de flujos imaginario anti-auto-duales, los cuales modifican la acción de DBI de D7-branas. Dado que nuestro análisis en supergravedad los efectos de *backreaction* del sector de Kähler son despreciables, esperamos que lo mismo ocurra en presencia de estos nuevos flujos. Sin embargo, sería interesante generalizar el cálculo realizado en la Sección 5.2 para incluir estos flujos y así verificar está suposición. Además, con motivo de obtener un análisis más preciso de los efectos de *backreaction* a lo largo de la trayectoria inflacionaria, sería interesante describir el potencial y los términos cinéticos de D7-branas provenientes de DBI+CS en términos de supergravedad en cuatro dimensiones. Debido a la complicada dependencia dada por la raíz cuadrada que aparece en la acción de DBI esto parece en general complicado, pero puede ser conseguido para las expresiones simplificadas obtenidas para la elección de parámetros hecho en la Sección 5.2.4.

A pesar del fracaso de los modelos presentados en el Capítulo 7 creemos que el análisis realizado proporciona varios puntos que merecen ser investigados en el futuro sobre la realización de campos estabilizadores en el tipo de cuerdas II. Primero, sería posible la ruptura de la simetría de cruce en el tipo de cuerdas IIA, donde las jerarquías de masas se encuentran bajo control? Sin sacrificar el límite de volumen grande, posibles fuentes de ruptura de esta simetría sería correcciones provenietnes de α' y g_s. En segundo lugar, existe algún mecanismo que pueda restablecer la necesaria jerarquía de masas en el tipo de cuerdas IIB, donde las direcciones taquiónicas pueden ser evadidas? Puede que la respuesta se encuentre en regiones altamente anisotrópicas del espacio de *moduli* del sector de estructura compleja.

Finalmente, del Capítulo 8, la primera pregunta natural que uno puede tratar de responder es si existe alguna manera de crear una jerarquía de masas suficientemente grande entre el sector inflacionario y el sector de cuerda cerrada que garantice la validez del desplazamiento transplanckiano del inflatón. Una posibilidad interesante es la discutida en la Sección 8.3, en la cual la simetría $SL(2,\mathbb{R})$ puede ser empleada para obtener un término μ pequeño. Pero si esa opción es satisfactoria se debería poder estimar la cantida de afinación necesaria en caso de que exista.

A

Type IIA four-diemensional supergravity analysis

As stressed in the main text, at small field values the inflationary potential can be described as a 4d F-term supergravity scalar potential containing all the scalars of the compactification. This allows to understand the interplay of the inflationary sector with all the other massive scalars of the compactification, and to see to what extent both sectors are decoupled.

The purpose of this appendix is to analyze the 4d supergravity potential of the type IIA compactifications discussed in the main text and to obtain an effective potential for the inflaton sector from it, applying the philosophy of section 4.3 to both of the scenarios described there. We will then use this result to analyze the stability of the inflationary trajectory against giving a vev to those scalars of the inflationary sector which are not the inflaton. As we will see near the trajectory one can show that these other scalars are more massive than the inflaton, at least in the small field regime where the supergravity approximation is valid. While in general inflation takes place outside this regime, we take the supergravity result as a good indicator on whether the inflationary trajectory is stable after Planck suppressed corrections have been taken into account. This intuition is partially tested in section 4.4.2, where it is indeed found that the supergravity stability bounds are very mildly corrected in the DBI potential.

A.1 Type IIA scalar potential and moduli fixing

Let us consider the 4d supergravity scalar potential

$$V = e^K \left(K^{\alpha\bar{\beta}} D_\alpha W D_{\bar{\beta}} \bar{W} - 3|W|^2 \right) \quad , \quad \alpha, \beta = N^K, T^a, \Phi \, , \tag{A.1.1}$$

where W is given by (4.3.1) and the Kähler potential is $K = K_K + K_Q$, with the first piece given by (3.3.32)c and the second by (4.1.3).[1] As discussed in the main

[1] In this appendix we will consider, for completitude, the scenario where the shift symmetry of the D6-brane Wilson line is broken (3.5.13) as discussed in Section 3.5.1.

text, we are interested in a superpotential of the form

$$W \;=\; W_{\rm mod} + W_{\rm inf}\,, \tag{A.1.2}$$

where $W_{\rm inf}$ is given by (4.2.6) and depends on a particular linear combination T of Kähler moduli, while $W_{\rm mod}$ is given by (4.1.4). For simplicity we will consider the case where the latter contains no linear terms in Φ or T, and so it can be written as

$$W_{\rm mod} \;=\; W_1 + W_2 T^2 + W_3 \Phi^2 + \ldots\,, \tag{A.1.3}$$

where W_i, $i = 1, 2, 3$ are such that $\partial_T W_i = \partial_\Phi W_i \equiv 0$, and the dots contain terms with higher powers on Φ and T. Finally, let us apply the assumption of section 4.3 and assume that K_K only depends on T via $(\text{Im}\,T)^2$. Then it is easy to see that the F-terms $D_T W$ and $D_\Phi W$ vanish at the point $\Phi = T = 0$.

In the following we would like to evaluate the scalar potential (A.1.1) dependence on (Φ, T) around the point $\Phi = T = 0$ and at point in closed string moduli space selected by $W_{\rm mod}$ and the Kähler potential $K = K_K + K_Q$. For simplicity we will choose an scenario where all their F-term vanish, namely we take N^K, T^a at a value such that they solve

$$[D_{N^K} W_{\rm mod}]_{\Phi=0} \;=\; [D_{T^a} W_{\rm mod}]_{\Phi=0} \;=\; 0\,, \tag{A.1.4}$$

assuming that all closed string moduli are fixed by these conditions, except perhaps the axionic component of T. Following the discussion in the main text, these set of equations can be interpreted as the conditions for a 4d supersymmetric vacuum in the absence of the D6-brane generating $W_{\rm inf}$. As in the main text we label by $W^0_{\rm mod}$ the value of $W_{\rm mod}$ at the point selected by (A.1.4), noticing that in order to connect with the framework in [132] we need to consider $|W^0_{\rm mod}|$ very small.

To proceed and analyze the scalar potential dependence on Φ, T around this point let us first split (A.1.1) as $V = V_Q + V_K - 3e^K|W|^2$, where

$$V_Q \;=\; e^K \left(K^{\alpha\bar\beta} D_\alpha W D_{\bar\beta} \bar W \right) \quad , \alpha, \beta = N^K, \Phi \tag{A.1.5}$$

$$V_K \;=\; e^K \left(K^{T^a \bar T^b} D_{T^a} W D_{\bar T^b} \bar W \right)\,. \tag{A.1.6}$$

To evaluate (A.1.5) we consider the F-terms F_{N_K} around $\Phi = T = 0$ and up to first order in such fields. Namely we have

$$D_{N^K} W \;=\; K_{N^K} W_{\rm inf} + D_{N^K} W_{\rm mod}\,, \tag{A.1.7}$$

where

$$D_{N^K} W_{\rm mod} \;=\; \partial_{N^K} W_1 + K^{\Phi=0}_{N^K} W_1 + \ldots\,, \tag{A.1.8}$$

where we have expanded up to linear order in Φ, $\bar\Phi$ and T. Due to (A.1.4) the rhs of (A.1.8) vanishes at this order of the expansion, and we can simply take $D_{N^K} W = K_{N^K} W_{\rm inf}$. Similarly, for the F-term F_Φ we find

$$D_\Phi W \;=\; \partial_\Phi (W_{\rm inf} + W^0_3 \Phi^2) + K_\Phi (W_{\rm inf} + W^0_{\rm mod}) + \ldots\,, \tag{A.1.9}$$

where W_3^0 is the value of W_3 at the point where closed string moduli are stabilized.

Plugging this into (A.1.5) and using the identities

$$K^{\Phi\bar{\Phi}}K_{\bar{\Phi}} + \sum_L K^{\Phi\bar{N}^L}K_{\bar{N}^L} \;=\; 0\,, \tag{A.1.10}$$

$$\sum_{\alpha,\beta=N^K,\Phi} K_\alpha K^{\alpha\bar{\beta}}K_{\bar{\beta}} \;=\; 4\,, \tag{A.1.11}$$

we are able to express V_Q as a sum of two squares

$$V_Q \;=\; e^K\left(K^{\Phi\bar{\Phi}}\left|\partial_\Phi W_{\rm inf} + 2\Phi W_3^0 + K_\Phi W^0_{\rm mod}\right|^2 + 4\,|W_{\rm inf}|^2\right)\,. \tag{A.1.12}$$

Identities (A.1.10) and (A.1.11) can be checked by direct computation, and they apply to both versions (4.1.3) and (3.5.13) of the Kähler potential. They can be understood from the fact that adding Φ to the Kähler potential (3.3.28) can be seen as a change of coordinates in the complex structure moduli space. Indeed, on the one hand and as pointed out in [122], the usual type IIA no-scale condition should also hold in this new coordinate system, and in our setup such condition translates into the identity (A.1.11).

Eq.(A.1.10), on the other hand, can be seen as follows. One may rewrite the Kähler potential (4.1.3) as $K_Q(Z) = -2\log\left(\frac{i}{4}\mathcal{F}_{KL}\text{Im }Z^K\text{Im }Z^L\right)$, with $Z^K(N^K,\Phi,\bar{\Phi})$. Then it is easy to see that the differential operator

$$X_{\bar{\Phi}} \;=\; \partial_{\bar{\Phi}} + 2i(\partial_{\bar{\Phi}}\text{Im }Z^K)\partial_{\bar{N}^K}\,, \tag{A.1.13}$$

is such that $X_{\bar{\Phi}}K_Q = 0$. Finally, by the results in subsection A.1.2 one can check explicitly that

$$\partial_{\bar{\Phi}}Z^K \;=\; \frac{K^{\Phi\bar{N}^K}}{K^{\Phi\bar{\Phi}}}\,, \tag{A.1.14}$$

and so (A.1.10) follows from applying $X_{\bar{\Phi}}$ on K_Q.

One may now evaluate (A.1.6) by using the following F-terms

$$D_{T^\alpha}W \;=\; K_{T^\alpha}W_{\rm inf} + D_{T^\alpha}W_{\rm mod} \;=\; K_{T^\alpha}W_{\rm inf} + \ldots \tag{A.1.15}$$

$$D_T W \;=\; \partial_T(W_{\rm inf} + W_2^0 T^2) + K_T[W^0_{\rm mod} + W_{\rm inf}] + \ldots\,, \tag{A.1.16}$$

where T^α are the Kähler moduli that W_i depend on, and where W_2^0 is the value of W_2 at the point where closed string moduli are stabilized. Again we have expanded up to linear order in T, $\bar{T}$ and Φ and imposed (A.1.4). Plugging these expressions into (A.1.6) and using the identities (A.1.59) we find that

$$\begin{aligned} V_K \;&=\; e^K\left(K^{T\bar{T}}|\partial_T W_{\rm inf} + 2TW_2^0 + K_T W^0_{\rm mod}|^2 + (2i\text{Im }T)^2|\partial_T W_{\rm inf}|^2 + 3\,|W_{\rm inf}|^2\right)\,, \\ &+\; e^K\left(\sum_a K_{T^a}K^{T^a\bar{T}}W_{\rm inf}(K_{\bar{T}}\overline{W}^0_{\rm mod} + 2\bar{T}\overline{W}^0_2) + \text{c.c.}\right)\,. \end{aligned} \tag{A.1.17}$$

Finally, adding (A.1.12) and (A.1.17) into $V = V_Q + V_K - 3e^K|W|^2$ we obtain

$$\begin{aligned} V &= e^K \left(K^{\Phi\bar{\Phi}} \left|\partial_\Phi W_{\rm inf} + 2\Phi W_3^0 + K_\Phi W^0_{\rm mod}\right|^2 + K^{T\bar{T}} |\partial_T W_{\rm inf} + 2TW_2^0 + K_T W^0_{\rm mod}|^2 \right) \\ &+ e^K \left(4({\rm Re}\, T)^2 |\partial_T W_{\rm inf}|^2 + (4i{\rm Im}\, T K_{\bar{T}} - 6)\, {\rm Re}\,(W_{\rm inf}\overline{W}^0_{\rm mod}) + {\rm Re}\,((8i{\rm Im}\, T)\bar{T} W_{\rm inf}\overline{W}^0_2) \right) \\ &- 3e^K |W^0_{\rm mod}|^2 . \end{aligned} \tag{A.1.18}$$

Notice that the first line of (A.1.18) contains the terms quadratic in Φ and T and hence determines the mass matrix for these fields. The third line contains a constant term which is nothing but the vacuum energy inherited from the closed string moduli stabilization process. Finally, the second line contains various terms with quartic dependence dependence on Φ and T. While at the level of approximation which we have taken one may in principle neglect these terms, they contain a non-trivial dependence on the inflaton candidates Re T and Re Φ, so they may become relevant in each of the two scenarios discussed in section 4.3. In the following we analyze both scenarios and adapt the computation that led to the expression (A.1.18) for each of them.

Stabilizer field without shift symmetry breaking in the Kahler potential

Let us fist consider the scenarios shown in sections 4.3.1 and 4.3.2, in which the inflaton candidate is either the B-field,ReT , or the D6-brane Wilson line Re Φ. As mentioned before K_Q is given by (4.1.3) and that $W_{\rm mod}$ does not depend on the inflaton superfield. In this case we obtain that the scalar potential is

$$\begin{aligned} V &= e^K \left(K^{\Phi\bar{\Phi}} \left|\partial_\Phi W_{\rm inf} + K_\Phi W^0_{\rm mod}\right|^2 + K^{T\bar{T}} |\partial_T W_{\rm inf} + 2TW_2^0 + K_T W^0_{\rm mod}|^2 + 4|a|^2 ({\rm Re}\, T)^2 ({\rm Re}\, \Phi)^2 \right) \\ &- e^K \left(6\, {\rm Re}\,(W_{\rm inf}\overline{W}^0_{\rm mod}) + 3|W^0_{\rm mod}|^2 \right) , \end{aligned} \tag{A.1.19}$$

where we have neglected terms of cubic order on the stabilizer field. One can check that otherwise the above expression is exact in Re T or Re Φ, and therefore it can be used along the inflationary trajectory up to the point where the supergravity approximation is not trustable. Finally, taking the limit of very small $|W^0_{\rm mod}|$ we obtain

$$V = e^K \left(K^{\Phi\bar{\Phi}} |\partial_\Phi W_{\rm inf}|^2 + K^{T\bar{T}} |\partial_T W_{\rm inf} + 2TW_2^0|^2 + 4|a|^2 ({\rm Re}\, T)^2 ({\rm Re}\, \Phi)^2 \right) + \mathcal{O}(W^0_{\rm mod}) . \tag{A.1.20}$$

Alternative B-field scenario

We now consider, for completitude, the scenario of section 4.3.1 considering the possibility of shift symmetry breaking terms in the Kahler potential for the Wilson line. In that case one could use the Kähler potential (3.5.13). This Kähler potential only allow the case where the B-field is the inflaton candidate. There, on top of the assumptions already taken it was assumed that $W_{\rm mod}$ does not depend on the

Kähler modulus T, so that the B-field direction Re T is a flat direction of the scalar potential if we switch off $W_{\rm inf}$.[2] Imposing such extra condition on (A.1.3) implies, in particular, that $W_2 \equiv 0$, and applying it to the computation above gives

$$\begin{aligned} V &= e^K\left(K^{\Phi\bar\Phi}\left|\partial_\Phi W_{\rm inf} + 2\Phi W_3^0 + K_\Phi W^0_{\rm mod}\right|^2 + K^{T\bar T}|\partial_T W_{\rm inf} + K_T W^0_{\rm mod}|^2 + 4({\rm Re}\ T)^2|\partial_T W_{\rm inf}|^2\right) \\ &+ e^K\left((4i{\rm Im}\ T K_{\bar T} - 6)\,{\rm Re}\,(W_{\rm inf}\overline{W}^0_{\rm mod}) - 3|W^0_{\rm mod}|^2\right) . \end{aligned} \tag{A.1.21}$$

One can check that this expression for the potential is exact in the inflaton candidate Re T, while it is quadratic in the fields Φ, $\bar\Phi$ and Im T. If we now take $|W^0_{\rm mod}|$ very small in order to connect with the setup of [132] the second line can be neglected, and one finds

$$V = e^K\left(K^{\Phi\bar\Phi}\left|\partial_\Phi W_{\rm inf} + 2\Phi W_3^0\right|^2 + (K^{T\bar T} + 4({\rm Re}\ T)^2)|\partial_T W_{\rm inf}|^2\right) + \mathcal{O}(W^0_{\rm mod}) . \tag{A.1.22}$$

Finally, if we impose the condition $\partial_\Phi W_{\rm mod} = 0$ then $W_3 \equiv 0$ and we recover the result in [130].

A.1.1 Effective potentials and stability bounds

Given the above scalar potentials, one must consider the stability of the inflationary trajectory for each of them. That is, since out the two complex fields Φ and T we have selected one real scalar as the inflaton candidate, we must insure that all the other three real directions remain non-tachyonic during inflation. Finally, in order to describe our system as a model of single field inflation these three scalars must have a mass higher than the Hubble scale, since otherwise they cannot be decoupled from the inflationary dynamics.

This sort of analysis was carried in [132] for a rather general class of supergravity chaotic inflation models with a stabilizer field. The main results were then encoded in two stability bounds expressed in terms of a normalised Kähler potential. For the models analyzed in [132], if such inequality bounds are satisfied then the three scalar fields beyond the inflaton are massive enough to be decoupled during inflation. The case of interest in this model is different from the models in [132], in the sense that the effective scalar potential is derived after a process of moduli stabilization that has been analyzed in the previous section. As a result extra terms appear in the potential as compared to the potentials in [132], and so the whole analysis must be reconsidered. In the following we will perform such analysis for the scalar potential derived above, both for the case where the inflaton is a B-field or a Wilson line axion. In both scenarios we will find that the extra terms obtained from the process of moduli fixing relax the stability bounds found in [132], making them easier to satisfy.

[2]Alternatively, one may consider the case where W_2^0 is very small, so that the mass contribution to Re T from $W_{\rm mod}$ is extremely small. This case, however, is quite analogous to the one analyzed in [71] and we would expect that it suffers from the problems of fine-tuning and backreaction there discussed.

Stabilizer field without shift symmetry breaking

As we have already mentioned one could consider either the case where the inflaton candidate is the B-field or the D6-brane Wilson line. Since the computations for both cases are essentially the same we will focus on the Wilson line scenario. In this case the inflationary trajectory is given by

$$\text{Traj} = \{\text{Re}\ \Phi \neq 0\ ,\ \text{Im}\ \Phi = 0\ ,\ T = 0\}\ , \tag{A.1.23}$$

and the scalar potential is (A.1.20). In this case W_2 can arise from a flux superpotential and it may be as large as any other term, but in order to simplify the discussion we will assume that $W_2^0 = 0$, leaving the more general case for future work. The effective potential then reads

$$V \ =\ |a|^2 e^K \left(K^{\Phi\bar{\Phi}}\, |T|^2 + K^{T\bar{T}} |\Phi|^2 + 4(\text{Re}\ T)^2 (\text{Re}\ \Phi)^2 \right)\ , \tag{A.1.24}$$

and one can easily check that

$$\partial_{\text{Im}\,\Phi} V|_{\text{Traj}} \ =\ \partial_T V|_{\text{Traj}} \ =\ \partial_{\bar{T}} V|_{\text{Traj}} \ =\ 0\,. \tag{A.1.25}$$

The first stability bound is now expressed in terms of

$$m^2_{\text{saxion}}\ |_{\text{Traj}} = \frac{1}{2K_{\Phi\bar{\Phi}}} \partial^2_{\text{Im}\Phi} V\ |_{\text{Traj}} \simeq\ 3H^2\,(\epsilon + 2)\ \simeq\ 6H^2\,, \tag{A.1.26}$$

where we have used that K splits as split as $K = K_K(T^a) + K_Q(N^K, \Phi)$, and now

$$3H^2 \ \simeq\ |a|^2 e^K K^{T\bar{T}} (\text{Re}\ \Phi)^2 \qquad ,\quad \epsilon \ =\ \frac{1}{K_{\Phi\bar{\Phi}} (\text{Re}\ \Phi)^2}\,. \tag{A.1.27}$$

The stability bound for the stabilizer field is turns to be different for the real and imaginary parts. Now defining $s_1 + is_2 = \sqrt{2K_{T\bar{T}}}T$ we have that

$$m^2_{s_1}\ |_{\text{Traj}} = \frac{1}{2K_{T\bar{T}}} \partial^2_{\text{Re}T} V\ |_{\text{Traj}} \qquad m^2_{s_2}\ |_{\text{Traj}} = \frac{1}{2K_{T\bar{T}}} \partial^2_{\text{Im}T} V\ |_{\text{Traj}}\,, \tag{A.1.28}$$

and so

$$m^2_{s_1}\ |_{\text{Traj}} = \ |a|^2 e^K (K_{T\bar{T}})^{-1} \left(K^{\Phi\bar{\Phi}} + 4(\text{Re}\ \Phi)^2 \right)_{\text{Traj}} \ \simeq\ 3H^2\,(\epsilon + 4)\ \simeq\ 12H^2\,, \tag{A.1.29}$$

where we have used that $K^{T\bar{T}}$ only depends on $\text{Im}\,T$. Similarly

$$\begin{aligned} m^2_{s_2}\ |_{\text{Traj}} \ &=\ |a|^2 e^K K^{T\bar{T}} \left(K^{\Phi\bar{\Phi}} + \left(2 + \frac{1}{2} \partial^2_{\text{Im}T} K^{T\bar{T}} \right) (\text{Re}\ \Phi)^2 \right)_{\text{Traj}} \\ &\simeq\ 3H^2 \left(\epsilon + 2 \left[1 + \partial_T \partial_{\bar{T}} K^{T\bar{T}} \right]_{\text{Traj}} \right) \ \simeq\ 6H^2 \left(1 + \partial_T \partial_{\bar{T}} K^{T\bar{T}} \right)_{\text{Traj}}, \end{aligned} \tag{A.1.30}$$

and so in the second case the mass will depend on the stabilization details for the Kähler moduli.

Alternative B-field scenario

In this scenario the inflationary trajectory is given by

$$\mathrm{Traj} = \{\mathrm{Re}\, T \neq 0 \ , \ \mathrm{Im}\, T = 0 \ , \ \Phi = 0\} \ , \tag{A.1.31}$$

and the scalar potential is (A.1.22). Because W_3 in (A.1.3) arises from either world-sheet or D-brane instanton effects it will be naturally suppressed with respect to other terms in the superpotential, and so we may approximate $W_3^0 \simeq 0$. The effective potential then reduces to

$$V \ = \ |a|^2 e^K \left(K^{\Phi\bar{\Phi}} \, |T|^2 + (K^{T\bar{T}} + 4(\mathrm{Re}\ T)^2)|\Phi|^2 \right) \ , \tag{A.1.32}$$

and one can check that the trajectory is an extremum in the non-inflationary directions, namely

$$\partial_{\mathrm{Im}\, T} V|_{\mathrm{Traj}} \ = \ \partial_{\Phi} V|_{\mathrm{Traj}} \ = \ \partial_{\bar{\Phi}} V|_{\mathrm{Traj}} \ = \ 0 \, . \tag{A.1.33}$$

A more constraining requirement arises from demanding that the masses of these three fields are beyond the Hubble scale. For the canonically normalised saxion partner of the inflaton we have that

$$m^2_{\mathrm{saxion}} \ |_{\mathrm{Traj}} = \ \frac{1}{2K_{T\bar{T}}} \partial^2_{\mathrm{Im} T} V \ |_{\mathrm{Traj}} \, , \tag{A.1.34}$$

and so

$$\begin{aligned} m^2_{\mathrm{saxion}} \ |_{\mathrm{Traj}} \ &= \ |a|^2 e^K K^{\Phi\bar{\Phi}} \left(K^{-1}_{T\bar{T}} + \left[2 + \frac{\partial^2_{\mathrm{Im} T} K^{\Phi\bar{\Phi}}}{2K_{T\bar{T}} K^{\Phi\bar{\Phi}}} \right] (\mathrm{Re}\ T)^2 \right)_{\mathrm{Traj}} \\ &\simeq \ 3H^2 \left(\epsilon + 2 + \frac{\partial^2_{\mathrm{Im} T} K^{\Phi\bar{\Phi}}}{2K_{T\bar{T}} K^{\Phi\bar{\Phi}}} \right)_{\mathrm{Traj}} \ , \end{aligned} \tag{A.1.35}$$

where we have used our assumption that K only depends on T via $(\mathrm{Im}\ T)^2$ which implies that

$$K_{T\bar{T}} = -K_{TT} = -K_{\bar{T}\bar{T}} \, , \tag{A.1.36}$$

and identified the cosmological parameters as

$$3H^2 \ \simeq \ |a|^2 e^K K^{\Phi\bar{\Phi}} (\mathrm{Re}\ T)^2 \qquad , \quad \epsilon \ = \ \frac{1}{K_{T\bar{T}} (\mathrm{Re}\ T)^2} \, , \tag{A.1.37}$$

evaluated at the trajectory. Because the Kähler potential split as $K = K_K(T^a) + K_Q(N^K, \Phi)$, $K^{\Phi\bar{\Phi}}$ does not depend on $\mathrm{Im}\, T$ and so the last contribution to (A.1.35) vanishes. Moreover, because during inflation $\epsilon \ll 1$ the first contribution can be neglected and so we arrive at

$$m^2_{\mathrm{saxion}} \ |_{\mathrm{Traj}} \simeq \ 6H^2 \, , \tag{A.1.38}$$

which satisfies the criteria drawn in [132].

Regarding the open string field that here plays the role of stabilizer we have that the normalised fields are s_1 and s_2 where $s_1 + is_2 = \sqrt{2K_{\Phi\bar{\Phi}}}\Phi$ and so

$$m^2_{s_1}\,|_{\rm Traj} = \frac{1}{2K_{\Phi\bar{\Phi}}}\partial^2_{{\rm Re}\Phi}V\,|_{\rm Traj} \quad , \quad m^2_{s_2}\,|_{\rm Traj} = \frac{1}{2K_{\Phi\bar{\Phi}}}\partial^2_{{\rm Im}\Phi}V\,|_{\rm Traj}\ . \tag{A.1.39}$$

The precise expressions for these two masses depends on the expression for the Kähler potential piece K_Q, and in particular on whether we should consider (3.5.13) or (4.1.3). For simplicity we here consider the first choice (3.5.13), for which we have that both masses are equal to

$$\begin{aligned} m^2_{\rm stab}\,|_{\rm Traj} &= |a|^2 e^K K^{\Phi\bar{\Phi}}\left(K^{T\bar{T}} + \left[4 + 1 - \frac{1}{2}(K^{\Phi\bar{\Phi}})^2 K_{\Phi\bar{\Phi}\Phi\bar{\Phi}}\right]({\rm Re}\ T)^2\right)_{\rm Traj} \\ &\simeq 3H^2\left(K^{T\bar{T}}K_{T\bar{T}}\,\epsilon + 5 - \frac{1}{2}(K^{\Phi\bar{\Phi}})^2 K_{\Phi\bar{\Phi}\Phi\bar{\Phi}}\right)_{\rm Traj}, \end{aligned} \tag{A.1.40}$$

where we have used that at $\Phi = \bar{\Phi} = 0$

$$K^{\Phi\bar{\Phi}} = (K_{\Phi\bar{\Phi}})^{-1} \qquad \text{and} \qquad \partial_\Phi\partial_{\bar{\Phi}}K^{\Phi\bar{\Phi}} = -\frac{1}{2}(K^{\Phi\bar{\Phi}})^2 K_{\Phi\bar{\Phi}\Phi\bar{\Phi}}\,, \tag{A.1.41}$$

as follows from the results of appendix A.1.2. On can also check that, because K_K only depends on T via $({\rm Im}\ T)^2$, $K^{T\bar{T}}K_{T\bar{T}} = 1$ at ${\rm Im}\ T = 0$ and so the first term in (A.1.40) can be neglected. We are then left with

$$m^2_{\rm stab}\,|_{\rm Traj} \simeq 3H^2\left(5 - \frac{1}{2}(K^{\Phi\bar{\Phi}})^2 K_{\Phi\bar{\Phi}\Phi\bar{\Phi}}\right)_{\rm Traj}. \tag{A.1.42}$$

Compared to the result in [132] there is an extra contribution of $15H^2$ that pushes the stabilizer mass above the Hubble scale. The second contribution is similar to the one found in [132], and it may be positive or negative depending on the parameters of the compactification.

Indeed, in order to evaluate this second term let us first rewrite (3.5.13) as

$$K_Q = -2\log\left(\mathcal{F}^0\right) - 2\log\left(1 + \frac{i}{2}\left(\Phi\bar{\Phi}\right)\frac{\partial_{N^K}\mathcal{F}^0 Q^K}{\mathcal{F}^0} - \frac{1}{16}\left(\Phi\bar{\Phi}\right)^2\frac{\partial_{N^K}\partial_{N^L}\mathcal{F}^0 Q^K Q^L}{\mathcal{F}^0}\right), \tag{A.1.43}$$

where we have defined

$$\mathcal{F}^0 = \frac{1}{16i}\mathcal{F}_{KL}\left[N^K - \bar{N}^K\right]\left[N^L - \bar{N}^L\right]. \tag{A.1.44}$$

We may now expand the second logarithm around $x = \Phi\bar{\Phi}$ as

$$-2\log\left(1 + Ax + Bx^2\right) \simeq -2Ax + \left(A^2 - 2B\right)x^2 + \mathcal{O}\left(x^3\right), \tag{A.1.45}$$

obtaining that the coefficient for $\Phi\bar{\Phi}$ is given by

$$K_{\Phi\bar{\Phi}}|_{\Phi=0} = -i\frac{\partial_{N^K}\mathcal{F}^0 Q^K}{\mathcal{F}^0} = -\frac{1}{2}\frac{\mathcal{F}_{KL}{\rm Im}N^L Q^K}{\mathcal{F}_{KL}{\rm Im}N^K{\rm Im}N^L} = (K^{\Phi\bar{\Phi}}|_{\Phi=0})^{-1}, \tag{A.1.46}$$

in agreement with eq.(4.2.21). From the coefficient of $(\Phi\bar{\Phi})^2$ one obtains that

$$-\frac{1}{2}\,(K^{\Phi\bar{\Phi}})^2 K_{\Phi\bar{\Phi}\Phi\bar{\Phi}}\Big|_{\rm Traj} = \frac{1}{4}\left(\frac{\mathcal{F}_{KL}Q^KQ^L\mathcal{F}_{RS}{\rm Im}N^R{\rm Im}N^S}{2\,(\mathcal{F}_{KL}{\rm Im}N^LQ^K)^2} - 1\right)\,, \tag{A.1.47}$$

the first term depending on where the complex structure fields are stabilized. Generically, one would expect that this term is an order one positive number, obtaining that the stabilizer field mass at the trajectory is above the Hubble scale. It would be however interesting to evaluate the quantity (A.1.47) for explicit models with concrete mechanisms and values for complex structure moduli stabilization.

A.1.2 Kähler metrics

The 4d Kähler metric in our setup is given by

$$\mathbf{K} \;=\; \begin{pmatrix} \mathbf{K_K} & \\ & \mathbf{K_Q} \end{pmatrix}\,, \tag{A.1.48}$$

where with a slight abuse of notation we have defined the matrices

$$(\mathbf{K_K})_{a\bar{b}} \;\equiv\; \partial_{T^a}\partial_{\bar{T}^b}K_K = K_{a\bar{b}}\,, \tag{A.1.49}$$
$$(\mathbf{K_Q})_{\alpha\bar{\beta}} \;\equiv\; \partial_\alpha\partial_{\bar{\beta}}K_Q \;=\; K_{\alpha\bar{\beta}} \qquad \alpha,\beta = N^K,\Phi\,, \tag{A.1.50}$$

where in the rhs of (A.1.49) K_K is given by (3.3.32) and in the rhs of (A.1.50) K_Q is given by either (4.1.3) or (3.5.13), and $K = K_K + K_Q$.

In order to find the inverse of the matrix $\mathbf{K_Q}$ notice that it is of the form

$$\mathbf{K_Q} \;=\; \begin{pmatrix} A & -AB \\ -B^\dagger A & B^\dagger AB + C \end{pmatrix} \;=\; \begin{pmatrix} \mathbb{I} & 0 \\ -B^\dagger & 1 \end{pmatrix}\begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}\begin{pmatrix} \mathbb{I} & -B \\ 0 & 1 \end{pmatrix}\,, \tag{A.1.51}$$

where

$$A_{KL} \;=\; K_{N^K\bar{N}^L} \qquad B^L \;=\; \partial_{\bar{\Phi}}Z^L \qquad C \;=\; \frac{i}{4}K_{N^K}Q^K\,, \tag{A.1.52}$$

and where as above we have defined Z^L by writing $K_Q(Z) = -2\log\left(\frac{i}{4}\mathcal{F}_{KL}{\rm Im}\;Z^K{\rm Im}\;Z^L\right)$. The inverse of (A.1.51) is given by

$$\mathbf{K_Q}^{-1} \;=\; \begin{pmatrix} A^{KL} + C^{-1}B^KB^{\dagger\,L} & C^{-1}B^L \\ C^{-1}B^{\dagger\,K} & C^{-1} \end{pmatrix}\,, \tag{A.1.53}$$

with A^{KL} the inverse of A_{KL}. From here we obtain that

$$\frac{K^{\Phi\bar{N}^K}}{K^{\Phi\bar{\Phi}}} \;=\; \partial_{\bar{\Phi}}Z^K \quad ,K^{\Phi\bar{\Phi}} \;=\; \left[\frac{i}{4}K_{N^K}Q^K\right]^{-1}\,. \tag{A.1.54}$$

To analyze the inverse of $\mathbf{K_K}$ it is useful to define the following quantities

$$\mathcal{K}_{ab} = \mathcal{K}_{abc}v^c \qquad ,\mathcal{K}_a = \mathcal{K}_{abc}v^bv^c \qquad ,\mathcal{K} = \mathcal{K}_{abc}v^av^bv^c\,, \tag{A.1.55}$$

with $v^a = e^{-\phi/2}\text{Im } T^a$. We then have the following derivatives of (3.3.32)

$$K_a = \frac{3i}{2\mathcal{K}} e^{-\phi/2}\mathcal{K}_a \qquad\qquad K_{a\bar{b}} = -\frac{3}{2\mathcal{K}^2} e^{-\phi}\left(\mathcal{K}\mathcal{K}_{ab} - \frac{3}{2}\mathcal{K}_a\mathcal{K}_b\right), \tag{A.1.56}$$

and so the inverse metric is given by

$$K^{a\bar{b}} = -\frac{2}{3}e^{\phi}\mathcal{K}\mathcal{K}^{ab} + 2e^{\phi}v^a v^b\,, \tag{A.1.57}$$

where $\mathcal{K}^{ab}$ is the inverse of $\mathcal{K}_{ab}$ which implies that

$$\mathcal{K}^{ab}\mathcal{K}_b = v^a\,. \tag{A.1.58}$$

One can check that indeed $K^{a\bar{b}}K_{c\bar{b}} = \delta^a_c$ and $K^{a\bar{b}}K_{a\bar{c}} = \delta^{\bar{a}}_{\bar{c}}$. Finally we also have that

$$K_a K^{a\bar{b}} K_{\bar{b}} = 3, \qquad\qquad K_a K^{a\bar{b}} = 2ie^{-\phi/2}v^b\,. \tag{A.1.59}$$

B

A simple background for the Wilson line scenario

Here we will consider a simple type IIA compactification that can be used as a toy model for implementing the scenario is the Wilson line. More precisely, we will consider the class of type IIA flux compactifications studied in [91] and see under which conditions one can have a closed string background with the properties described in section 4.3.2.

For simplicity let us consider a type IIA compactification with two Kähler moduli, which we dub T_1 and T_2. We may then define the linear combinations

$$T_+ \;=\; \frac{1}{2}\,(T_1 + T_2) \qquad \text{and} \qquad T_- \;=\; \frac{1}{2}\,(T_1 - T_2)\ , \tag{B.0.1}$$

and identify T_- with the combination of Kähler moduli (4.2.7) that will appear in the the bilinear superpotential W_{inf} when we add the D6-brane, and which we have dubbed T in the main text. From this example it is easy to see that $T_- = 0$ does not imply that any volume of the of the compactification vanishes, but rather that two compactification volumes are related.

One of the requirements for both scenarios of section 4.3 is that the Kähler potential of the compactification only depends on T through $(\text{Im}\ T)^2$. In the case at hand and taking $K = K_K + K_Q$ we see that this is easily achievable by imposing the following relations for the triple intersection numbers

$$\mathcal{K}_{111} \;=\; \mathcal{K}_{222} \qquad \text{and} \qquad \mathcal{K}_{122} \;=\; \mathcal{K}_{211}\,. \tag{B.0.2}$$

From here we obtain

$$K_K \;=\; -\log\left(\frac{i}{6}\mathcal{K}_{+++}(T_+ - \bar{T}_+)^3 + \frac{i}{2}\mathcal{K}_{+--}(T_- - \bar{T}_-)^2(T_+ - \bar{T}_+)\right)\ , \tag{B.0.3}$$

where we have defined

$$\mathcal{K}_{+++} \;=\; 2(\mathcal{K}_{111} + 3\mathcal{K}_{112}) \qquad \text{and} \qquad \mathcal{K}_{+--} \;=\; 2(\mathcal{K}_{111} - \mathcal{K}_{112})\,. \tag{B.0.4}$$

An extra requirement of the Wilson line scenario is that there appear no linear terms in $T = T_-$ in $W_{\rm mod}$. To evaluate this condition let us consider the class of type IIA compactifications considered in [91], in which

$$W_{\rm mod} \,=\, W_{\rm flux} \,=\, W_K + W_Q\,, \tag{B.0.5}$$

where W_K is given by (4.1.1) and

$$W_Q = \int \Omega_c \wedge H_3 = -N^K p_K \,=\, -\left(\xi^K + i{\rm Re}\left(e^{-\phi}CZ^K\right)\right)p_K \,=\, -\left(\xi^K + il^K\right)p_K\,, \tag{B.0.6}$$

with the moduli N^K defined as in (3.3.20). In this case obtaining a superpotential with no linear term in T_- is achievable by imposing the following relations among RR background fluxes

$$e_1 \,=\, e_2 \,=\, e \qquad \text{and} \qquad m_1 \,=\, m_2 \,=\, m\,, \tag{B.0.7}$$

from which we obtain that

$$\begin{aligned} W_K \;&=\; e_0 + 2eT_+ + \frac{m}{2}\mathcal{K}_{+++}T_+^2 + \frac{m}{2}\mathcal{K}_{+--}T_-^2 - \frac{1}{6}m_0\left(\mathcal{K}_{+++}T_+^3 + 3\mathcal{K}_{+--}T_+T_-^2\right) \\ &=\; W_1(T_+) + \frac{1}{2}\mathcal{K}_{+--}\left(m - m_0T_+\right)T_-^2\,, \end{aligned} \tag{B.0.8}$$

as required in the main text.

Moduli stabilization

Let us now compute the point in moduli space in which the closed string moduli are stabilized with vanishing F-terms. That is, we impose the conditions

$$D_{T^a}W_{\rm mod} \,=\, D_{N^K}W_{\rm mod} \,=\, 0\,, \tag{B.0.9}$$

with the superpotential above and the Kähler potential $K = K_K + K_Q =$(B.0.3)+(3.3.28). Following the general discussion saw in 3.4.1 and given in [91] we first consider the stabilization of the complex structure moduli, whose F-term is given by

$$D_{N^K}W_{\rm mod} = -p_K + K_{N^K}W_{\rm mod} = p_K + 4e^{2D}\mathcal{F}_{KL}l^LW_{\rm mod} = 0\,. \tag{B.0.10}$$

Note that $\mathcal{F}_{KL}$ is pure imaginary by definition [122]. Looking at its imaginary part we arrive to

$${\rm Re}\,W_{\rm mod} = 0 \quad \Rightarrow \quad -p_K\xi^K + {\rm Re}\,W_K = 0\,, \tag{B.0.11}$$

which implies that only a linear combination of RR three-form axions will be stabilized by the fluxes. Notice however that when we include D6-branes some other linear combinations will be eaten by open string gauge bosons and become massive via Stückelberg mechanism, and therefore they should not appear in the superpotential [93]. Hence the lack of stabilization of some of these axions should not be seen as a flaw of the model but rather as a necessary condition to introduce D6-branes, which is important for our purposes. The real part of (B.0.10) will give us

$$p_K + 4ie^{2D}\mathcal{F}_{KL}l^L \mathrm{Im}\, W_{\mathrm{mod}} = 0\,. \tag{B.0.12}$$

Where we have used that $e^{-2D} = 2il^K\mathcal{F}_{KL}l^L$. Note that Im $W = 0$ implies zero H_3 flux. For Im$W \neq 0$ we see that for every p_{K_i} different from zero and rearranging the former expression we arrive to

$$\frac{ip_{K_i}}{\mathcal{F}_{K_iL}Z^L}e^{-K_{CS}/2} \;=\; \frac{p_{K_i}}{\mathrm{Im}\mathcal{F}_{K_i}}e^{-K_{CS}/2} \;:=\; Q_0\,, \tag{B.0.13}$$

where we have used the the definition $l^K := e^{-D}e^{\frac{1}{2}K_{CS}}Z^K$ and the relation $\mathcal{F}_K = \mathcal{F}_{KL}Z^L$, note that Q_0 is a fixed quantity. The above system of $h^{2,1}$ equations, generically, will stabilize all the complex structure saxions to a specific value. Finally using that $e^D = e^{\phi+\frac{1}{2}K_K} = \frac{e^{\frac{\phi}{4}}}{\sqrt{\frac{4}{3}\mathcal{K}}}$ we find that the dilaton is stabilized at

$$e^{-\phi} = 4\frac{e^{K_K/2}}{Q_0}\mathrm{Im}\, W_{\mathrm{mod}}\,. \tag{B.0.14}$$

Regarding the F-terms for the Kähler moduli, first of all we will derive that the superpotential evaluated in the vacuum can be written only in terms of the Kähler moduli

$$W_{\mathrm{mod}} = -i\mathrm{Im}\; W_K\,, \tag{B.0.15}$$

this can be see taking into account the following: if we multiply (B.0.10) per l^K and sum over K, and using the definition of D we arrive to

$$-\, iW_{\mathrm{mod}} = \frac{1}{2}\mathrm{Im}\; W_Q\,. \tag{B.0.16}$$

Imposing the above relations for the complex structure moduli we find that the F-term equation for the Kähler moduli is given by

$$D_{T_a}W_K - iK_{T_a}\mathrm{Im}\, W_K = 0\,, \tag{B.0.17}$$

whose imaginary part fixes the B-field axions to

$$\mathrm{Re}\, T_+ \;=\; m/m_0 \quad \text{and} \quad \mathrm{Re}\, T_- \;=\; 0\,. \tag{B.0.18}$$

Moreover, one can see that the real part of the F-term for T_- fixes Im $T_- = 0$ while that for T_+ imposes the relation

$$20e + \mathcal{K}_{+++}\left(3m_0\,\mathrm{Im}\, T_+^2 + 5\frac{m^2}{m_0}\right) = 0\,, \tag{B.0.19}$$

so the volume modulus is stabilized at

$$\mathrm{Im}\, T_+ = \frac{\sqrt{5}}{\sqrt{3}m_0}\sqrt{-\frac{4em_0}{\mathcal{K}_{+++}} - m^2}\,, \tag{B.0.20}$$

which is positive as long as $e < 0$ and $|e|m_0 > \frac{1}{4}m^2\mathcal{K}_{+++}$.

C

Other flux-flattened potentials

In this appendix we perform an analysis for the D7-brane single field potential of subsection 5.2.3 along the lines of subsection 5.2.5, but for different values of $\hat{G}$ and Υ that may arise in different setups from the one of subsection 5.3.4. We considered two regions in the $\hat{G}$ parameter space, namely $\hat{G} \sim 0.003$ and $\hat{G} \sim 3$, and vary Υ which is the parameter that controls the deviation from the model of [83]. We show how the cosmological observables vary in the two regimes for $0 \leqslant \Upsilon \leqslant 20$ in the figures C.1 and C.2 .

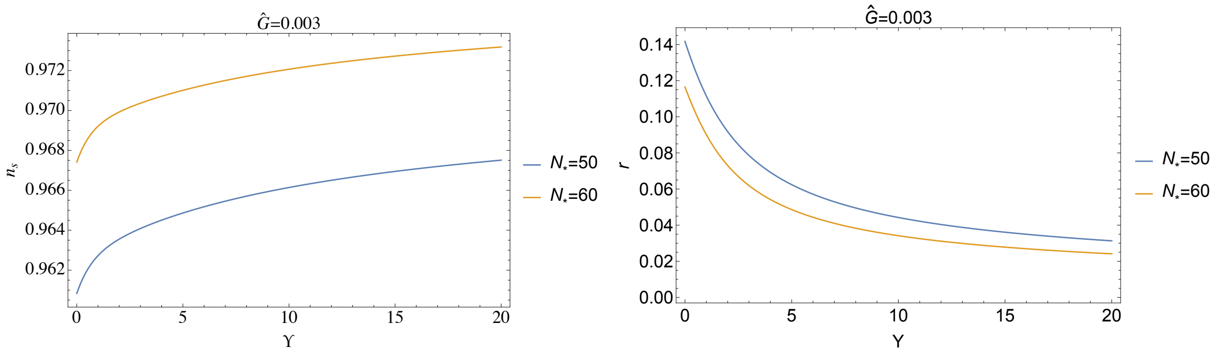


Figure C.1: Spectral index n_s and tensor-to-scalar ratio r in terms of Υ with $\hat{G} = 0.003$ for $N_* = 50$ and $N_* = 60$ e-folds.

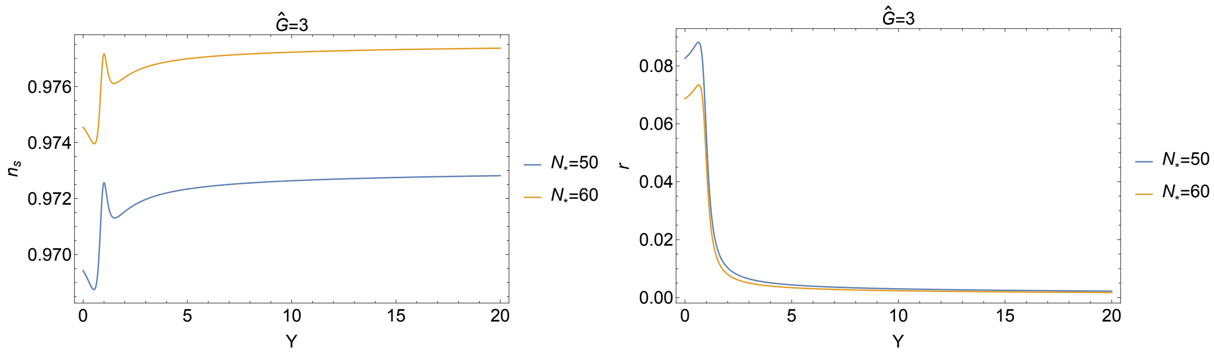


Figure C.2: n_s and r in terms of Υ with $\hat{G} = 3$ for $N_* = 50$ and $N_* = 60$ e-folds.

We see that in both cases the effect of the parameter Υ is quite dramatic: it leads to a significant lowering of the tensor-to-scalar ratio r as expected from the flattening induced by the self-dual component of the flux $\mathcal{F}$. At the same time the spectral index n_s generally moves closer to 1 as Υ increases. This behaviour occurs for both regimes of $\hat{G}$ that we chose to explore. The rôle of this second parameter is to provide (at $\Upsilon = 0$) an interpolation between models with quadratic and linear potential as already observed in [83] (a similar interpolation between quadratic and linear potentials was also observed in [31, 130]). Therefore if we allow for more general values of $\hat{G}$ and Υ than the ones used in section 5.2.4 we see that it is possible to explore additional regions of the $n_s - r$ plane, namely we may start with any potential interpolating between quadratic and linear (the exact interpolation being set roughly by $\hat{G}$) and by increasing Υ access regions with a lower value of the tensor-to-scalar ratio r. To show this more explicitly we chose to superimpose over the Planck collaboration results [4] the two regions explored in the $n_s - r$ plane, showing the result in figure C.3.

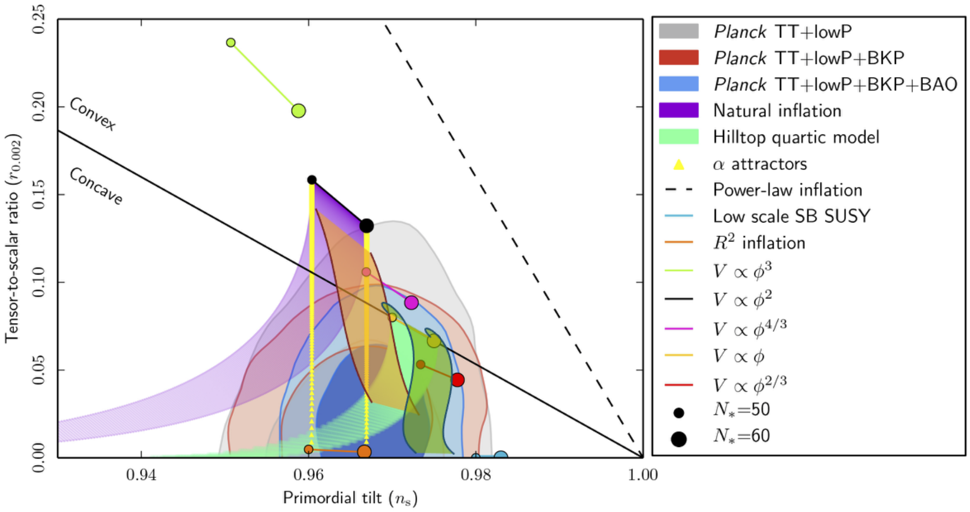


Figure C.3: Region for the spectral index n_s vs tensor-to-scalar ratio r for the two values of $\hat{G}$ (orange region corresponds to $\hat{G} = 0.003$ and green to $\hat{G} = 3$) and $0 \leqslant \Upsilon \leqslant 20$.

D

$\mathcal{N} = 1$ supergravity analysis of the D6/D7 brane model

In this appendix we are going to show the technical details about the $\mathcal{N} = 1$ supergravity analysis of models with the stabilizer field proposed by in Chapter 4. The results shown here apply for both the IIA original case and its type IIB dual version analyzed in Section 7.2.2. For simplicity we will compute it in the type IIA case, but the final result could be dualized.

The main difference between this computation and the one shown in Appendix A is the way the Wilson Line appears in the Kahler potential. In this case we will use the Kähler potential derived in (7.2.15a).[1]

$$K = K_{\mathrm{K}}\left(T_a, S\right) + K_{\mathrm{cx.str.}}\left(N_a, \frac{\left(\Phi - \bar{\Phi}\right)^2}{f\left(T_a\right) - f\left(\bar{T}_a\right)}\right) , \tag{D.0.1}$$

where we denoted S as the stabilizer field, which is a Kahler modulus. The superpotential is given by the standard expression

$$W = W_{\mathrm{bil}}\left(\Phi, S\right) + W_{\mathrm{mod}}\left(N_a, T_a\right) , \tag{D.0.2}$$

where

$$W_{\mathrm{bil}} = a\Phi S , \tag{D.0.3}$$

and $W_{\mathrm{mod}}\left(N_a, T_a\right)$ is given by the Gukov-Vafa-Witten flux superpotential plus euclidean D-brane instantons.

D.1 Scalar potential

Let us first start from the usual F-term scalar potential

$$V = e^K \left(K^{a\bar{b}}\left(D_a W\right)\left(D_{\bar{b}}\bar{W}\right) - 3\left|W\right|^2\right) . \tag{D.1.1}$$

[1]For technical details we refer the reader to [120].

Where $a, b = T_a, N_a, \Phi, S$, it runs over all the possible moduli. It can be shown that the Kahler potential (D.0.1) satisfies the following relations

$$K^{a\bar{b}}K_a K_{\bar{b}} = 7\,, \tag{D.1.2}$$

$$K^{a\bar{b}}K_{\bar{b}} = -2i\mathrm{Im}\Psi^a\,, \tag{D.1.3}$$

where, Ψ^a denotes the moduli with a index.

We assumen that the Kahler moduli and the complex structure moduli T_a and N_a are stabilized supersymmetrically at a higher scale. Moreover we assume that W_{mod} is negligible or that the heavy moduli are stabilized supersymmetrically in a Minkowski vacua $W_{mod} = 0$. With these assumptions we see that the scalar potential is only given by

$$e^{-K}V = 4\left|W_{\mathrm{bil}}\right|^2 + \left(\partial_\alpha W_{\mathrm{bil}}\right)\left(-2i\mathrm{Im}\Psi^\alpha \bar{W}_{\mathrm{bil}}\right) \tag{D.1.4}$$

$$+ \left(\partial_{\bar{\alpha}}\bar{W}_{\mathrm{bil}}\right)\left(-2i\mathrm{Im}\Psi^{\bar{\alpha}}W_{\mathrm{bil}}\right) + K^{\alpha\bar{\beta}}\left(\partial_\alpha W_{\mathrm{bil}}\right)\left(\partial_{\bar{\beta}}\bar{W}_{\mathrm{bil}}\right) + \mathcal{O}\left(W_{\mathrm{mod}}\right). \tag{D.1.5}$$

So, the scalar potential is given by

$$e^{-K}V = 4\left|\Phi\right|^2\left|S\right|^2 + \left|\Phi\right|^2\left(\bar{S}^2 - \left|S\right|^2\right) + \left|S\right|^2\left(\bar{\Phi}^2 - \left|\Phi\right|^2\right) \tag{D.1.6}$$

$$+ \left|\Phi\right|^2\left(S^2 - \left|S\right|^2\right) + \left|S\right|^2\left(\Phi^2 - \left|\Phi\right|^2\right) + K^{S\bar{S}}\left|\Phi\right|^2 + K^{\Phi\bar{\Phi}}\left|S\right|^2 + \mathcal{O}\left(W_{\mathrm{mod}}\right). \tag{D.1.7}$$

We can simplify this expression and arrive to

$$e^{-K}V = \left|\Phi\right|^2\left(K^{S\bar{S}} + \left(S^2 + \bar{S}^2\right)\right) + \left|S\right|^2\left(K^{\Phi\bar{\Phi}} + \left(\Phi^2 + \bar{\Phi}^2\right)\right) + \mathcal{O}\left(W_{\mathrm{mod}}\right)\,. \tag{D.1.8}$$

If we want to write it in terms of the real scalar fields we see that

$$e^{-K}V = K^{S\bar{S}}\left|\Phi\right|^2 + K^{\Phi\bar{\Phi}}\left|S\right|^2 + 4\mathrm{Re}\left(\Phi\right)^2\mathrm{Re}\left(S\right)^2 - 4\mathrm{Im}\left(\Phi\right)^2\mathrm{Im}\left(S\right)^2 + \mathcal{O}\left(W_{\mathrm{mod}}\right)\,. \tag{D.1.9}$$

D.2 Masses

Next, one may compute the masses of the dynamical fields S and Φ. Recall that we assume that there is no kinetic mixing between S and Φ so, this means that terms like $K_{T\bar{T}\Phi\bar{\Phi}}$, i.e. with mixed derivatives are exactly 0.

First we are going to compute the mass of the inflaton and the saxionic partner. Due to the fact that Φ is in the Kahler potential like (D.0.1) there are several symmetric relations that follow during the inflationary trajectory $\mathrm{Trj} = \{\mathrm{Im}\left(\Phi\right) = 0, S = 0\}$

$$K_\Phi|_{\mathrm{Trj}} = 0 = K_{\bar{\Phi}}|_{\mathrm{Trj}}\,, \tag{D.2.1}$$

$$K_{\Phi\Phi}|_{\mathrm{Trj}} = -K_{\Phi\bar{\Phi}}|_{\mathrm{Trj}} = K_{\bar{\Phi}\bar{\Phi}}|_{\mathrm{Trj}}\,. \tag{D.2.2}$$

Using these properties we arrive to the following masses

$$m^2_{\mathrm{Re}(\Phi)}|_{\mathrm{Trj}} = e^K a^2 \frac{K^{S\bar{S}}}{K_{\Phi\bar{\Phi}}}\,, \tag{D.2.3}$$

$$m^2_{\mathrm{Im}(\Phi)}|_{\mathrm{Trj}} = e^K a^2 \frac{K^{S\bar{S}}}{K_{\Phi\bar{\Phi}}}\left(1 + 2\mathrm{Re}\left(\Phi\right)^2 K_{\Phi\bar{\Phi}}\right)\,. \tag{D.2.4}$$

We see that the mass hierarchy between the axion and the saxion is trivially satisfied because the saxionic partner is proportional to the inflaton vev during inflation. Moreover, we see that in the vacuum both masses are the same since supersymmetry is restored. Now, we are going to compute the mass of the the stabilizer field

We assume that the stabilizer field is given by a complex structure modulus in the type IIB case and a Kahler modulus in the IIA. The stabilizer field is inside a logarithm which share the same structure for the large volume/large complex structure limit. TOne could see that, in general, there are no symmetries between the derivatives of the inverse Kahler metrics. One may focus on the Kahler potential for the stabilizer field (4.1.3). There the stabilizer field appears quadratically in the Kahler potential due to some choice of intersection numbers. This fact is translated into some symmetries on the derivatives of the Kahler potential

$$K_S|_{\text{Trj}} = 0 = K_{\bar{S}}|_{\text{Trj}}\,, \tag{D.2.5}$$

$$K_{SS}|_{\text{Trj}} = -K_{S\bar{S}}|_{\text{Trj}} = K_{\bar{S}\bar{S}}|_{\text{Trj}}\,, \tag{D.2.6}$$

$$\partial_S K_{S\bar{S}} = 0 = \partial_{\bar{S}} K_{S\bar{S}}\,, \tag{D.2.7}$$

$$\partial_S \partial_S K^{S\bar{S}} = -\partial_S \partial_{\bar{S}} K^{S\bar{S}} = \partial_{\bar{S}} \partial_{\bar{S}} K^{S\bar{S}}\,, \tag{D.2.8}$$

with these properties associated to the decision of a quadratic stabilizer en the Kahler potential the masses are given by

$$\begin{aligned} m^2_{\text{Re}(S)}|_{\text{Trj}} &= \frac{e^K a^2}{K_{S\bar{S}}} \left(K^{\Phi\bar{\Phi}} + 4\text{Re}\,(\Phi)^2 \right)\,, && \text{(D.2.9)} \\ m^2_{\text{Im}(S)}|_{\text{Trj}} &= \frac{e^K a^2}{K_{S\bar{S}}} \left(K^{\Phi\bar{\Phi}} + 2\text{Re}\,(\Phi)^2 \left(K_{S\bar{S}} K^{S\bar{S}} + \partial_S \partial_{\bar{S}} K^{S\bar{S}} \right) \right)\,. && \text{(D.2.10)} \end{aligned}$$

Looking at (D.2.10) we see that in order to assure any tachyonic direction we have to impose that, during inflation

$$K_{S\bar{S}} K^{S\bar{S}} > \partial_S \partial_{\bar{S}} K^{S\bar{S}}\,. \tag{D.2.11}$$

D.3 Backreacted scalar potential and mass terms

Here we are going to see how our model behaves under backreaction. Here we are going to use the shortcut shown in Section 6.2. The leading order of the backreaction effects of the supersymetric moduli stabilization in a Minkowski vacua of the heavy moduli can be understood as computing the effective scalar potential using an effective Kahler potential and an effective superpotential, where the heavy moduli stabilized are settled at its vev.

$$K_{\text{eff}} = K_{\text{K}} \left(T_a^0, S \right) + K_{\text{cx.str.}} \left(N_a^0, \frac{\left(\Phi - \bar{\Phi} \right)^2}{f\left(T_a^0 \right) - f\left(\bar{T}_a^0 \right)} \right)\,. \tag{D.3.1}$$

This means that now, there is no no-scale structure to use in the computation of the F-term scalar potential. As we did in the last section we assume that there is no kinetic mixing between the only dynamical fields S, Φ

$$V = e^{K}\left(K^{S\bar{S}}\left|\Phi\right|^{2} + K^{\Phi\bar{\Phi}}\left|S\right|^{2} - 3\left|\Phi S\right|^{2}\right) + \mathcal{O}\left(W_{\text{mod}}\right) . \tag{D.3.2}$$

We see that during inflation we have exactly the same scalar potential as we had in the last section up to order W_{mod}. Computing the masses of the inflation and its saxionic partner we see

$$m^2_{\text{Re}(\Phi)}|_{\text{Trj}} = e^{K}a^{2}\frac{K^{S\bar{S}}}{K_{\Phi\bar{\Phi}}} , \tag{D.3.3}$$

$$m^2_{\text{Im}(\Phi)}|_{\text{Trj}} = e^{K}a^{2}\frac{K^{S\bar{S}}}{K_{\Phi\bar{\Phi}}}\left(1 + 2\text{Re}\left(\Phi\right)^{2}K_{\Phi\bar{\Phi}}\right) . \tag{D.3.4}$$

We see that the mass of the inflaton is tis saxionic partner are the same as in the former case. The difference comes from the stabilizer. Here, we assume the same symmetries in the Kahler potential as in the last case. If we use other Kahler potential for the stabilizer field the following results will change.

$$m^2_{\text{Re}(S)}|_{\text{Trj}} = \frac{e^{K}a^{2}}{K_{S\bar{S}}}\left(K^{\Phi\bar{\Phi}} - 3\text{Re}\left(\Phi\right)^{2}\right) , \tag{D.3.5}$$

$$m^2_{\text{Im}(S)}|_{\text{Trj}} = m^2_{\text{Im}(S)}|_{\text{Trj}} = \frac{e^{K}a^{2}}{K_{S\bar{S}}}\left(K^{\Phi\bar{\Phi}} + \text{Re}\left(\Phi\right)^{2}\left(-2 + \left(K^{S\bar{S}}\right)^{2}K_{S\bar{S}S\bar{S}}\right)\right) . \tag{D.3.6}$$

Here, we see that the there is a tachyonic direction in the stabilizer field during inflation in general. As we can see on Section 7.1.2, using the large volume Kahler potential $K_{S\bar{S}S\bar{S}}$ is negative in general so there will be 2 tachyonic directions in this concrete case.

This is a clear statement about how backreaction destroys the Wilson line case because of the shift symmetry of the stabilizer field on the Kahler potential (D.0.1). Backreaction effects of the heavy moduli washes the appearance of the 'uplifting' mass terms for the stabilizer field due to the no-scale structure that we enjoyed with the kahler potential. We see straightforwardly the necessity the breaking of the shift symmetry of the stabilizer field.

D.4 Masses and backreaction in the small complex structure limit

Here we will see in detail why we need to break the shift symmetry for the stabilizer field. The point is that the absence of shift symmetry induces some symmetries in the derivatives of the Kahler potential that avoids the tachyonic direction in the axion of the stabilizer field. Now, consider the following effective Kahler potential in the small complex structure limit

$$K_{\text{sm cx}} = -\log\left(A_0 + A_1 S\bar{S} + A_2\left(S\bar{S}\right)^{2} + \cdots\right) . \tag{D.4.1}$$

Now, there symmetries of the derivatives of the Kahler potential are

$$K_S|_{\mathrm{Trj}} = 0 = K_{\bar{S}}|_{\mathrm{Trj}}\,, \tag{D.4.2}$$

$$K_{SS}|_{\mathrm{Trj}} = 0 = K_{\bar{S}\bar{S}}|_{\mathrm{Trj}}\ ,\ K_{S\bar{S}}|_{\mathrm{Trj}} \neq 0\,, \tag{D.4.3}$$

$$\partial_S K_{S\bar{S}} = 0 = \partial_{\bar{S}} K_{S\bar{S}}\,, \tag{D.4.4}$$

$$\partial_S \partial_S K_{S\bar{S}} = 0 = \partial_{\bar{S}} \partial_{\bar{S}} K_{S\bar{S}}\ ,\ \partial_S \partial_{\bar{S}} K_{S\bar{S}} \neq 0\,, \tag{D.4.5}$$

The masses for the inflation and its partner are the same as in the former case

$$m^2_{\mathrm{Re}(\Phi)}|_{\mathrm{Trj}} = e^K a^2 \frac{K^{S\bar{S}}}{K_{\Phi\bar{\Phi}}}\,, \tag{D.4.6}$$

$$m^2_{\mathrm{Im}(\Phi)}|_{\mathrm{Trj}} = e^K a^2 \frac{K^{S\bar{S}}}{K_{\Phi\bar{\Phi}}} \left(1 + 2\mathrm{Re}\,(\Phi)^2\, K_{\Phi\bar{\Phi}}\right)\,. \tag{D.4.7}$$

But there is an important change in the masses of the stabilizer field, in this case both masses are the same and are given by

$$m^2_{\mathrm{Re}(S)}|_{\mathrm{Trj}} = m^2_{\mathrm{Im}(S)}|_{\mathrm{Trj}} = \frac{e^K a^2}{K_{S\bar{S}}} \left(K^{\Phi\bar{\Phi}} + \mathrm{Re}\,(\Phi)^2 \left(-3 + K_{S\bar{S}} K^{S\bar{S}} + \partial_S \partial_{\bar{S}} K^{S\bar{S}}\right)\right)\,. \tag{D.4.8}$$

We see that this mass term could be positive definite as long as

$$\left(K^{S\bar{S}}\right)^2 K_{S\bar{S}S\bar{S}} > -2\,. \tag{D.4.9}$$

This proves us why we need both, non shift symmetric Kahler potential for the stabilizer field (changes the symmetries of the derivatives) and the famous quartic term $\left(S\bar{S}\right)^2$, because without this term $K_{S\bar{S}S\bar{S}} = 0$ and we cannot satisfy the bound.

But explicitly what happened to have this difference, the key point are the induced symmetries in the derivatives of the Kahler potential. The axionic mass of the stabilizer field in general is given by

$$m^2_{\mathrm{Re}(S)}|_{\mathrm{Trj}} e^{-K} \frac{K_{S\bar{S}}}{a^2} = K^{\Phi\bar{\Phi}} + \mathrm{Re}\,(\Phi)^2 \left(-3 + \frac{K^{S\bar{S}}}{2}\left(2K_{S\bar{S}} + K_{SS} + K_{\bar{S}\bar{S}} + K_S^2 + K_{\bar{S}}^2\right)\right) \tag{D.4.10}$$

$$+ \frac{1}{2}\left(\partial_S \partial S K^{S\bar{S}} + \partial_S \partial_S K^{S\bar{S}} + 2\partial_S \partial_{\bar{S}} K^{S\bar{S}}\right) \tag{D.4.11}$$

$$+ \partial_S K^{S\bar{S}} \left(K_S + K_{\bar{S}}\right) + \partial_{\bar{S}} K^{S\bar{S}} \left(K_S + K_{\bar{S}}\right) + K_S K_{\bar{S}} K^{S\bar{S}}\,. \tag{D.4.12}$$

We see that in the shift symmetric case (when we consider backreaction of the heavy moduli) the symmetries conspire in order to vanish all terms that are multiplying $\mathrm{Re}\,(\Phi)^2$ where the only term that survives is $-3\mathrm{Re}\,(\Phi)^2$ which comes from the $-3\,|\Phi S|^2$.

E

Details on the Picard-Fuchs basis

In this appendix we present a few details about how to obtain the effective theory in the Picard-Fuchs basis, as employed in Section 7.2.3.

E.1 The periods of Fermat hypersurfaces

Let us begin by recalling a few details about the manifolds to which this technique was first applied. A hypersurface $\mathcal{M}$ defined as the zero locus of a Polynomial

$$P = \sum_{j=1}^{4} x_j^{d/k_j} \,, \tag{E.1.1}$$

in the projective space $\mathbb{P}^4_{[k_0,k_1,k_2,k_3,k_4]}$ is a Calabi-Yau three-fold when the degree of the defining Polynomial satisfies $d = \sum_{i=0}^{4} k_i$. On the mirror manifold of $\mathcal{M}$, defined by

$$\mathcal{W} := \frac{\{P=0\}}{H} \,, \tag{E.1.2}$$

where H is the maximal group of scaling symmetries, the number of complex structure moduli is given by the number of possible monomial degree-d deformations of P. For a manifold with two such possible deformations, denoted by S and U, we can write the deformed polynomial as follows,

$$P = \sum_{j=1}^{4} x_j^{d/k_j} - dSx_0x_1x_2x_3x_4 - \frac{d}{q_0} U x_0^{q_0} x_1^{q_1} x_2^{q_2} x_3^{q_3} x_4^{q_4} \,. \tag{E.1.3}$$

In a similar fashion, the holomorphic three-form Ω is deformed by S and U.

As mentioned in Section 7.2.3, the periods of $\mathcal{W}$ can be obtained by direct integration of Ω along a suitable contour. The fundamental period close to the Landau-Ginzburg point, where $S = U = 0$, reads

$$\varpi_0\left(S,U\right) = -\frac{2}{d}\sum_{n=1}^{\infty} \frac{\Gamma\left(\frac{2n}{d}\right)\left(-dS\right)^n u_{-\frac{2n}{d}}\left(U\right)}{\Gamma\left(n\right)\Gamma\left(1-\frac{n}{d}\left(k_1-1\right)\right)\Gamma\left(1-\frac{k_2 n}{d}\right)\Gamma\left(1-\frac{k_3 n}{d}\right)\Gamma\left(1-\frac{k_4 n}{d}\right)} \,, \tag{E.1.4}$$

where, for $|U| < 1$,

$$u_\nu(U) = \frac{e^{i\pi\nu/2}\Gamma\left(1+\frac{\nu(k_1-1)}{2}\right)}{2\Gamma(-\nu)} \sum_{m=0}^{\infty} \frac{e^{i\pi m/2}\Gamma\left(\frac{m-\nu}{2}\right)(2U)^m}{m!\Gamma\left(1-\frac{m-\nu k_1}{2}\right)} \,. \tag{E.1.5}$$

The remaining entries of the period vector are then constructed via

$$\varpi_j(S,U) = \varpi_0\left(\lambda^j S, \lambda^{jq_0} U\right) \,, \qquad j = 0, \ldots, d-1 \,. \tag{E.1.6}$$

There are only $2\left(h^{2,1}+1\right) = 6$ linearly independent entries, as discussed in [195]. We can now express the periods in terms of S and U as follows, cf. (7.2.28),

$$(\varpi)^j = 2 \cdot (2\pi i)^3 \sum_{n=1}^{\infty}\sum_{m=0}^{\infty} f_{n,m}\lambda^{nj}(-1)^{jm} S^{n-1}U^m \,, \tag{E.1.7}$$

and $f_{n,m}$ is given by the impressive expression

$$f_{n,m} = \frac{\Gamma\left(\frac{2n}{d}\right)\Gamma\left(1-\frac{n(k_1-1)}{d}\right)e^{-2i\pi\frac{n}{d}}(-d)^{n-1}e^{i\frac{\pi m}{2}}\Gamma\left(\frac{m+2\frac{n}{d}}{2}\right)2^m}{\Gamma(n)\Gamma\left(1-\frac{n}{d}(k_1-1)\right)\Gamma\left(1-\frac{k_2 n}{d}\right)\Gamma\left(1-\frac{k_3 n}{d}\right)\Gamma\left(1-\frac{k_4 n}{d}\right)m!\Gamma\left(1-\frac{m+2\frac{n}{d}k_1}{2}\right)} \,. \tag{E.1.8}$$

E.2 The Kähler potential

Before we can compute the Kähler potential for S and U via (7.2.1), we must perform a basis change from the Picard-Fuchs basis to the symplectic basis (7.2.27). For simplicity, let us define the skew-symmetric matrix Λ by

$$\Lambda = m_{\mathrm{PF}}^T \cdot \Sigma \cdot m_{\mathrm{PF}} \,, \tag{E.2.1}$$

where m_{PF} encodes the basis change and depends on the geometric details of the specific manifold. Then, to find K_{cs} we must compute

$$-\log\left(-i\Pi^\dagger\Sigma\Pi\right) = -\log\left(-i\varpi^\dagger\Lambda\varpi\right) \,. \tag{E.2.2}$$

Using (E.1.7) we find

$$\begin{aligned}\varpi^\dagger\Lambda\varpi = &\left(\bar{f}_{n,m}(-1)^{jm}\bar{U}^{mj}\right)\left(f_{n,l}(-1)^{lk}U^l\right)\bar{\lambda}^{nj}\lambda^{nk}(\Lambda)_{jk}|S|^{2(n-1)} \\ &+\left(\bar{f}_{\tilde{n}_1,m}(-1)^{jm}\bar{U}^{mj}\right)\left(f_{\tilde{n}_2,l}(-1)^{lk}U^l\right)\bar{\lambda}^{\tilde{n}_1 j}\lambda^{\tilde{n}_2 k}(\Lambda)_{jk}\bar{S}^{\tilde{n}_1-1}S^{\tilde{n}_2-1} \,,\end{aligned} \tag{E.2.3}$$

where $\tilde{n}_1, \tilde{n}_1 \in \mathbb{N}\setminus\{0\}$ and the last line holds if $\tilde{n}_1 \neq \tilde{n}_2$. Note that m and l are summed from zero to infinity for each n. For all two-parameter manifolds where m_{PF} is known one can verify that

$$\bar{\lambda}^{\tilde{n}_1 j}\lambda^{\tilde{n}_2 k}(\Lambda)_{jk} = 0 \,, \tag{E.2.4}$$

and that $\bar{\lambda}^{nj}\lambda^{nk}\left(\Lambda\right)_{jk}$ is purely imaginary. This is related to the properties of the monodromy matrices of the Landau-Ginzburg point in the known manifolds. Whenever this is satisfied, the Kähler potential is a function of $|S|^2$. This means that the shift symmetry for S is completely broken in the vicinity of the Landau-Ginzburg point. Finally, making contact with (7.2.29), we find

$$\alpha\left(U,\bar{U}\right) = \left(\bar{f}_{1,m}\left(-1\right)^{mj}\bar{U}^{mj}\right)\left(f_{1,l}\left(-1\right)^{lk}U^{l}\right)\bar{\lambda}^{j}\lambda^{k}\left(\Lambda\right)_{jk}\,, \tag{E.2.5}$$

$$\beta\left(U,\bar{U}\right) = \left(\bar{f}_{2,m}\left(-1\right)^{mj}\bar{U}^{mj}\right)\left(f_{2,l}\left(-1\right)^{lk}U^{l}\right)\bar{\lambda}^{2j}\lambda^{2k}\left(\Lambda\right)_{jk}\,, \tag{E.2.6}$$

$$\gamma\left(U,\bar{U}\right) = \left(\bar{f}_{3,m}\left(-1\right)^{mj}\bar{U}^{mj}\right)\left(f_{3,l}\left(-1\right)^{lk}U^{l}\right)\bar{\lambda}^{3j}\lambda^{3k}\left(\Lambda\right)_{jk}\,. \tag{E.2.7}$$

Therefore, in all cases where (E.2.4) is satisfied we find

$$K_{\text{cs}} = -\log\left[-i\sum_{n=1}^{\infty}\left(\bar{f}_{n,m}\left(-1\right)^{mj}\bar{U}^{mj}\right)\left(f_{n,l}\left(-1\right)^{lk}U^{l}\right)\bar{\lambda}^{nj}\lambda^{nk}\left(\Lambda\right)_{jk}|S|^{2(n-1)}\right]\,, \tag{E.2.8}$$

close to the Landau-Ginzburg point. Note that, again, m and l are summed from zero to ∞ and j and k from zero to five for each n.

E.3 The superpotential

Let us now shift our attention to the superpotential for the variables S and U. The relevant superpotential we have in mind for our D-brane inflation model has two contributions. First, the bilinear superpotential introduced in Section 3,

$$W_{D7} = a\tilde{z}\Phi\,, \tag{E.3.1}$$

where, since we consider the dual type IIB theory of the original D6-brane inflation model, $\tilde{z}$ is a complex structure modulus and the term is sourced by a suitable D7-brane instead of a D6-brane. Second, there is the Gukov-Vafa-Witten flux superpotential [177]

$$W_{\text{GVW}} = \int\left(F_3 - \tau H_3\right)\wedge\Omega\,. \tag{E.3.2}$$

In this notation $\tilde{z}$ is a linear combination of complex structure moduli of the symplectic basis. This way $\langle\tilde{z}\rangle = 0$ is a possible vacuum without the cycle wrapped by the brane shrinking to zero volume. One of the results of [143] is that the open-string modulus Φ couples linearly to the entries of the periods in the symplectic basis. Under certain circumstances, this does not change in the mirror-dual type IIB description where Φ is associated with a D7-brane. Unluckily for us, it also means that Φ will almost certainly never couple linearly to S or U in the Picard-Fuchs basis, because of the non-linear relation (7.2.27).

So, how is $\tilde{z}$ related to S and U? As mentioned above, it is linearly related to the z^a. Let us assume the basis change from the z^a to the $\tilde{z}^b$ is determined by a matrix m_{lin}. Recall that the period vector, Π, is invariant under $Sp\left(6,\mathbb{Z}\right)$

transformations. So we define a transformation G which matrix representation is block diagonal

$$G = \begin{pmatrix} m_{\mathcal{G}} & 0 \\ 0 & m_{\mathrm{lin}} \end{pmatrix} , \tag{E.3.3}$$

where m_{lin} is a orthogonal 3×3 matrix which acts on the complex structure moduli in the symplectic basis and $m_{\mathcal{G}}$ acts on the derivatives of the prepotential. In order to preserve the Kahler potential under this transformation we impose that $m_{\mathcal{G}} = -m_{\mathrm{lin}}^T$.

Then, in terms of the Picard-Fuchs basis we have

$$\tilde{z}^b = (m_{\mathrm{lin}})^{ba} (m_{\mathrm{PF}})^{(a+3)j} (\varpi)^j = (A)^{bj} (\varpi)^j \ . \tag{E.3.4}$$

Let us focus on the bilinear superpotential first. Using the above we can write it as

$$W_{\mathrm{D7}} = a\Phi \, (A)^{1j} (\varpi)^j \ , \tag{E.3.5}$$

where we have identified $\tilde{z}^1 \equiv \tilde{z}$. Using (7.2.28) we can express the $\tilde{z}^b$ in terms of the Picard-Fuchs basis,

$$\tilde{z}^b = g_0^b(U) + g_1^b(U)\, S + g_2^b(U)\, S^2 + \cdots \ , \tag{E.3.6}$$

after expanding around $S = 0$. The functions g_i depend on U as follows,

$$g_{n-1}^b(U) = (m_{\mathrm{lin}})^{ba} (m_{\mathrm{PF}})^{(a+3)j} \left(f_{n,m} (-1)^{mj} U^m \right) \lambda^{nj} \ . \tag{E.3.7}$$

With this we can write W_{D7} in terms of S and U,

$$W_{\mathrm{D7}} = a\Phi \left((m_{\mathrm{lin}})^{1a} (m_{\mathrm{PF}})^{(a+3)j}\, 2 \cdot (2\pi i)^3 \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} f_{n,m} \lambda^{nj} (-1)^{jm} S^{n-1} U^m \right) . \tag{E.3.8}$$

Assuming, for now, that U is stabilized at a high scale, we can extract the leading-order dependence on S to be

$$W_{\mathrm{D7}} = a\Phi \left(g_0 + g_1 S + g_2 S^2 + \dots \right) \ , \tag{E.3.9}$$

where the g_i are now constant coefficients which depend on the vacuum expectation value of U.

Let us now turn to the flux superpotential (E.3.2). In terms of the flux and period vectors it reads

$$W_{\mathrm{GVW}} = (2\pi)^2 \alpha' \left(f^a - \tau h^a \right) \Pi^a \, , \tag{E.3.10}$$

where f^a and h^a are the entries of different vectors of quantized fluxes. Following the steps above as for W_{D7}, we find in the Picard-Fuchs basis

$$W_{\mathrm{GVW}} = (2\pi)^2 \alpha' \left(\hat{f}^b - \tau \hat{h}^b \right) (G)^{ba} (m_{\mathrm{PF}})^{aj}\, 2 \cdot (2\pi i)^3 \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} f_{n,m} \lambda^{nj} (-1)^{jm} S^{n-1} U^m \, . \tag{E.3.11}$$

In a more compact form this becomes

$$W_{\mathrm{GVW}} = (2\pi)^2 \alpha' \left(\hat{f}^b - \tau \hat{h}^b\right) \left(f_0^b\,(U) + f_1^b\,(U)\,S + f_2^b\,(U)\,S^2 + \ldots\right) . \tag{E.3.12}$$

Note that the sum over b is implicit in this expression and we have redefined the fluxes in the new basis. In analogy with the functions g_i we have defined

$$f_{n-1}^b\,(U) = (G)^{ba}\,(m_{\mathrm{PF}})^{aj} \left(f_{n,m}\,(-1)^{mj}\,U^m\right) \lambda^{nj} . \tag{E.3.13}$$

In total, using (E.3.9) and (E.3.12), we can write the effective superpotential as

$$W = a\left[\Phi + \left(\tilde{f}^3 - \tau \tilde{h}^3\right)\right]\left[g_0 + g_1 S + g_2 S^2 + \ldots\right] + W_{\mathrm{mod}}(\tau, U) , \tag{E.3.14}$$

for an appropriate choice of fluxes. Here, $\tilde{f}^b = \frac{(2\pi)^2\alpha'}{a}\hat{f}^b$ and $\tilde{h}^b = \frac{(2\pi)^2\alpha'}{a}\hat{h}^b$. This coincides with (7.2.32) studied in the supergravity analysis of Section 7.2.3. Note that we have collected the parts of W_{GVW} that do not depend on S in W_{mod}. In most of the suitable compactifications that have been studied in the literature, the fluxes F_3 and H_3 offer enough freedom to stabilize both τ and U supersymmetrically.

F

Transplanckian field range

F.1 Analytic approximation

In this appendix we will show an analytic approximation to the results observed in Section 8.3.2. In that section due to the large amount of scalar fields it was impossible for us to obtain an analytic expression for the backreaction of the complex structure sector. In order to obtain some analytic insight of the computations done, we will oversimplify the system considered in Chapter 8. As a remark, the computations done in this section are done by illustrative means.

First of all, one could consider U^1, U^2 integrated out since these moduli does not arise in the kinetic term of the inflaton (8.3.14). Afterwards we will consider U^3 and S on equal-footing. The second approximation that one may argue is to consider only leading order volume corrections. This assumption automatically will imply that the vev of the volume form $\mathcal{V}$ should be large in order to trust the following analytic approximation. With these assumptions we consider the following supergravity lagrangian

$$K = -3\log\left(T+\bar{T}\right) - \log\left(\left(S+\bar{S}\right)^2 - \frac{1}{2}\left(\Phi+\bar{\Phi}\right)^2\right) + X\bar{X}\,. \quad \text{(F.1.1)}$$

$$W = W_{\text{flux}}\left(S\right) + Ae^{-aT} + \mu\Phi^2 + \Delta X\,, \quad \text{(F.1.2)}$$

where we have considered, by means of simplicity, a F-term uplifting through a nilpotent goldstino. As we have done before, one could minimize the scalar potential and obtaining a Minkowski or de Sitter vacuum where the vevs of the closed string sector could be fixed to $S = s_0 + i0$ and $T = t_0 + i0$. Next, we compute the mas of the canonically normalized fields in the vacuum obtaining

$$m_S^2 = \frac{2s_0^2 W''_{\text{flux}}(s_0)^2}{16t_0^3}, \tag{F.1.3}$$

$$m_T^2 = \frac{a^2 W_{\text{flux}}(s_0)^2}{8s_0^2 t_0}, \tag{F.1.4}$$

$$m_{\text{Re}\Phi}^2 = \frac{16\mu^2 s_0^2 + 3\mu W_{\text{flux}}(s_0)}{8t_0^3}, \tag{F.1.5}$$

$$m_{\text{Im}\Phi}^2 = \frac{8\mu^2 s_0^2 + 2\mu W_{\text{flux}}(s_0) + W_{\text{flux}}(s_0)^2}{4t_0^3}, \tag{F.1.6}$$

$$\tag{F.1.7}$$

where W' denotes $\partial_S W$ and W is evaluated at the minimum found. We see that naturally the mass of the saxionic partner fo the inflaton is bigger that the inflaton because of a renmant that arises because of the uplifting, this gives a soft mass that makes it to go as $W_{\text{flux}}(s_0)^2$. In our case the mass of the axionic component is around the Kahler moduli scale. In order to obtain this mass terms, apart from the simplification of taking leading order in the expansion $\mathcal{V}$ we have considered the following.

Since in in KKLT, naturally, W_0 has to be small in order to obtain a large volume looking at the F-term of the complex structure we see that

$$D_S W + K_S W = 0 = W'_{\text{flux}}(s_0) + \frac{W_{\text{flux}}(s_0)}{2s_0}. \tag{F.1.8}$$

Naturally, in a toroidal orientifold the dilaton is stabilized at order 1. So we assume that $1 > g_s > 0.1$. Since $W_{\text{flux}}(s_0) \ll 1$ because of KKLT, naturally we see that $W'_{\text{flux}}(s_0) \ll 1$. So the main assumption regarding complex structure moduli is that $W''_{\text{flux}}(s_0) \gg W'_{\text{flux}}(s_0), W_{\text{flux}}(s_0)$. With that assumption we achieve the former mass terms. Note that the effect of the gravitino mass who separates the mass of the axionic and saxionic components is subleading in volume, that explains that the masses of both real fields are the same.

With these assumptions at hand one could compute the backreaction of the surviving closed-string sector. Perturbing the scalar potential around the minimum found, up to quadratic order, one could obtain, at leading order in $\frac{1}{\mathcal{V}}$ and $\frac{1}{W''_{\text{flux}}}$

$$\delta T = \frac{2\mu s_0^2}{a W_{\text{flux}}(s_0)^2}\varphi^2, \tag{F.1.9}$$

$$\delta S = -\frac{2\mu s_0}{W''_{\text{flux}}(s_0)^2}\varphi^2. \tag{F.1.10}$$

We explicitly see that the backreaction of the complex structure sector is strongly dominated by the mass hierarchy between the inflaton and the complex structure sector. Paying attention to the mass of the complex structure, we see that

W'' has to be big enough in order to se this scale above the Hubble scale. But comparing the complex structure moduli mass scale and the inflaton one could see that both have the same powers of $\mathcal{V}$. I we compare the numerators of both, we see that $W_0'' \sim N \gg W_0, \mu \sim 10^{-4}$ Where one could consider W_0'' as an order 1 coefficient which depends on the flux quanta.

With the former results one could compute explicitly the backreacted scalar potential. However, in this appendix we will focus on the interplay between backreaction and the kinetic term of the inflaton. Plugging (F.1.10) into (8.3.14) one could find

$$K^{\Phi\bar{\Phi}}|_{\text{inf}} = 2\left(s_0^2 - 2\frac{2\mu s_0^2}{W''_{\text{flux}}(s_0)^2}\varphi^2\right) . \tag{F.1.11}$$

The first result that one can see is that taking $\frac{H}{m_{\text{mod}}} \to 0$ (which is the same as $W'' \to \infty$) one could recover the results that one could obtain using the shorcut shown in Section 6.2. Where in this limit the leading order backreaction effect is the same as freezing the closed string moduli at its vev in the Kähler potential and superpotential. Obviously, this naive approach is unrealistic since the complex structure scale will be above the KK scale. Computing the field range for a finite mass scale one obtains

$$\begin{aligned}
\Delta\varphi &= \int\sqrt{\frac{1}{4\text{Re}\,(S)\,\text{Re}\,(S)}}d\phi && \text{(F.1.12)}\\
&= \frac{1}{2}\frac{1}{\sqrt{K_0^{\Phi\bar{\Phi}}}}\int\sqrt{\frac{1}{1-\frac{2f}{W''_{\text{flux}}(s_0)^2}\phi^2}} && \text{(F.1.13)}\\
&= \frac{1}{2^{\frac{3}{4}}}\frac{1}{\sqrt{K_0^{\Phi\bar{\Phi}}}}\frac{\arcsin\left(\sqrt{2\frac{\mu}{W''_{\text{flux}}(s_0)}}\phi\right)}{\sqrt{\frac{f}{W''_{\text{flux}}(s_0)}}} && \text{(F.1.14)}\\
&= \frac{1}{2^{\frac{3}{4}}}\frac{1}{\sqrt{K_0^{\Phi\bar{\Phi}}}}\frac{\log\left(\sqrt{2\frac{\mu}{W''_{\text{flux}}(s_0)}}\phi+\sqrt{2\frac{\mu}{W''_{\text{flux}}(s_0)}\phi^2+1}\right)}{\sqrt{\frac{\mu}{W''_{\text{flux}}(s_0)}}}\,, && \text{(F.1.15)}
\end{aligned}$$

where $K^{\Phi\bar{\Phi}}$ is (F.1.11) evaluated at $\phi = 0$. One could see that the logarithmic behavior appears modulated by the ratio $\frac{\mu}{W''_{\text{flux}(s_0)}}$. One could see that if one is able to assure a sufficient mass hierarchy between the closed-string sector and the inflaton the logarithmic dependence could be avoided during inflation but, as stressed along the text, the microscopical origin of these tunable μ-term is beyond the scope of the text.

Using the example For completitude we will show a similar computation based on the supergravity setup analyzed in Chapter 8. In order to analyze this behavior one could cancel the F-terms for the complex structure moduli during inflation explicitly. For simplicity, again, we set the saxionic component of the inflaton superfield at the origin. Then, imposing the following relations between background fluxes

$$m_3 = 0 \ , \ \hat{m}_1 = \hat{m}_2 = \hat{m}_3 = 0 \ , \ \hat{n}_3 = 0 \ , \ n_1 = n_2 = n_3 = 0 \ , \tag{F.1.16}$$

$$m_0, m_1, \hat{m}_0, \hat{n}_0, \hat{n}_2 < 0 \ , \ \hat{n}_1, n_0, m_2 > 0 \, , \tag{F.1.17}$$

one could cancel explicitly the F-terms of the complex structure moduli during inflation to the following vevs

$$S \ = \ 0 + i\sqrt{\left|\frac{m_0}{\hat{m}_0 \hat{n}_0}\right|}\sqrt{n_0 + \mu\phi^2} \, , \tag{F.1.18}$$

$$U^3 \ = \ 0 + i\sqrt{\left|\frac{\hat{n}_1 \hat{n}_2 m_0}{m_1 m_2 \hat{m}_0 \hat{n}_0}\right|}\sqrt{n_0 + \mu\phi^2} \, , \tag{F.1.19}$$

$$U^1 \ = \ 0 + i\sqrt{\left|\frac{m_2 \hat{n}_0}{m_0 \hat{n}_2}\right|} \, , \tag{F.1.20}$$

$$U^2 \ = \ 0 + i\sqrt{\left|\frac{m_2 \hat{n}_0}{m_0 \hat{n}_2}\right|} \, . \tag{F.1.21}$$

Plugging these results into (8.3.14) one could arrive to

$$K^{\Phi\bar{\Phi}} = 4\frac{|m_0 n_0|\sqrt{|\hat{n}_1 \hat{n}_1|}}{|\hat{m}_0 \hat{n}_0|\sqrt{|m_1 m_2|}}\left(1 + \frac{\mu}{n_0}\phi^2\right) \, . \tag{F.1.22}$$

During the inflationary trajectory, setting $\mathrm{Im}\,(\Phi) = 0$ one could see that the field range is given by

$$\Delta\varphi \ = \ \int \sqrt{\frac{1}{4\mathrm{Im}\,(S)\,\mathrm{Im}\,(U)}} d\phi \tag{F.1.23}$$

$$= \ \frac{1}{2}\sqrt{\frac{|\hat{m}_0 \hat{n}_0|\sqrt{|m_1 m_2|}}{|m_0 n_0|\sqrt{|\hat{n}_1 \hat{n}_1|}}}\int \sqrt{\frac{1}{1 + \frac{\mu}{n_0}\phi^2}} \tag{F.1.24}$$

$$= \ \frac{1}{2}\sqrt{\frac{|\hat{m}_0 \hat{n}_0|\sqrt{|m_1 m_2|}}{|m_0 n_0|\sqrt{|\hat{n}_1 \hat{n}_1|}}}\frac{\arcsin\left(\sqrt{\frac{\mu}{n_0}}\phi\right)}{\frac{\mu}{n_0}} \tag{F.1.25}$$

$$= \ \frac{1}{2}\sqrt{\frac{|\hat{m}_0 \hat{n}_0|\sqrt{|m_1 m_2|}}{|m_0 n_0|\sqrt{|\hat{n}_1 \hat{n}_1|}}}\frac{\log\left(\sqrt{\frac{\mu}{n_0}}\phi + \sqrt{\frac{\mu}{n_0}\phi^2 + 1}\right)}{\frac{\mu}{n_0}} \, . \tag{F.1.26}$$

We see that, again, the logarithmic behavior is controlled by the μ-term over an order one flux. Thus, again, we see that the logarithmic behavior could be delayed by tuning this coefficient sufficiently small. In the following figure we show the field range for different values of μ.

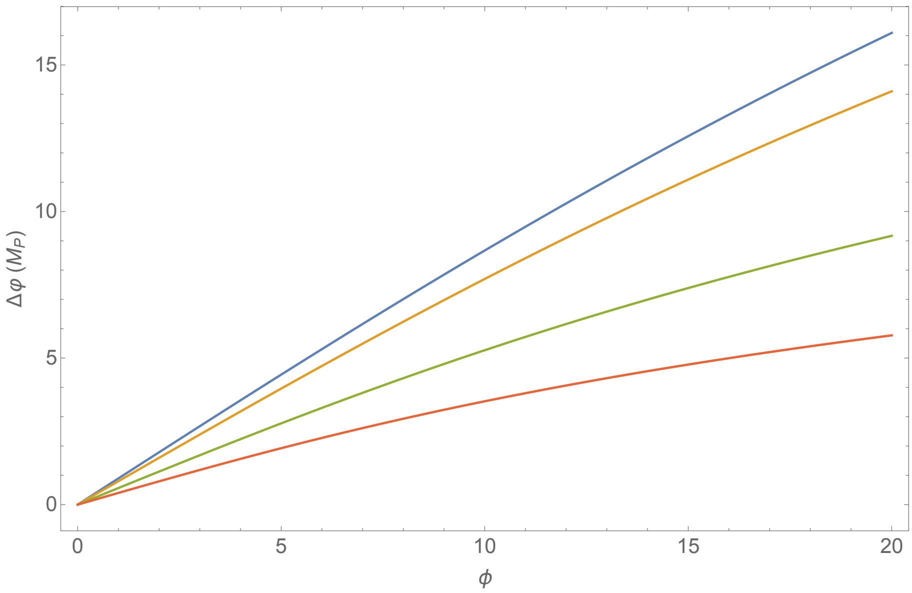


Figure F.1: General field range for the axion, for (from up to down) $\frac{\mu}{n_0} = \left\{\frac{1}{500}, \frac{1}{400}, \frac{1}{200}, \frac{1}{100}\right\}$ for $\frac{1}{2}\sqrt{\frac{|\hat{m}_0\hat{n}_0|\sqrt{|m_1 m_2|}}{|m_0 n_0|\sqrt{|\hat{n}_1\hat{n}_1|}}} = 0.04$

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